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Editors:
Douglas E. McDougall
John A. Ross

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USING DESIGN RESEARCH TO SUPPORT THE LEARNING OF A PROFESSIONAL TEACHING COMMUNITY OF MIDDLE-SCHOOL MATHEMATICS TEACHERS

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In this paper, we document the iterative cycles of design research conducted to support the learning of a group of middle school mathematics teachers. The goal of the research team was to support the teachers' ability to reason statistically about data. In this process, understanding the teachers' instructional reality became crucial for conducting productive ongoing collaborations.

Introduction

Our purpose in this paper is to document the process of supporting mathematics teacher learning through iterative cycles of design research (cf. Brown, 1992; Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). Data is taken from our ongoing collaboration with a group of middle school mathematics teachers in a large urban district in the southeast United States. This school district serves a 60% minority student population and is located in a state with a high-stakes accountability program. In the third year of our collaboration, the group consisted of 12 teachers in total, 6 of whom had participated from the beginning of the collaboration and the rest joining the group at the beginning of year three. Our long-term goal in working with the teachers is to support their development of instructional practices that place students' reasoning at the center of their instructional decision making. To this end, we have engaged the teachers in activities from a statistical data analysis instructional sequence that was designed, tested, and revised during prior NSF funded classroom design experiments conducted with middle grades students (Cobb, 1999; McClain & Cobb, 2001). During the three years of our collaboration, we have conducted monthly work sessions and extended summer sessions focused on instructional practices that place students' diverse ways of reasoning in problem situations at the forefront of instructional planning. Our analysis will document how the iterative process of testing and revising conjectures about ways to support the teachers' development informed our interactions with the teachers and contributed to a better understanding of what we describe as their *instructional reality*.

Methodology

The general methodology falls under the heading of a design experiment (Brown, 1992; Cobb, Confrey et al., 2003). Following from Brown's characterization of design research, our collaboration with the teachers involved *engineering* the process of supporting teacher change. This involved iterative cycles of design and research where conjectures about the learning route of the teachers and the means of supporting it were continually tested and revised in the course of ongoing interactions. In this highly interventionist activity, decisions about how to proceed are constantly being analyzed against the current activity of the teachers.

Our design was guided by the formulation of an initial conjectured learning trajectory (cf. Simon, 1995) for supporting the learning of the PTC. This trajectory, based on the research literature and the results of prior classroom design experiments, encompassed starting points, envisioned overarching goals, and conjectured ways of proceeding towards these goals via means of support. The initial learning trajectory was subject to continual revisions made against the background of the researchers' conjectures and with the added information from ongoing analyses.

Orientation to Professional Development

The overarching goal of our collaboration with the teachers was to orient them towards their students' reasoning and place this at the center of their pedagogical decision-making. To this end, we engaged with the teachers in activities such as planning and orchestrating whole class conversations in the context of statistics. We aimed to support the teachers not only in focusing on their students' reasoning but, more importantly, in capitalizing on the diversity of ways of reasoning by using it as a primary resource in building their instructional agenda.

Aligned with Ball and Cohen's (1999) image of professional development, part of our collaboration with the teachers was devoted to producing classroom-related materials and teacher experiences "immediate enough to be compelling and vivid" (p.12) that would later serve as resources for discussions within the PTC. To this end, the teachers typically (1) engaged in the selected task from the statistics sequence as problem-solvers during our work session, (2) taught this task with their classes, and (3) brought their students' written work to the following work session. For each session, a pair of teachers would co-teach and videotape their statistics lesson, which then became subject to "careful scrutiny, unpacking, reconstruction" (p.12) during the next session.

Each classroom statistics activity was typically comprised of (1) a whole-class discussion in which the teacher and students talked through the data creation process (Cobb & Tzou, submitted), (2) individual or small-group activity in which the students worked to analyze data, and (3) a whole-class discussion of the students' data analyses (Cobb, McClain, & Gravemeijer, 2003; McGatha, Cobb, & McClain, 1999, April). In the third year of our collaboration with the teachers, we focused specifically on the data creation process and subsequent data analysis discussion.

Data Creation Conversations and Data Analysis Discussions

From the researchers' perspective, the data creation conversation with students "involved discussing the particular problem or question to be investigated, clarifying its social or scientific significance, delineating aspects of the situation that might be relevant to the question at hand, and developing procedures for measuring them. The data the students are to analyze are then introduced as having been generated in this way" (Cobb, McClain, & Gravemeijer, 2003, p.8). From the researchers' perspective, the purpose for organizing the ensuing data analysis discussion as a whole-class event was to provide the teachers with means of capitalizing on the diversity in the students' reasoning about data by identifying analyses that, when discussed directly or compared with other analyses, might lead to substantive mathematical conversations that would advance the teacher's pedagogical agenda (Cobb, McClain, & Gravemeijer, 2003).

Our initial conjecture was that by examining these two aspects of instruction the teachers would come to see in their current practice as problematic. In particular, they would focus on what was entailed in carefully orchestrating a deliberately facilitated whole-class discussion where students' diverse ways of reasoning are taken as resources for supporting learning. Consequently, this would make it possible to create situations where students' reasoning could be a focus of discussion within the PTC. Based on our understanding of the significance of these two aspects of instruction we also conjectured that the teachers themselves would see an initial focus on data creation conversation and data analysis discussion as relevant to their instruction and therefore worth pursuing.

However, analysis of the work session data indicated that our discussions of these two aspects of instruction with the teachers yielded different learning experiences for them. The teachers were engaged during the work sessions activities focused on conducting data creation conversations; they also became increasingly sophisticated in conducting these conversations with their students--both indicated to us that these activities were valued by the teacher and viewed as relevant to their classroom instruction.

In contrast, our conversations about how to conduct whole-class discussions of data analysis remained, in our view, unproductive for the teachers. The level of teachers' engagement during these discussions suggested that most of them did not consider conversations of this kind relevant and pragmatically beneficial for their classroom instruction.

Will: Does anybody in here have their kids up, the entire class up at front of classroom [to present their solutions to their classmates] once a week?
Rita: No. Don't have time.
Lily: No. My kids can't handle it.
Rachel: Having them every two weeks or so.
Will: I find it to be very effective discipline tool.
Teachers: (chuckles)
Will: I really do. Because they'll respect each other more than they'll respect me.
Lily: I guess not.
Cathy: Not with 7th graders.
Rita: Not with 7th graders.

The teachers' image of a whole-class discussion at that time involved a series of student presentations where little or no attempt was made to capitalize on students' current understandings. Instead, the teachers saw value in these presentations in terms of building students' social skills and self-esteem.

Our conversation with the teachers about the data analysis discussion also involved analyzing students' work collected from the teachers' classrooms. Although many teachers became increasingly proficient in categorizing students' solutions according to their levels of sophistication, they seemed to view the purpose of this activity to be an assessment of their students' reasoning rather than an opportunity to build on it in their instructional planning.

The research team repeatedly struggled to capture the nature of the differences in teachers' experiences against the operative research conjectures. It became clear to us that beyond understanding *how* the teachers valued different activities, it was *why* they valued these activities within their perspectives that we needed to understand better.

Reflection

Teachers' lack of interest in conversations about data analysis was unexpected and led us to question the validity of the initial conjectures that guided our design of the work sessions. We came to the realization that our relative ineffectiveness in working with the teachers was a result of our lack of critical understanding of the teachers' *instructional reality.* We use the notion of instructional reality in an attempt to capture the intertwined system involving teachers' instructional practices and perspectives from an observer's point of view (Simon & Tzur, 1999) together with teachers' experiences of those as they are trying to accomplish certain instructional goals within institutional setting in which they work. Such experiences highlight the immediate challenges that teachers encounter, the frustrations they go through and the valuations they hold towards specific aspects of their instructional reality. This notion helped us to seek interpretations of teachers' instructional practices and perspectives by situating them in the boarder institutional context in which they work.

In order to generate data to inform our collaboration with the teachers and revisions to our conjectured learning trajectory for supporting the learning of the PTC, we conducted a series of modified teaching sets (Simon & Tzur, 1999) with all participating teachers. Our purpose was to use these teaching sets as contexts for gaining insights into teachers' instructional reality in order to identify problems and issues that the teachers would find relevant to their classroom instruction and that, at the same time, could provide potential means of supporting teachers' learning via which we could advance our research agenda.

A central principle that guided our analysis of the teaching sets was to assume that teachers' perspectives of teaching and learning and specific instructional practices they developed in their classrooms are always reasonable and coherent in the context of their instructional reality. Operating with this assumption enabled us to avoid the deficit view when examining the collected data and instead brought to the fore the necessity of generating a reasonable interpretation of the teachers' instructional reality against which their perspectives and practices can be understood.

Analysis of the modified teaching sets revealed that most teachers were unable to explicate the learning process of their students. For these teachers, there was a black box between teaching and learning. For example, one of the teachers' talked about the "aha" moments through which students came to understand mathematics:

> Cathy: I don't know [how "aha" moments happen]. I like it when [students] finally get it. I like it when they tell me that this is easy and that they wanted do more [similar problems]. … Sometimes it is a struggle. [In] lot of the units … they struggle in the first three lessons, maybe four, and by the time they get to the fifth they finally understand all the other ones. Sometimes it's hard to convince them that they're gonna have to struggle through the first ones… I wish I had that control [over "aha" moments]. I don't.

From the teachers' perspective, it was the students who should be responsible for their own learning and who should be held accountable to make the most out of the teacher's instruction. The fact that the same classroom instruction always resulted in different learning outcomes with different students—a phenomenon that the teachers dealt with on daily basis—served as a justification for the teachers to contribute learning outcomes to the personal qualities of individual students, for instance, to their intelligence.

As teachers attributed the primary agency of learning to their students, they also expressed a sense of limited control in the process of supporting students' learning of mathematics. The teachers' only perceived realm of influence over learning was in creating situations and experiences where it *could* happen. Mathematics instruction for these teachers seemed to involve two equally important and complementary aspects. The first aspect centered on making sure that students were provided with sufficient opportunities to engage in instructional activities as intended by the teacher. The common strategies employed by the teachers to achieve this goal usually encompassed utilizing different forms of presentations (e.g. different visuals or manipulatives), providing students with enough problems to practice or enough time to process the information, or breaking down the mathematics problems into small steps. Not only did we observe the teachers using such strategies during their classroom instruction, they were also explicit about deliberately using these strategies to promote students' learning:

> Rachelle: [When my students are experiencing difficulties, I] descale [the problem], break it down, and let them know "yes, there are rules and once you do enough of the problems you can generate your own rules or you will pick up on [these] rules." …

Interviewer: How could you tell if this kind of approach is helping the students or not?

Rachelle: Usually after they do enough problems, they [see] visually that "I don't have to go through this process, I know there are only gonna be two left." …it comes after they do enough problems with it. They have enough exposure.

The second important identified aspect of teachers' instructional practices was that of making sure the students would attend to such learning opportunities. The teachers' primary focus was therefore ensuring that the students engaged in the mathematical activities. This is not surprising when we consider that student engagement constituted an important criterion by which these teachers' instructional practices got assessed in their schools (cf. Cobb, McClain, Lamberg, & Dean, 2003). Students' paying attention was highly valued in all of the observed classrooms and for many teachers it was synonymous with learning. Students' failure in understanding the mathematics was typically accounted for in terms of their lack of focused attention or, sometimes, their unwillingness to concentrate on the mathematics. Not only did the teachers stress on the importance of paying attention to their students, they also arranged the instruction in a way to achieve this goal. For example, many teachers explained their preferences of having either whole-class conversations or small-group discussions in terms for gaining more control over students' engagement and therefore creating a better chance to help their students learn.

As teachers struggled regularly to work on these two aspects of their instruction, they valued students' willingness to cooperate and regarded it as not only a premise but also a precursor for learning. The normative strategies the teachers used to engage students followed mostly a sugar-coating approach (Cobb & Hodge, 2003, April) that involved getting students engaged in something they might enjoy (e.g., video games, internet, or free time) to buy their attention to mathematics that came later. These strategies were all aimed at evoking students' interest and engagement that were external to the mathematical activities. Nevertheless, many teachers expressed their concern for having limited control over students' willingness to engage and sought an explanation grounded in terms of students' motivation.

Ben: …it's not that the concept is that difficult. It's that they chose to tune out and then didn't hear the end, didn't care. …The biggest problem is motivational. And paying attention in class. …the differences between [various instructional] methods in terms of the outcomes of the kids' understanding are not big compared to the difference between a kid that's unmotivated and a kid who is motivated. The kid who is motivated is gonna get it no matter which way you teach it. And the kid that's unmotivated is *not* gonna get it no matter which way you teach it. … One thing, and it's very frustrating and difficult, it's incredible how hard it is to motivate the kids. All the standard motivators for a big chunk of my kids don't matter. I mean grades, disciplinary stuff.

The issue of students' motivation was brought to the foreground in that it constituted an explanation for students' engagement or lack there-of for these teachers, and more importantly, it became a useful index for them to predict whether their instruction could be effective or not with certain groups of students.

Rethinking Data Creation Conversations and Data Analysis Discussions

Based on this analysis, we formulated revised conjectures about the instructional reality in which the teachers operated. This allowed us to generate an account of the contrasting experiences that the teachers had with the data creation conversation and the data analysis discussion during our work sessions. We conjectured that the teachers valued the data creation conversation mainly because the scenarios of the statistics activities were usually appealing to the students and therefore, from the teachers' perspective, helped to evoke students' engagement in the instructional activities. The data creation conversation seemed to fit with teachers' expectation of an effective opening of a lesson and therefore, the teachers viewed discussing them as pragmatically valuable.

We could also better understand teachers' lack of interest in the work session activities that focused on conducting the data analysis discussion. The analysis of the teaching sets indicated that the teachers considered the diversity among students' reasoning as undesirable in their classrooms. Their normative ways of dealing with diversity in students' understanding involved individualized instruction, which the teachers could rarely afford because of the additional constraints it placed on their already limited instructional time. Attempting to gain understanding of student reasoning by analyzing students' work was therefore not the teachers' priority. The image that many teachers had about the whole-class data analysis discussions was that of students presenting the results of their learning, not an event that constituted valuable learning opportunities for their students.

Modified Conjectures

Based on our analysis of the teaching sets, we chose to center on the issue of motivation as a new starting point for our modified conjectures. First, issues of motivation were viewed by the teachers as highly problematic and intimately related to the students' engagement, an issue that many teachers were struggling with in their classroom instruction. Considering institutional context in which these teachers operated in addition to what we learned from the interviews, it seemed reasonable to expect that most teachers would find the issue of motivation a sensible and relevant topic of conversation. We also expected that some teachers would be interested in and eager to participate in such conversations given the prevalent concern that they had for engaging students in instructional activities.

Second, it provided leverage for us to challenge the normative notion among the teachers that motivation is inherent, determined mainly by societal or economical factors beyond the classroom. We viewed this notion of motivation restricting and problematic in that it deprived the teachers of opportunities to effectively teach "unmotivated" children and thus depriving these children of opportunities to learn. We conjectured that challenging this notion would create a perturbation for the teachers that could not be accounted for by their current understandings of motivation and its relationship to students' learning. This perturbation would therefore generate a need for an alternative perspective — one in which we would help teachers to focus on their students' experiences and in which students' diverse ways of reasoning statistically would come to the foreground. As a result, addressing the issue of motivation would make our original overarching agenda, that of developing instructional practices that place students' diverse ways of reasoning at the forefront of instructional planning, also meaningful to the teachers.

The modified starting point proved to be viable for our interactions with the teachers. It supported teachers' increasing engagement in discussions, during which they recognized alternative ways to account for students' motivation. The teachers realized that they could influence students' motivation and that different mathematical instruction would result in different levels of students' engagement in the mathematical activities. This shift in the teachers' perspective on students' motivation allowed us to highlight issues of supporting students' ability to reason statistically by placing *teaching and learning* in the forefront and issues of motivation in the realm of supports. Our analysis therefore explicates how a design research approach to professional development caused the research team to question the appropriateness of its conjectures and search for new starting points that would be personally relevant to the teachers. This process then made it possible for the research team to cultivate teachers' interest in ways that supported their students' learning.

This analysis also allows for reflection on the relation between teachers' beliefs and practice, bringing into question causality in that relationship. Instead of viewing teachers' beliefs or practices as inherent, we is to understand teachers' instructional reality. Not only did this orientation to analysis help us to discover reasonableness and coherence in the teachers' perspectives; it also enabled us to see how teachers' beliefs and practices become an inseparable system. Reconstructing teachers' instructional reality enhanced our ability to design for more accurate trajectory for learning of the PTC.

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BUILDING A MATHEMATICS EDUCATION COMMUNITY THAT FACILITATES TEACHER SHARING IN AN URBAN ELEMENTARY SCHOOL

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This paper describes how teachers, as members of a mathematics education community, interpret their professional development experiences designed to facilitate teacher sharing. Project SIPS (Support and Ideas for Planning and Sharing in Mathematics Education) is a school-based professional development project designed to help teachers improve the quality of their mathematics instruction by establishing a mathematics education community within their school. Analysis of data from focus group interviews revealed that SIPS enabled teachers to learn from and support each other and helped them to collectively think about mathematics teaching and learning. Through their interpretations we learned that developing a community creates several possibilities and tensions.

Theoretical Framework

Since the early 1990s, educational researchers have highlighted the importance of working with schools as organizations (Fullan, 1990), considering schools as a unit of change (Wideen, 1992). Clarke (1994) suggest that professional development opportunities should "involve groups of teachers rather than individuals from a number of schools, and enlist the support of the school and district administration, students, parents, and the broader community" (p. 39). In mathematics education, researchers have reiterated the importance of teachers working with colleagues within their school in the implementation and development of reform efforts (e.g., Campbell & White, 1997; Franke & Kazemi, 2001; Stein & Brown, 1997; Stein, Silver, & Smith, 1998). As Loucks-Horsley, Hewson, Love and Stiles (1998) explain, "effective professional development experience builds a learning community" (p.37). But, we know very little about how these communities operate and the ways in which teachers participate in them (Gutierrez, 2000).

Research on professional learning communities suggests these communities provide teachers with opportunities to communicate with colleagues and encourages teachers to continuously learn together and share. Paramount to these communities is teacher's engagement in professional conversations. Professional conversations are "discussions among those who share a complex task or profession in order to improve their understanding of and efficacy in what they do" (Britt, Irwin & Ritchie, 2001, p.31). Britt et. al. (2001) found for many teachers "professional conversations provided an important vehicle for the examination of their beliefs and practices" (p. 50). Teachers working together and sharing their mathematics teaching experiences through professional conversations are the bedrock of Project SIPS.

Methods

SIPS is a school-based professional development project designed to help elementary teachers improve the quality of their mathematics instruction by building a mathematics education community within their school. Building a community meant that school staff and two university-based mathematics educators met regularly during school hours to explore, discuss, and plan the direction of mathematics education in the school. With teachers' input, activities were designed to nurture professional conversations where teachers share and plan mathematics-teaching strategies for the school's diverse student body. While building a community, SIPS aimed to help teachers increase their mathematical content and pedagogical knowledge. SIPS is currently in its third year. However, this paper focuses on the activities during the first year that facilitated teacher sharing within professional conversations.

Adams Elementary (pseudonym) is a Prek- 5 urban school where 90% of the children qualify for free or reduced lunch. During 2001-2002 the school enrolled about 400 children: 57% African American, 29% Hispanic, and 14% White. During this year, SIPS worked with 27 teachers at the school: all 18 homeroom teachers and 9 Resource teachers.

Three assumptions guided our work: (1) A mathematics education community is anchored on the exchange of knowledge and ideas; (2) Teachers need opportunities to share their mathematics teaching experiences with colleagues, and (3) Teachers *want* to share and learn from their colleagues. Therefore, SIPS included several activities to facilitate teacher sharing and to help teachers collectively think about mathematics content and pedagogy. SIPS first began by establishing a *Mathematics Leadership Team (MLT)*. This team included a teacher from each grade level, the Special Education Teacher and the Gifted Education Teacher. In the Spring of 2001, the MLT conducted a mathematics needs assessment for every grade level to identify the topics teachers wanted to learn more about or needed help teaching. For example, fifth-grade teachers wanted to learn more about teaching fractions and decimals while second grade teachers needed help teaching place value. Teaching problem solving was mentioned as a need for all grade levels. Based on the needs assessment, the MLT met with the mathematics educators, the school administrators and the mathematics consultant to plan the professional development topics for the year.

In the Fall 2001, SIPS was inaugurated with a four-hour workshop for all the teachers, paraprofessionals, and school administrators at the school. The focus of this workshop was to introduce teachers to the ideas espoused in the *Principles and Standards for School Mathematics* (NCTM, 2000), to discuss children's mathematical learning, teaching mathematics via problem solving and teaching mathematics for understanding. Following this initial workshop, teachers met throughout the year in grade-specific worksessions and monthly faculty meetings.

Grade-specific worksessions were held during school hours at the school. Teachers worked in four grade groups: PreK-K, 1^{st} – 2^{nd} , 3^{rd} – 4^{th} , 5^{th} – /Gifted ed. Each grade group met for half a day every other month. Substitute teachers were hired to allow teachers to attend the worksessions. During the worksessions, mathematics educators presented the research on children's learning of specified mathematics topics and introduced activities and ideas for teaching the topic with a grade-level focus. These worksessions encouraged teachers to explore their knowledge of the topic, to share their classroom experiences and teaching strategies, and to collectively plan mathematics lessons. These lessons were to be taught and shared at the faculty meetings.

Monthly mathematics faculty meetings were held after school and attended by the entire school staff. These meetings enabled vertical integration across grades where teachers shared their classroom activities with colleagues and discussed the mathematics expectations for students across grades. The meetings also provided an opportunity for teachers to engage in mathematics problem solving. Teachers solved adult mathematics problems, shared their solutions and discussed how the problems and concepts could be adapted for their classrooms.

In addition to the aforementioned activities, a *SIPS web page* was also established to highlight mathematics topics, to post teachers' teaching ideas and student work, and to provide links for related math sites. All these activities were designed to create a mathematics education community within Adams Elementary. This paper describes how teachers, as members of the mathematics education community, interpret these professional development experiences.

Data Sources

SIPS' research is interested in unveiling teachers' perceptions about the development of the mathematics education community. Data were collected from videotapes of all monthly faculty meetings, teachers' written reflections after worksessions and faculty meetings, and the mathematics educator's field notes. Semi-structured interviews, conducted by an external evaluator at the end of the first year, provided another source of data. Interviews provided an opportunity for teachers to reflect on their experiences, freely voice their opinions, and make suggestions for changes in the project. The focus group approach allowed "participants to talk to one another, asking questions, exchanging anecdotes, and commenting on each others' experiences and points of view" (Kitzinger & Barbour, 1999, p.4). Interviews were conducted with groups of three or four teachers, organized by grade level (seven groups), and some resource teachers. They lasted approximately 45 minutes and were all audio-taped and transcribed.

The main data source for this report is participants' language during the interviews. Through content analysis of the interview transcripts, we searched for patterns in the teachers' discussion of SIPS and for recurring words and themes that expressed teachers' interpretations of and engagement with the project. We looked within interviews and across the seven interviews to identify issues that were important to many teachers, trying to represent an overall view of the teachers instead of particular aspects commented by one or two teachers only. We contrasted and augmented these findings with those reported by the external evaluator.

Results

Results from interview data revealed that teachers found the mathematics education community afforded them with opportunities to learn, share, and think about mathematics learning as a collective community. Through their interpretations we learned that the word sharing was used in a broad sense to represent a variety of opportunities.

Sharing To Learn

Teachers found sharing with colleagues allowed them to hear what their peers were doing in their classrooms and to learn new teaching approaches from each other. A kindergarten teacher spoke of the value of sharing in grade-specific worksessions while a fifth-grade teacher spoke of the cross-grade sharing during faculty meetings:

> There were some things that other teachers were doing that I had never seen before and I've taught for 20 years. You know, it was like- that's I think the biggest thing in teaching is that teachers need to share. And that [worksessions] gave us that time to sit down, and you know, say now "What are you doing in your class?"
>
> We never have a chance to sit down and watch someone else do anything. And you learn something. It's not just the University—we need to do it at Adams, not just within but also across grades. I was amazed to see the advanced geometry they're doing in kindergarten- that's what we're doing in 5th grade too!

For experienced teachers, sharing helped them revisit familiar ideas and teaching approaches:

> [S]ometimes you can come up with something that was just a little bit different from how you had done it in the past. And it was just to get an idea across to a student that you may

not have thought about. You know, two brains are better than one, you know, when bouncing ideas.

Sharing to learn was especially important to teachers that have witnessed a change in the school's demographics. As a kindergarten teacher explained:

> [H]earing what other people were doing lets you know what was going on and how their children were reacting and I think that is important because over the years at Adams our population has changed greatly, our approaches with teaching them have changed, and I think it is important that we share those experiences with others.

Sharing For Support

Both new and experienced teachers found the opportunity to share gave them support as they thought about and tried new teaching strategies in their classroom. Two teachers, a kindergarten and first grade teacher, expressed how the SIPS mathematics community gave them the needed support to teach:

> As a new teacher coming from a different country, it helped a lot to understand how to teach math here, how to help [the children], you know, and listen to my peers.
>
> As a new teacher in the schools in the United States, you gave me a level of comfort and awareness that I was on target, you know, on track, doing what I was supposed to be doing.

By consulting with other teachers and the mathematics educators, most teachers were more willing to try new approaches. For example, a fourth grade teacher spoke of her confidence attempting new ideas in her classroom because "everybody was trying"

> I just like the fact [that] in talking about it and sharing, I personally felt a lot more confident to do.. an idea or attempt to, not take it directly, but I'll get an idea and jump, you know. All of the work that we did hands-on was so physically engaging. One of the things that I did that I thought could help.. the kids remember,, the multiples of certain numbers…[W]e did goofy things like jumping jacks and stuff with counting the multiples instead of 1,2,3,4, 5. And just silly things like that, I would have possibly thought of before but wouldn't have felt daring enough to try it. But because I knew everybody was trying..

Sharing To Think About Mathematics Together

Many teachers spoke of their experiences as "learners of mathematics" and how they valued hearing about the different approaches their colleagues used to solve problems. As a second grade teacher stated:

> It is amazing how they would say work through this activity, and you would see how she would work through it opposed to how I would work through it and you come up with the same answer. It's just our approaches was *so* different but yet they were not.

As teachers collaborated on problem solving activities, they saw how their ways of thinking were similar to, or different from, their students' mathematical thinking. A second grade and a first grade teacher suggested:

> [Y]ou actually got to feel how the kids felt and even doing the projects on a larger adult scale we still came up with a lot of the same concepts in the same orders that our students do.
>
> One of the goals that really helped me this year was to think about math not necessarily as the teacher teaching the math, but also from the students' points[s] of view and their perspective and how they do take in math concepts and process them and the many different ways there are for students to do that.

The professional development activities also helped teachers to think about how they encouraged their students to share and think through mathematics problems. A third-grade teacher explained:

> I think we emphasize, even more, student sharing…Sharing solutions or ways to [problem solve].. I mean they were allowed to draw the picture on the board and they were allowed to solve it in any way and then they explained it to the class, how they did it.

Sharing Beyond The Worksessions

Teachers found that they shared their learning experiences with their students especially after a SIPS worksession.

> My students are always asking me what's your meeting about. So I have to tell them.
>
> I would come back and actually, you know, if there was something I thought was particularly interesting or cool, I would share it. And they just loved the fact that I was a student, something, that really got them.

Some teachers also made more of an effort to talk to colleagues about mathematics at different times of the day (e.g., at recess on the playground or after school in the hallway). As a second grade teacher explained:

> [S]he's right next door to me, and it's amazing, we don't see each other. I may hear her going up and down the steps, but you know now I make an effort; I try to go out and talk to people during different times when I see them on the playground.

Although teachers in this project expressed agreement with the premise that a mathematics community is anchored on the sharing of ideas, they also expressed concerns about how well the sharing was being fostered. As a kindergarten teacher suggested:

> I don't think we have done as much sharing on a routine basis as I would have hoped. One of the things we were supposed to do was to be able to visit in each other's classrooms. And we really didn't find ways to make that work and I think that would have been a valuable thing to do.

Teachers were often confused about when and what to share and did not feel that enough teachers shared to truly have everyone as active members in the community. A second grade and special education teacher explained:

> It's real hard at the end of the day, and it got kind of confusing in there too, as to who was supposed to be, what grade level was supposed to be sharing for that month and everything so people weren't prepared because they weren't sure if it was supposed to be fourth and fifth grade that was sharing this month or if it was kindergarten or pre-K. I think that once we got into the staff meetings a lot of people shared, but I think sometimes you don't want to share because of a large group. It's better to have smaller groups, you know, and then maybe that person wants to share.

Discussion and Conclusion

The findings presented in this paper highlight the possibilities and tensions that manifest when establishing a mathematics education community. With respect to possibilities, we believe teachers saw sharing ideas and strategies a worthwhile activity, where they learned from and supported each other and collectively thought about mathematics teaching and learning. Teachers valued both the grade-specific and mixed-grade sharing and the time that they were afforded to think and plan. By engaging in professional conversations, teachers learned what their colleagues expect across the grades and worked on ways to attend to these expectations. Teachers also began to examine the ways their students thought about mathematical concepts. We are also optimistic that teachers will continue to engage in professional conversations about mathematics and to share after the SIPS project. Thus, we concur with Britt et. al. (2001) that professional conversations provide an important vehicle for teachers to examine their beliefs and practices.

Our work also highlights several tensions when establishing a mathematics education community. In spite of our efforts to maintain an emphasis on student's mathematical knowledge, teachers' comments focused more on students' flaws than on their actual teaching. For example, teachers would share how students could not solve specific problems when we really wanted them to talk about what the answers said about students thinking and instruction. Similarly, teachers rarely spoke about teaching mathematics to poor students of color. Some teachers would imply that their students were different and that they had to teach differently without any elaboration. In both situations we opted not to probe teachers further because we placed more emphasis on establishing trusting relationships with the teachers and wanted teachers to feel comfortable sharing their thoughts.

Another tension that arose was that teacher sharing was usually "*by invitation only*." That is, teachers rarely initiated conversations but would share their ideas and "success stories" after we started talking about specific instances. Teachers were more comfortable sharing their strategies for solving problems than they were for talking about their teaching.

In closing, our research on a mathematics education community underscores the importance of providing opportunities and time for teachers to engage in professional conversations to share knowledge and ideas and how difficult it is to focus that sharing on mathematics teaching and learning. As university mathematics educators we valued teachers opinions and wanted them to have ownership of the community while trying to maintain the goal of improving student learning. Therefore, teachers usually determined the scope and direction of their sharing which often focused on other school issues. However, the realities of schools, especially schools that have specific challenges, have to be an integral part of the community. Therefore, teachers must have the freedom to express their concerns and suggestions.

As we continue to analyze the data from SIPS we are examining other aspects of teacher sharing and building trusting relations with teachers when establishing a mathematics education community. In particular, we are examining what specific mathematical ideas teachers share, how the sharing evolves over time, and how to facilitate teachers sharing their unique views on teaching mathematics to poor students of color.

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TEACHER LEARNING IN LESSON STUDY: DEVELOPING PROFESSIONAL CAPACITY AND RESOURCES

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This study examined teacher learning during two cases of lesson study. In contrast to conceptions of lesson study as instructional improvement that occurs mainly through refinement of lesson plans and requires teachers to have content knowledge and collaborative skills as a prerequisite to participation, the two cases suggest that teacher learning occurs during lesson study through the interactive development of resources and professional capacity. Specifically, teachers' content knowledge and other professional capacities grew over time, supported by and supporting changes in the resources (e.g., lesson plans) they created. An implication of the study is that teachers' content knowledge and curricular materials should be considered both contributors to lesson study as well as outcomes of it.

Lesson study is a form of teacher professional development that originated in Japan (Lewis, 2002a, b; Lewis and Tsuchida, 1998; Stigler and Hiebert, 1999; Yoshida, 1999) and has recently emerged in at least 32 states of the US (Lesson Study Listserv, 2004; Fernandez, et. al., 2002; Lewis, 2002; Stepanek, 2001). Lesson study consists of a cycle of collaborative teacher activities: 1) considering goals for student learning and development, 2) studying existing instructional materials, 3) planning a lesson designed to make the goals visible in the classroom, 4) having one team member teach the lesson while others observe, 5) debriefing the lesson, and optionally, 6) revising the lesson for re-teaching. This cycle provides multiple opportunities for teachers to collaboratively consider their teaching, student learning, and the connections between them.

This study analyzes two cases of lesson study in order to identify pathways from lesson study to instructional improvement. We chose two cases that differ in characteristics of the lesson study (e.g., with or without guidance from an outside researcher, in one's own classroom or as a guest teacher in a collaborating school). The cases were analyzed for evidence of development of professional capacity (qualities of educators such as knowledge, motivation, and collaborative skill) and of resources for teaching (physical tools such as lesson plans, assessment tools, and instructional materials), as shown in Figure 1.

Figure 1, grounded in the literature on program theory in evaluation outlines the lesson study process in terms of intermediate and long-term outcomes. Program theory is useful because it enables users to clearly articulate the "theory of action" and may help identify why and how innovations such as lesson study work or fail to work (Rogers et. al., 2000). Lesson study activities ultimately improve classroom practice through intermediate outcomes such as increased professional capacity and development of resources for teaching, and these intermediate outcomes interact synergistically. The model laid out in Figure 1 contrasts with some existing ideas about lesson study that instructional improvement in lesson study occurs mainly through refinement of lesson plans, and that content knowledge and collaborative skill are *prerequisites* for lesson study (e.g., Columbia university lesson study listserv).

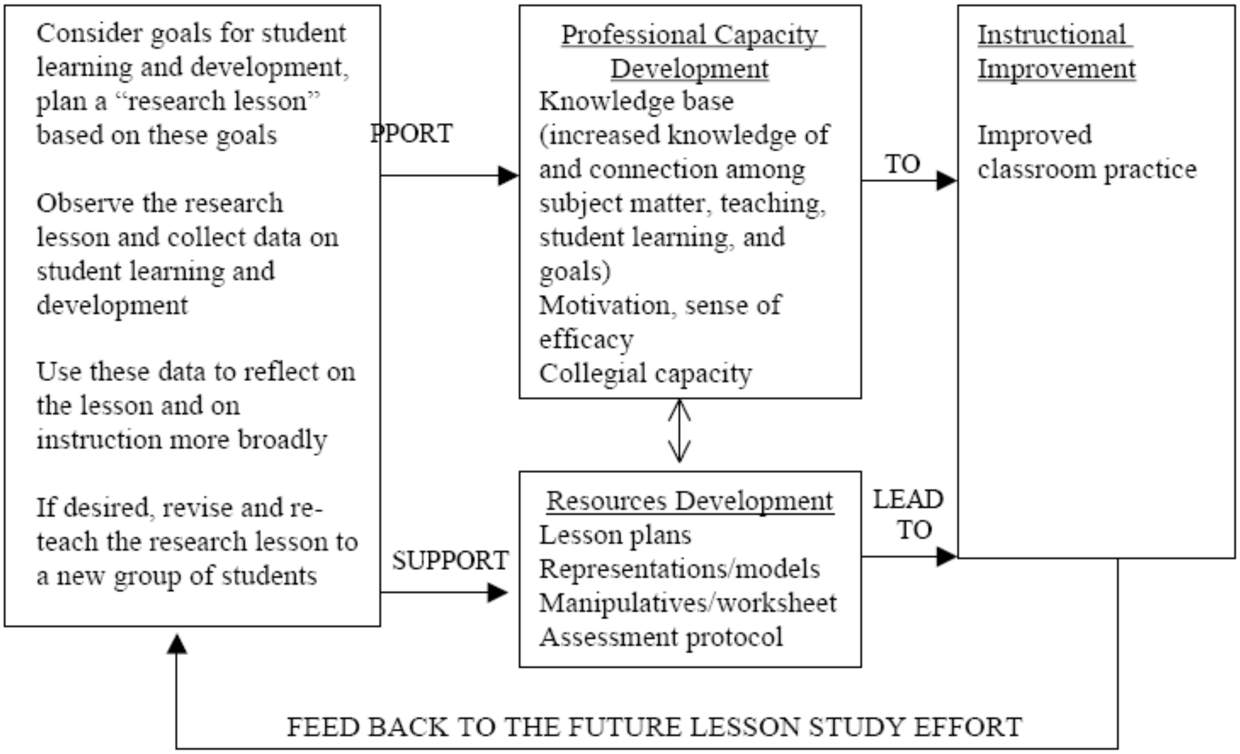

Figure 1: Teacher learning and lesson study activities and intended outcomes

Methods

This paper describes two cases from ongoing data collection in one K-8 school district in the Western United States. This district is at the forefront of the US lesson study effort, having engaged in lesson study since 2000. Lesson study participants in the district have different ways to be involved in lesson study, either by participating in lesson study groups during the school year or during two-week summer institutes organized to support teachers' development of content and lesson study knowledge. Data collected include videotapes of meetings and research lessons, fieldnotes, audiotapes of meetings and of teacher and student interviews, lesson plans (in multiple revisions), student work, written teacher reflections, and lesson study forms (e.g., lesson study agenda sheet). The cases were selected because the lesson study participants themselves, researchers within and outside our research project team, and outside educators who have viewed case artifacts (e.g., video, lesson plans, and excerpts from teachers' discussions) judged that teacher learning occurred that influenced or could be expected to influence instructional improvement. These judgments were obtained by presenting video excerpts (case 1) or a tabular summary of the evidence (case 2) to audiences of educators and educational researchers and soliciting written responses and ratings from audience members.

For case 1, teachers attended a summer workshop in algebra and lesson study where they worked together closely for a two-week period. The six teachers in the group under study came from different schools in the district but many knew each other from prior professional development occasions. The teachers in this lesson study case taught middle- to upper-elementary grades (Grades 3 to 6), and planned a Grade 4 algebra lesson together. During the summer institute, teachers solved and discussed algebra problems, studied mathematics standards and curricula, and planned, taught, revised, and re-taught a research lesson designed to help

Grade 4 students find and mathematically represent patterns. One team member taught each iteration of the research lesson, with the remaining team members closely observing and recording student activities. Over the two-week summer workshop, group members spent roughly 20 hours in the study of curriculum, planning, teaching, revising, re-teaching, and reflection on the research lesson.

For case 2, three Grade K teachers participated in a year-long lesson study with an outside researcher (the first author of this paper), within a school-wide lesson study effort. These teachers had worked at the same school for years and knew each other well. They planned and taught a Kindergarten lesson on number decomposition. They met once a month for the entire school year (9 months) and spent approximately 25 hours in the process. The research lesson was taught in Month 6, and all three teachers taught the lesson in their own classrooms while other members observed. The teachers reflected on and discussed their experience for the remainder of the year. Data were drawn from the entire year.

Results

Figure 2 presents details regarding the two cases of teacher learning from lesson study.

	Case 1: Triangle Table Algebra Lesson	Case 2: Alien Decomposition Lesson
Teachers	6 Teachers, Grades 3 to 6, From different schools in the same district	3 teachers, Grade K, Same school
Lesson	Grade 4	Grade K
Content	Algebra	Number Sense
Standards	Algebra and Functions, Grade 4: Understand that an equation such as y = 3x + 5 is a prescription for determining a second number when a first number is given.	Number Sense, Grade K: Students use concrete objects to determine the answer to addition and subtraction problems (for two numbers that are each less than 10).
Goals of the lesson	Students will: • Discover a pattern • Represent the pattern with numbers and symbols • Begin to understand what a mathematics rule is • Be introduced to the idea of representing a rule in an equation • Be curious about future explorations of patterns and rules	Students will increase their understanding of embedded nature of numbers and problem solving skills by engaging in open-ended problem situation
Lesson Study Context	2-week summer lesson study institute, worked together most days, for a total of about 20 hours	A year-long lesson study effort, met once a month for 9 months, for a total of 25 hours
Brief description of the student task in the lesson	When equilateral triangles are arranged in a long row with edges touching, what is the number of perimeter units for any given number of triangles? (Context: triangle tables with students seated at edges.) [Answers can be summarized with a y = x+2 equation, where y = perimeter units and x = number of triangles.]	Students find different number combinations to make 5 (e.g., 1 + 4, 2 + 3) by using red and blue crayons to color the legs of an Alien picture.
Activities that	Live observation of student learning during lesson; Requirement to re-teach 2 days later;	Students interviews; Discussion of teaching and learning trajectories (Japanese

supported	Observation of student counting methods by	curriculum, conception of quantities
teacher learning	one teacher; Discussion of two data sources that table "spoonfed" answer to students.	development); Observation of student learning during lesson; Discussion of data collected during the lessons to compare with existing teaching and learning trajectories.
Tools that supported teacher learning	Two data sources (provoked doubt that while data table was filled out correctly, students could not describe results in words or equation)	Student interview data, Levels of conceptions of quantities (research-based learning trajectory for young children's conception of numbers), Japanese teaching-learning trajectory for number sense.

Figure 2. Two cases of teacher learning in lesson study

Case 1: Triangle-table Algebra Lesson

As Figure 2 shows, case 1 teachers taught a research lesson to two different classes, in both of which students were asked to find the number of perimeter units for any given number of equilateral triangles arranged in a long row with edges touching. Teachers revised the lesson after the first teaching when they confronted puzzling student data: Few students could verbally describe the pattern relating number of triangles to number of perimeter units, despite the fact that all students filled out the worksheet tables (showing the plus-two finding) correctly. Specifically, all 22 students filled out the data table on their worksheet correctly, but only five noted the "plus two" pattern in response to a question asking them to note patterns in the problem, and few students raised hands when asked to describe the results in words. The teachers' observations during the lesson suggested that the table "spoonfed" the pattern to students. The table was organized with the number of tables in the left-hand column (increasing by 1 in each row), and students were to fill out the right-hand column with the corresponding numbers of seats. The organization of the table enabled students to fill in the table looking only at the vertical pattern in the right-hand column without referring to the left-hand column (5 students noted the "goes up by one" pattern) or to notice the plus-two pattern without understanding how it related to the problem. As the instructor noted after the lesson, "When I was trying to get them to say the number of [triangles] plus two equals the number of [perimeter units], there was a lot of confusion. It's easy for them to just go plus two, plus two, plus two, as they complete the worksheet, and they sort of lose the whole picture of what the plus two is representing." For the second research lesson, the group eliminated the data table and gave students strips of paper with various (non-sequential) numbers for tables. Students found the number of seats individually, shared data in small groups, and discussed and wrote about the pattern, resulting in a larger number of students who noticed and explained the plus-two pattern. In lesson two, all six groups' posters included descriptions of the plus two pattern ("There will be 2 more seats…than youre (sic) tables," "However many tables there are, there's 2 more seats.")

Case 2: Alien Decomposition Lesson

In planning their decomposition lesson (described in Fig. 2), the Grade K teachers initially struggled, as they believed the new state standard was inappropriate for their young students. Compared to what they had taught previously, the standard appeared to require a huge leap forward in student learning. In prior years, the standard required that students learn to manipulate numbers up to 5 using concrete objects and to count up to 20. For the new state standard, students were expected to solve addition and subtraction problem up to a total of 18, using concrete objects. Teachers were disinclined to teach the new standard, which they believed required them to teach the concept by drill and memorization. One teacher expressed her feeling by saying, “There’s no way our students can do this!”

One of the authors of this paper joined the lesson study group as a collaborating educator to support the teacher learning. At the beginning of the school year, in order to investigate the students’ current level of understanding to better formulate a lesson that would help them meet the goals outlined in the standard, the group decided to conduct an informal interview assessment. The collaborating researcher interviewed a representative sample of students from each classroom for simple decomposition and addition/subtraction problems to stimulate the teachers’ thinking about student problem-solving methods (Three interview tasks were chosen: find two numbers that make 5, 2 + 6, and 8 – 2). The sample of students was chosen to reflect high, middle, and low levels of understanding (as identified by teachers) so that a range of different methods could be observed. When asked to find two numbers that make 5, 50% of students could identify at least one pair. For the 2 + 6 (addition problem), 33% of students needed step-by-step guidance from the researcher to solve it by counting, 25% of students did so without guidance, and 8% recalled the answer without using an apparent method. Thirty three percent of students solved the problem incorrectly. For the 8 – 2 (subtraction problem), 42% of the students needed step-by-step guidance to solve by counting, 25% did so without guidance, and 8% recalled the answer. Fifteen percent of students solved the problem incorrectly. When the interview results were shared, the teachers were surprised to see how well their students did with the seemingly difficult tasks. Teachers examined and analyzed the results in relation to a research-based developmental trajectory for children’s conception of quantities provided by a collaborating researcher, and they saw how their students’ thinking fit in the universal trajectory, thus understanding how the standard aligned with their own teaching strategies. Furthermore, when a Japanese learning trajectory for number sense was shared, and when teachers saw how decomposition activities they were familiar with were used as an important foundational step for young students’ learning in Japan, this also helped them to see the standard differently. These materials helped the teachers view student learning on decomposition in relation to the long-term developmental process and thus helped teachers plan a lesson to assist their students in this area.

Teachers’ discussions after planning and teaching the lesson on decomposition revealed that their thinking about the standard and student learning had changed (Murata, 2003). One teacher commented, “What changed (in my understanding) is seeing new ways to discuss and teach this concept … despite what we thought, watching our children demonstrate their ability to do it is the key to our understanding. That made me change my mind about the standard.” A second teacher reported, “If we set this right (with developmentally appropriate activity), all can be successful.” Experiencing student learning and success in the classroom were critical in changing their perspectives.

Discussion

Cases 1 and 2 provide examples of how professional capacity and resources for teaching may increase over the course of lesson study, with each supporting the development of the other. For example, in Case 1, the improved lesson plan was just one of several intermediate outcomes of this case (See Figure 1). Others include teachers’ increase in mathematical knowledge (i.e., understanding of the plus-two pattern), the recognition that students’ methods of counting can reveal their mathematical thinking, a rethinking of what it means to “understand” a mathematical pattern, and a recognition that students learn something from organizing data themselves. Teachers’ final reflections included comments such as, “I learned that a worksheet can be a dangerous thing,” “I learned that students need to do the work, not the teacher,” and the following comment about mathematical knowledge and motivation: “… (A) personal aha for me. When you had said, Teacher J…in the first debriefing,

> that, we should really spend some time on having the students share [their counting], at first I thought, “who cares about that?” I did not see that as an important thing because I personally did not see the pattern that the ends [end triangles] are the plus two. I did not see that. So it just shows that in all this math, well, in everything we teach, we’re only as effective as our level of understanding. So we have to keep pushing ourselves to delve into…the why, the how come.”

Case 1 suggests an interactive relationship between resource and professional capacity development. After teachers adopted the teaching worksheet (resource development), they experienced through live observation of students how that structured worksheet seemed to limit student learning, since students could fill it out easily without being able to explain the meaning of the results. This conflict led to their further reflection on their lesson and teaching (development of professional capacity) and to the development of a second teaching resource that allowed students to represent their thinking and ideas differently (resource development). Next, seeing student learning with the new teaching resource enabled teachers to think further about student thinking and about the mathematics of the lesson (development of professional capacity). Thus, professional capacity and resource development each provided the base for further development of the other, in an interactive process, while teachers’ goals shifted from having students notice numerical patterns to having them be able to explain how the numerical pattern relates to the problem’s geometric characteristics. Research suggests that the expansion of mathematical proficiency to include conceptual understanding as well as procedural skills is of key importance (National Research Council, 2001).

For case 2, despite the teachers’ initial disagreement with the standard, their thinking changed when they saw students’ responses to the interview tasks in relation to the research-based levels of quantity conception and the Japanese learning trajectory of the mathematics content. They began to make connections between the standard and their classroom practice through the interview (development of professional capacity). And this ability to make instructional connections supported them to develop their lesson plan (resource development). Teaching and observing the lesson and experiencing student learning in relation to their shared lesson plan further supported teachers to think more deeply about student learning (development of professional capacity). For this case, the interactions between professional capacity development and resources development also clearly illustrate how the interactive cycle supported the improvement of classroom practice. Teachers changed their thinking that students could reach the learning goals outlined in the standard when they were able to design age-and developmentally-appropriate learning activities for their students. The complex mathematical goals presented in the standard may be achieved through a simple hands-on activity and not necessary paper-and-pencil practices in the classrooms. They realized that the activities with which they were already familiar (decomposition using concrete objects) were important and foundational learning step for future student learning.

Conclusions and Implications

As the two cases illustrate, the development of professional capacity and instructional resources interact and support each other through the process of lesson study. New teaching resources (e.g., the interview protocol, the revised lesson plan, the content trajectory) might have enabled teachers to increase their professional knowledge. Conversely, increased professional knowledge enabled them to use old tools (e.g., standards, decomposition activities) in more effective ways. Opportunities were provided to compare forms of data about student thinking (e.g., verbal/written responses, interview data), and such comparisons enabled them to simultaneously grow professionally and develop resources that reflect increased professional capacity. Although lesson study is sometimes described as a set of procedures for creating better lessons (resource development), the cases suggest it is better described as an interactive process of resource development and professional capacity development. Implications include (1) lesson plans are an inadequate measure of lesson study progress, and (2) content knowledge may be conceived as an outcome of, as well as a contributor to, lesson study, (3) resources such as curriculum materials should be considered inputs as well as outputs of lesson study.

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SCHISMS, BREAKS, AND ISLANDS - SEEKING BRIDGES OVER TROUBLED WATERS: A SUBJECTIVE VIEW FROM GERMANY

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The far-flung community of professionals dedicated to the practice or teaching of mathematics is cleft by at least three kinds of divisions: (a) the schism between mathematics and math education, (b) the barrier between school and research mathematics, (c) the rifts between mathematical subdisciplines. While latter are in some sense inherent and could at most be made less jagged, we shall argue that the former two could in principle be bridged, albeit slowly and gradually. The small space allowed will, however, often force us to be sketchy and use metaphors to compress meaning.

Cultures, Schisms, Breaks

Two Cultures

The "gulf of mutual incomprehension" separating the exact sciences and the humanities has neither narrowed nor been bridged in the half century since C.P. Snow deplored it in his famous lecture (Snow, 1959) on the Two Cultures (cf. the assessment by Davis, P., 1990). The same is true for the present situation in Germany. A recent bestseller (Schwanitz, 1999) dismissed mathematical culture in a brief sketch on the last pages, and thereby - fortunately - provoked another bestseller (Fischer, 2001) which deals with the broad cultural influence of mathematics and the exact sciences. We do not intend to add to the ink already spilled on this topic, except to recall that the fruitful metaphor of different cultures has been frequently applied to the various forms of mathematics instruction, starting at the latest with the work of Lerman (1990), and has demonstrated its explanatory value (cf. Cobb, 1991; or Cobb & Bauersfeld, 1994; Yackel & Cobb, 1996; and many others). Hence we shall allow ourselves to look through this filter at the makers, users and communicators of mathematics.

However, we shall also remember that an overly facile handling of this tool can lead to embarrassing misjudgments, which are particularly visible in the old dichotomy of "pure" versus "applied". They come from of three sources: distortion by contraction and caricature, ever-changing subject-boundaries (e.g., knot theory is now "applied"), and the classical dynamics of human behavior (tribalism and empire building justified by "objective" necessity).

For the sake of completeness one should not forget the distinction between "academic" and "industrial" mathematics, which shows similar features. With these caveats in mind, we shall approach mathematic and math education.

Since Snow's time, the main change in the landscape surrounding his gulf is that more material has accumulated on both sides: for instance, mathematics education on the side opposite a greatly enlarged mathematics. Since, in the short run, no bridge is likely to pop out of nowhere to connect the two, any hope for future communication requires that each of them understand, if the not the substance, then at least the form of the other. Both also need to know how far they are apart.

Surely, any understanding of 'math education' depends on some notion of what is 'mathematics'. This apparent tautology is expressed by the geometer René Thom (1972), as follows: *In fact, whether one wishes it or not, all mathematical pedagogy, even if scarcely* *coherent, rests on a philosophy of mathematics.* Constructivists might object to the implicit hierarchy and asymmetry of A resting on B, but we are more interested in the claim that a philosophy of mathematics - that is, an idea of its nature - is required.

Where, then, do we find such a philosophy? Courant (1941) tries to answer the question 'What is mathematics?' in a famous book (updated in 1996 by Ian Stewart) with just that title - and Hersh (1997) has a very different answer in a book with the same title extended by the word 'really'. Tomorrow there might be yet another answer, in stark contrast to the belief, held by not a few math educators, that mathematics is monolithic and, in its core, eternally unchangeable. In reality, every generation of mathematicians must be reminded to look after the coherence of their science, which is in constant danger. In the words of Sir Michael Atiyah (1978): *we must continually strive ... to unify.* He and his colleague Isadore Singer, have just (2004) received the prestigious Abel Prize for "building new bridges ...". A more recent Fields Medalist, Timothy Gowers, even sees "two cultures" within mathematics itself (wwww.dpmms.cam.ac.uk/~wtg10/2cultures.pdf): the vast and weighty tradition versus newer and less prestigious branches such as graph theory. Needless to say, all of this is again divided into pure and applied areas, which are, however, not as far apart as they may appear.

Though aware of their own diversity, mathematicians by and large tend to regard math education as a rather dull but straightforward affair (cf. Fischbein (1990)). Hence they see math educators as most of the latter see them: as a clique of uniformly narrow-minded and feckless academics. Assigning to mathematicians an appropriate role in the transmission of their science (cf. Bass (1997)) might, under these circumstances, do more harm than good. It is therefore more urgent than ever to slow down (perchance to stop) the further drifting apart of these two communities – for instance in meetings like this one -- by tracing the roots of this double myopia. Here are some details from Germany.

Schisms

When parts of a professional community find their aims and interests increasingly diverging from the rest, they naturally tend to form separate entities. Thus, in 1890, under the leadership of Georg Cantor, the newly created German Mathematical Society (DMV) (cf. http://www.mathematik.unibielefeld.de/DMV/) split off from its more generally science oriented parent, the GDNÄ (cf. http://www.gdnae.de), which had still included medicine (whence the last letter of its acronym). The next century brought schisms within mathematics itself: statistics and computer science, for instance, are no longer seen as belonging to it, and more schisms seem to be in the offing. Mathematicians with a strong commitment to education had also founded their own professional organization, the Society for Mathematical Didactics (GDM) and thereby prepared their gradual drifting away from mathematics. In this manner, simple common interest groupings can eventually lead to the formation of new disciplines, with separate aspirations, assessment criteria, administrative structures, and degree granting status.

Until well into the second half of the twentieth century, the main qualification for a secondary school teacher was to pass the "State Exam", a natural milestone on the way to a doctorate. It was taken by the majority of future mathematicians, if only as employment insurance, because academic jobs were rare and often unavailable even to the most talented. Some of the great mathematicians of the nineteenth century began their careers as school teachers (e.g. Weierstrass) and sometimes even stayed there (e.g. Grassmann). While this system provided high school teachers with a strong background in university mathematics, it left them to work out their own ways around pedagogy and school mathematics. After the Second World War, as attendance of secondary schools expanded to a much more general public, these haphazard methods were seen to be insufficient, and lecture courses in mathematical didactics became a useful, ultimately mandatory, part of teacher training.

With its necessary presence in *academe* established, the new science soon discovered fresh fields of exploration which were not necessarily concerned with helping prospective teachers find their way into the class room. Though pedagogy could arguably be improved by any serious reflection on it, mathematical competence requires *practice* more than theorizing, which is the central activity of any academic discipline.

Breaks

For individual careers of teachers these organizational and social separations are deepened by the mind-boggling discontinuities already described by Felix Klein (1908) -- one of the few major mathematicians of his time to worry about education: *'The young student sees himself at the beginning of his university course confronted with problems which in no point remind him of things he was concerned with at school; of course this is why he forgets all these things rapidly and thoroughly. However, when he enters a teaching position after completion of study, he is expected to teach traditional elementary mathematics in the traditional school manner; as he can hardly relate this to his university mathematics, he will in most cases embrace traditional teaching within a short time, and the university course will remain to him only a more or less pleasant memory that has no influence on his lessons. This twofold discontinuity …'*, This description is unfortunately still valid now, a full century later, and - alas - not only in Germany, but world-wide.

A naive observer, heedless of word order, might expect that mathematical didactics would step into the breach, not realizing that this would be a more natural task for *didactical (or better: educational) mathematics*, which is unfortunately a non-entity in the academic world. It exists only as a kind of amateur sport or hobby, with its own books, magazines, and competitions, but without any central organization or recognition as a scholarly pursuit. There is no reason why it could not find a niche in the increasingly inclusive modern university, except for the fact that it is looked down upon by most mathematicians and actively ignored by most educators.

In both cases, we believe, the source of this attitude is what is known as *math anxiety*, which is at least partly due to a fear of public embarrassment, *'a failure of nerve'* , according to Tobias (1994). To test this by an easy example, pick any four points on a circle and mark the midpoints of the four resulting arcs. If you ask a random mathematician to explain why the lines connecting opposite midpoints cross at right angles, you will usually receive an evasive answer, because most mathematicians believe that school material is "trivial" and would feel uncomfortable in not being able to handle it effortlessly. On the other hand, most educators would profess indifference because this concerns *content*, whereas their expertise lies in *method.* To find someone taking it seriously, your best bet is to ask a high school teacher or student.

Islands, Prejudice, Bridges

Islands

As rifts widen, islands are created. In mathematics, this general pattern is exacerbated by a coast line of daunting cliffs. Even inside the subject -- which is really an archipelago – intelligent inhabitants of different parts are very often mutually unintelligible. In lieu of bridges, a system of tight ropes, used only by a handful of acrobats, holds the subject together. Sir Michael is one of these, but neither he nor any of his peers would claim a working familiarity with the whole. Some of the rank and file make an effort to know their own island thoroughly, a few even try to get acquainted with a neighboring one, but most are busy digging for theorems in their own back-yards. They could use some of their time to whittle down the cliffs, but such work is scorned. Addressing the International Congress of Mathematicians, the poet Enzensberger (1999, p.18) uses the metaphor of a fortress surrounded by a moat, with all drawbridges up and out of order. This, it seems, is how mathematics appears to the outsider. However, the image of a fortress connotes the kind of tangible unity which mathematics has gradually lost over the last 300 years. Sure enough, its islands -- euphemistically called "fields" -- are solidly united under the surface, and share a subtle yet easily recognizable atmosphere, but crossing the rifts between them is strenuous even for the experienced practitioner, and not always successful. The beginner's the first foothold on a ledge of one of the cliffs is still more elusive -- but probably more exhilarating when it happens. The teacher's job is to help find it.

If the reader were to chide us for building an elaborate metaphor to explain why mathematics is the perennial problem child of education, we would answer that the metaphor replaces an even more laborious explanation. It has often been said that mathematics is particularly *hard*, but we are trying to point out that it is particularly *hermetic* -- not by caprice but by nature. In a recent paper, Doerfler (2003) suggests that the gap between mathematics and math education might eventually be filled, at least in part, by a subject he calls *mathematicology*, to be developed as an analogue to musicology. The comparison can be drawn in amazingly close detail until it is stopped by the fact that most musicians can hear, while mathematicians are constrained to reading scores. In both cases, the main "action" takes place in the brain, but in one of them, the latter functions with minimal outside guidance. Hersh (1986) described the situation as follows: *'Anyone who has even been in the least interested in mathematics, or has even observed other people who were interested in it, is aware that mathematical work is work with ideas. Symbols are used as aids to thinking just as musical scores are used as aids to music. The music comes first, the score comes later.'* In other words, the symbols which try to convey mathematics are like notes on a page read by a deaf audience. It is no wonder that they so often fail to elicit applause.

Mathematicology and educational mathematics, however difficult they may be to structure, could constitute a pleasant terrain for mathematicians and educators to meet and exchange ideas. A starting point already exists: school mathematics, which, though not quite at sea level, is a single island whose base has not much changed in the last three centuries. Unfortunately, it has been badly neglected, to the point where Thompson (1994) could write: *'Indeed, the converse of Hersh's statement can be used to characterize typical school mathematics - first comes the score, but the music never follows.'* To remedy this seems to us an urgent and doable task. It would be unrealistic to expect the music to waft suddenly through every class room, but there is no reason not to begin with rehearsals immediately.

Prejudice

What seems to separate the two communities at the deepest level is the mutual underestimation of each other's scholarship. Isn't mathematics the hardest of sciences? Instead of playing on words such as 'hard' versus 'soft' or 'easy' (cf. Berliner (2003)), why don't we just agree that it is one of the oldest and least popular? Its hermetic nature makes it difficult for outsiders -- and even "insiders" of another field -- to identify trashy mathematics, while almost anyone can easily dig up shallow and ill-written papers in the more accessible social sciences, to which mathematical didactics has come to belong.

When Horup (1994, p. 277) observes: 'mathematicians, however, also tend to have a handicap. The particularly important position of the logical argument in mathematics easily leads to the opinion that everything not belonging to mathematics, particularly political and moral thought and convictions, is illogical and beyond argument' , he certainly and unfortunately has a point, but when he continues: 'Furthermore it is not uncommon that mathematicians mistake this epistemological dichotomy between demonstration and subjectivity for a social dichotomy and, [...] take their own inveterate persuasions and prejudices for objective truth', he unduly specializes a common human tendency (mistaking one's opinions for truth) to mathematicians, thereby supporting the widespread prejudice that they are narrow-mindedly wedded to logic. Mathematics not only has logical argument, but a directness of insight which moves it closer to art than to sciences built on external evidence. Barry Mazur, one of today's master mathematicians, says it this way (cf. LA Times, March 18, 2003): 'In experiencing the impact of a work of art, or understanding a piece of mathematics, you are -- or at least you can be -- entirely on your own, with no authority in sight.' He illustrates this with a purely diagrammatic proof of the first statement in "The Book of Squares" by Leonardo da Pisa (1225), which conveys an inescapable sense of certainty.

Mathematics education, by contrast, seems to be at a stage comparable to that of biology in Darwin's time: confronted with a multitude of avenues to explore with an enthusiasm which is inversely proportional to the scientific rigour it imposes on itself. How do we learn, reflect on things, grasp concepts, have intuitions, get ideas, and so on? Fine questions to ponder, but not ripe to answer with any certainty. The Procrustes bed of statistics will only cripple them, since they are too complex for it, as the mathematician H. Wilf points out in a 1998 lecture (cf. http://www.cis.upenn.edu/~wilf/ and follow the link "Other"). In one of the cases he considers, Wilf concludes: 'The authors would have made their points much more effectively by writing a thoughtful five page essay describing their thesis. The points ... are good ones ... [and] benefit not at all from being forced into a pseudo-research straightjacket.' Indeed, double-blind studies are difficult, when there is no analogue of a placebo for a changed curriculum, and ill-applied statistics smell of pseudoscience. Many other social scientists are in the same quandary: in an effort to obtain hard evidence, they tend to misuse the only tool available, namely statistics, and the system of peer-review legitimizes whatever procedure is accepted in their respective communities. But mathematical didactics has the misfortunate of standing next to mathematics, which not only has some expertise in statistics but also in systematic doubt. A few run-ins with questionable methods could easily lead to hardened prejudice.

All this quibbling within the ivory tower takes its greatest toll among those in the outside world who have no choice but to swallow what is handed down to them: the preservice teachers and, through them, the students.

Bridges

But there are hopeful signs: this PME-NA conference, for example. As far as we know, every meeting of the AMS or CMS as well as the DMV now has an education section. In the case of the DMV, they often give the impression of having largely ceremonial character: mathematicians cultivating their favourite field in public. However, even that is progress and, above all, shows good will. Moreover, DMV and GDM are in the process of compiling a cooperative issue of the Notices of the German Mathematical Society, which will constitute as a mutual display window.

In North America, the Notes of the CMS have a regular education column, and the Notices of the AMS support a steady trickle of articles dealing with mathematics education as a separate discipline, not as a branch of mathematics. In one of these, Schoenfeld (2000), after first quoting Pollak's dictum that 'there are no proofs in mathematics education', turns around and describes the subject in analogy to natural science, as having "applied" branches concerned with improving what goes on in schools and "pure" ones seeking to *'understand the nature of mathematical thinking, teaching and learning'.* However, he admits and explains the systemic weaknesses of its empirical endeavours, for instance, *why* classical statistics cannot work, and adds: *'Findings are rarely definitive. They are usually suggestive. Evidence is cumulative ... moving toward conclusions that can be considered to be beyond a reasonable doubt*.' Such openness goes a long way toward dispelling the "pseudo-science" prejudice, and might even provoke interest on the other side of C.P.Snow's "gulf". In a similar vein, Ralston (2003) writes: *'Work in education or the social sciences will* almost never *lead to provable, ironclad results'*, and endorses Berliner's (2003) distinction of hard versus easy sciences, without going as far as to claim that educational research is the hardest science of them all.

Despite its impulse toward pure inquiry, however, mathematical didactics is more action oriented than its parent, and is likely to pull it into a more active engagement in education. Like Bass (1997), Stiff (2003) calls on the mathematical community to fulfill that part of its mandate: mathematicians working *with* educators *as* educators. Indeed, prejudices on both sides would be most effectively softened by collaboration on concrete tasks -- of which there is indeed no shortage. Nobody will expect mathematicians to show consummate pedagogical skill, nor demand great theorems from educators. But the public has a right to expect each side to show a little more interest in what the other one is doing, and both communities to put their shoulders to the wheel.

Wake-up Calls, Official Inertia, New Directions

Wake-up Calls

In the particular case of Germany, the discussion about these problems was given new impetus by TIMSS. Its sobering results were suitable neither for the usual finger-pointing between specific groups nor for the classical clichés dear to the media: incompetent teachers, uninterested students, and other-worldly professors. One of the traditional tacit assumptions of German society was its good standing in the world of science and technology, and now its teenagers turned out to be below average, barely ahead of the USA, distinctly behind other countries of northern, western, and eastern Europe, "not to mention the Asian countries" whose level was, for German students, "at an unreachable height", according to the official report. For the first time in a long while, joint declarations (cf. www.mathematik.uni-bielefeld.de/DMV/archiv/memoranda/timms3.html) were issued by the professional societies concerned: the DMV and the GMD, as well as the Association of Mathematics and Science Teachers (MNU). Each of them admitted the urgent need for correction and action in its domain.

They agree in stressing that '... mathematics itself has changed significantly since the time of the instructional reforms in the sixties. It seems that the corresponding changes ... in its outlook ... have not yet everywhere reached teaching practice.' One can only hope that this will not be interpreted as a call to pile additional exotic material onto the curriculum, instead of backing away from formalism without sacrificing precision. At any rate, such changes must translate into modifications in teacher training both pre- and in-service. The pressure to act was further increased by the OECDfindings known as PISA (cf. www.pisa.oecd.org/), thus moving the two associations closer together and causing them to organize a common annual congress of German mathematicians and math educators, to be held for the first time in 2007, at the Humboldt-University in Berlin. It will provide each of them with a professional forum for research results, but also display the numerous interfaces in terms of both material and personnel. We find this indeed encouraging.

Official Inertia

Unfortunately, this new coalition was not recognized as a chance for positive change by the educational administrators on either the state or federal level. A promotional campaign for math and science, launched by a commission formed by them, completely ignored the teacher training role of universities. Honest burden-sharing was thereby aborted, and so was the multilateral potential in the discussion of standards, which was more recently triggered by PISA. With very few exceptions, the responsible actors, mostly teachers, teacher educators, textbook authors, and ministerial curriculum designers remained largely among themselves, with universities and learned societies serving at best as alibis.

In these circles, an old and nefarious prejudice seems chronic, namely that research mathematicians have nothing to offer in educational matters; the message of Schoenfeld (and others), that educators are not necessarily interested or competent in policy, has not yet reached them. Thus it appears that the ministerial apparatus is a self- stabilizing system with its own dynamics and rarely open to outside suggestions, except those which happen to be in favour with politicians second-guessing the public. Experience with it is apt to confirm many a university mathematician in an attitude of resigned disinterest; the few laudable exceptions to this statement do not suffice to negate it. In the longer run, math educators are also likely to find that open-ended inquiry is not of much immediate interest to decision makers.

In fairness to administrators, we should admit that they have good reason to expect our newly found common voice to be short-lived. But even if it lasted, it would make such unaccustomed financial, temporal, and organizational demands that immediate action is impossible for lack of a blue-print. They would affect the following areas.

Curriculum. This is the area most familiar to education administrators: they firmly and almost exclusively believe in the salutary effects of curricular change - in spite of international evidence to the contrary as shown by many failed reforms (cf. also Schoenfeld, 1994). Its attraction consists of its relatively low cost coupled with its unquestionable, albeit limited, short term effectiveness. In the long term, it can even be counter-productive (witness: the New Math).

In-service Training. Of course, we would also ask for the means to improve professional development, since the classical forms must be regarded as failures, while successful ones (cf. Cooney & Krainer, 1996) are relatively rare. One obvious weakness of these schemes is that they reach only individual teachers, who do return to their schools with renewed enthusiasm, but soon buckle under the daily pressures, with little support from colleagues or school administrations. The lack of psychological insight in such designs is enough to make even a mathematician blush.

Pre-service Training. The mathematical education of teachers offers the greatest room for improvement, but is also the most shackled by tradition. An outside observer might conclude that, in this domain, mathematicians and educators stand in each other's way, the former by remaining stuck their professional perspective even where it does not apply, the latter by refusing to get involved in "content" even where it is urgently needed. Until this split is repaired, education administrators can rest assured that there is no need to hurry.

New Directions

A more positive impact on the educational scene, especially as regards mathematics, comes through outside initiatives by the private sector (cf. http://www.mint-ec.de) and various support programmes offered by foundations (cf. http://www.nat-working.de). Implicitly, these programmes contain a belief in progress through paradigmatic change: research scientists transmitting science in schools, professional teachers on leave to act as serious participants in university research teams, students in mixed teams sharing learning experiences with teachers and having direct contact with professional researchers, and so on. Emotional benefits and social contacts are, of course, part of the plan.

The goal of the most ambitious private sector initiative, known as MINT (M = mathematics, I = informatics, i.e., computer science, N = natural science, T = technology), is to identify schools which are particularly innovative and effective in mathematics and science teaching, and to give them special support and public status. Today almost 100 schools carry the challenging, nonpermanent designation as Centers of Excellence for MINT. In these schools, teachers as well as students are given special opportunities to develop their mathematical interests; in-service training courses, for instance, can be organized at a high level. Moreover, the sponsors aim at enhancing the teachers' all too often low esteem of their own work by providing generous furnishings and equipment. There are, after all, not only talented and highly motivated students, but teachers cut of exactly the same cloth, and any encouragement of *them* has an immediately multiplied effect on the whole system.

Unless special efforts are made, these currents could easily by-pass both professional mathematics and mathematics education, the former because of its innate hermeticism, the latter because of its redundancy in an environment which addresses teachers and students directly, without mediation. For mathematics, this would mean putting its practical foot forward: much of what is actually used in industry is within comfortable reach of even the "purest" abstract mathematician. Care would have to be taken to show this foot as clearly belonging to a larger body with many more attractions. Similarly, educators would be called upon to turn toward the "applied" aspect of their discipline, i.e., pedagogy and communication, and also seize the opportunity to open windows showing aspects of the social sciences. If the two communities rise to this occasion, it could be the beginning of building a durable bridge -- around which mathematicology and educational mathematics might grow as well.

At the moment, even the professional societies -- who began the schisms -- seem to come around to a new way of thinking. They realize that, all across Germany, the problem is too massive and too serious to allow small groups any chance of effecting socially significant changes. The separations of the past are now to be at least partially reversed. Nevertheless, a certain hesitancy and fear of being reabsorbed is palpable among mathematics educators, while mathematicians seem to welcome all the pedagogical help they can get, as long as -- *noli tangere circulos meos* -- they get enough time to indulge their brains.

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STRENGTHENING INSERVICE SECONDARY MATHEMATICS TEACHERS' UNDERSTANDING AND STRATEGIES IN MATHEMATICAL PROBLEM SOLVING

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The study reported here is part of a larger yearlong study on inservice secondary mathematics teachers' knowledge and understanding of problem solving and the teaching of problem solving. The report here focuses on examining the outcomes on a problem solving assessment that indicate that the most significant gains in the abilities of the inservice teacher participants to demonstrate mathematical understanding in problem solving and to use complex strategies in mathematical problem solving occurred after a mathematical problem-solving (MPS) course focusing on reading articles from research and practice in problem solving, applying and discussing Polya's problem solving strategies in the context of in-depth problems, and examining rubrics for assessing students' work in problem solving.

As a Process Standard in the *Principles and Standards for School Mathematics* (NCTM, 2000), problem solving plays a prominent role in reform efforts in mathematics education. However, many teachers feel ill-prepared to teach problem solving and have had little experience solving open-ended or open-middle problems that require more than simple applications of algorithms and formulas. For this reason, courses for mathematics teachers at the graduate and undergraduate levels must be refined to address mathematical problem solving in a rich manner.

Teachers need to be the first to become problem solvers in their classroom (Wilson, Fernandez, & Hadaway, 1993).Wilson et al. (1993) also discuss different aspects of mathematical problem solving in secondary classrooms and some of the inconsistencies in instruction. Comparing the emphasis on problem solving in the NCTM Standards and the manner in which it is taught in the classroom raises questions about how teachers' beliefs about problem solving affect their teaching behavior in the classroom. Teachers often cite various reasons for not incorporating more problem solving in their teaching: time factors, difficulty for students, curriculum issues, complicated to assess, not easy to find appropriate mathematical tasks.

To be competent problem-solving practitioners, teachers need knowledge that includes both procedural (Eisenhart, Borko, Underhill, Brown, Jones & Agard, 1993) and conceptual or content knowledge (Leinhardt, 1988; Eisenhart et al., 1993). Procedural knowledge denotes the rules, procedures, and skills necessary for completing a task. However, procedural knowledge may or may not be supported by conceptual knowledge (Hiebert & Lefevre, 1986). Conceptual or content knowledge denotes the ability to understand the concept and connect or apply several different ideas. For mathematics teachers, conceptual knowledge also includes ability to generalize, determine multiple representations, describe relationships, and exhibit higher order reasoning skills. In addition, developing problem-solving practitioners need to explore the role of metacognition in mathematical thinking or problem solving (Schoenfeld, 1985).

Information about teachers' problem solving progress in the classroom can be gathered by discussions with teachers about the scores they assign to student problem-solving work and their rationale in assigning the scores (Vasquez-Levy, Garofalo, Timmerman and Drier, 2001). Though teachers expressed the same rationales they weren't valued to the same degree. Hiebert and Leferve (1986) claim that competency in mathematics involves knowing how concepts, symbols, and procedures relate. This underscores that both conceptual and procedural knowledge are needed to be a successful mathematical problem solver.

This report focuses primarily on the results from the problem solving test which was designed by the researcher and piloted prior to this study. The work of White and Michelmore (1996) in studying students' conceptual knowledge in calculus motivated the design of the problem solving tasks and methodology. Their framework characterizes a goal of having students move away from applying only procedural knowledge (procedures learned by cues) and move toward applying conceptual knowledge (grasping the relationships between mathematical objects in context).

Method

The study was conducted at a midsize (25,000 students) Texas university with one-fourth of the student body enrolled at the graduate level. A group of sixteen inservice middle school (n=3) and secondary (n=13) mathematics teachers participated in a yearlong professional development program focused on mathematical problem solving. All of the participants were certified teachers and were employed as full-time teachers. Fifteen held secondary (grades 7-12) certification and one held an elementary grades certification (grades 1-6). Four different school districts were represented in this group; however, half of the participants came from a single school district. Half of the participants had more than six years of experience teaching, 5 of the 16 had taught more than12 years, and five were in their first two years of teaching. The average number of college hours in mathematics for the group was 36 hours. Nine participants had taken a college level mathematics course within two years of participation in the study; however, there were five participants who had not been in a college level class in over 20 years. Twelve of the participants were female and four were male.

Participants began the program in a course in Discrete Mathematics that focused upon various modes of group learning and leveling concepts about functions, mathematical reasoning, and mathematical inquiry. After the initial forty-five hours of instruction in Discrete Mathematics, the participants enrolled in a course in mathematical problem solving. In the MPS course, participants read and reported upon articles in research and practice in problem solving, applied and discussed Polya's problem solving strategies in the context of their approaches to in-depth problems from discrete mathematics, geometry, algebra, and calculus. They also examined rubrics for assessing students' work in problem solving. Using a guide created by the researcher, they engaged in creating their own in-depth problems to elicit mathematical problem solving behaviors in their students on difficult concepts. The MPS course was followed by a course in probability and statistics with an emphasis on problem-based learning and real-world applications using technology. Data was collected systematically throughout the program in the form of student interviews, journal entries, student class work, pre- and post- tests of baseline skills and pedagogical strategies used in the classroom, and a problem solving assessment that was administered at four different intervals during the yearlong program. As a group, the courses consisted of some short lectures, small group and whole class discussions, student and group presentations and constant feedback and reflection for the participants enrolled in the courses; this model is characterized by Santos-Trigo (1998) and Schoenfeld (1991) as indicative of a successful problem solving classroom setting.

To focus on the development of teachers as problem solvers, the researcher designed and field tested a series of tasks that had been developed into four items each to delineate between various levels of sophistication in problem solving. Four tasks were the focus of the test items for this study. The mathematics needed to solve the tasks did not go beyond a typical second-year algebra high school course. Each task was structured in four versions so that the manipulation (procedural knowledge) required to solve each version was essentially the same. However, the difference in the versions (items) was that at each successive level the depth of conceptual understanding of the mathematics and the sophistication of the modeling and generalization decreased. Thus, Item 1 was the most challenging version of the task, while item 4 was the version that required almost no translation into mathematics but simply a straightforward manipulation because almost all of the translation to mathematics had already been completed.

In total, there were sixteen different items created from the original four distinct tasks. Descriptive names were assigned to each task that suggest the focus of the task: Projectile Task, Graphs Task, Fractals Task, and the Probability Task. The Projectile Task focused on a projectile motion problem and is given in Figure 1. The Graphs Task focused on a task from Schoenfeld (1998, p. 98). Its solution is extremely tedious algebraically and is more readily obtained graphically. The Fractals Task developed the Koch snowflake curve from a given hexagon. Participants were asked to make predictions and reason through them. Finally, the Probability Task addresses conditional probabilities.

Item 1	A projectile is launched and travels according to the law $s(t) = at - bt^2$, where a and b are constants, t is the time in seconds after it is launched, and $s(t)$ is the height in feet above the ground at time t. We know that after 1 second the projectile is 80 feet high and that after 4 seconds it is 128 feet high. There is a pavilion structure over the launch site that extends 7 feet and is 140 feet high. Does the projectile hit the roof of the pavilion?
Item 2	A projectile is launched and travels according to the law $s(t) = 96t - 16t^2$, t is the time in seconds after it is launched, and $s(t)$ is the height in feet above the ground at time t. There is a pavilion structure over the launch site that extends 7 feet and is 140 feet high. Does the projectile hit the roof of the pavilion?
Item 3	A projectile is launched and travels according to the law $s(t) = 96t - 16t^2$, t is the time in seconds after it is launched, and $s(t)$ is the height in feet above the ground at time t. Find the maximum height reached by the projectile.
Item 4	Given $s(t) = 96t - 16t^2$. Find the vertex.

Figure 1. Four versions of the Projectile Task.

Although the mathematical skills required for solving the projectile problem are typically learned in a second-year high school algebra course, the modeling and conceptual connections necessary for solving Item 1 of this set of items require more than a procedural application of finding the vertex of a quadratic function. For each item of the Projectile Task, the final step requires finding the vertex or the maximum for the same quadratic function. Note that the most challenging item, Item 1, requires translation of the situation into mathematics to determine the function and its applicability to the situation. A participant must conclude that the vertex must be found in order to complete the task in contrast to Item 4 which tests procedural fluency. For Item

4, no mathematical connections must be made other than recalling how to find a vertex for a quadratic function. In Item 2, the quadratic function is given, but students must discern how to use the information given. For Item 3, the problem situation is the same, yet the problem asks for the maximum height of the projectile—leading student thinking rather than their having to see the connection to the vertex of the parabola.

Participants were tested on four occasions: before the Discrete Mathematics course, before the Problem Solving course, before the Probability and Statistics course, and at the end of all courses. The participants were divided into four parallel groups of 4. They were not aware which group they were in. Four tests were constructed (test A, test B, test C, and test D) and each test included four items: one version of each of the four tasks. Each version of each task occurred on one and only one test and each test had only one question in each version. As in White and Michelmore (1996), a cyclic scheme was used to administer the tests to each of the four groups over the four data collections.

Participants were randomly placed into four different groups: A, B, C, and D. Their group name corresponded to the test they took first. In the successive administrations of the exam, the following cyclic scheme was used: (A,B,C,D) → (B, C, D, A) → (C, D, A, B) → (D, A, B, C).

Note in Figure 2 that Test A consisted of Item 1 from the Projectile Problem, Item 2 from the Fractals Problem, Item 3 from the Probability Problem and Item 4 from the Graphs Problem. Also, note that the items on the Probability Problem labeled Item 3A, Item 4B, Item 1C, and Item 2D denote that the most difficult (with respect to problem solving sophistication needed for solution) version of the probability problem is on Test C. Thus, the label Item 1C is given to the highest level probability problem. A similar scheme is used in enumerating the other items. Each item has a unique identifier that indicates the test on which is occurs and its problem-solving level.

Test A	Item 1A: Projectile Task	Item 2A: Fractals Task	Item 3A: Probability Task	Item 4A: Graphs Task
Test B	Item 1B: Graphs Task	Item 2B: Projectile Task	Item 3B: Fractals Task	Item 4B: Probability Task
Test C	Item 1C: Probability Task	Item 2C: Graphs Task	Item 3C: Projectile Task	Item 4C: Fractals Task
Test D	Item 1D: Fractals Task	Item 2D: Probability Task	Item 3D: Graphs Task	Item 4D: Projectile Task

Figure 2. Problem solving test composition per group.

Purposefully, the tasks were chosen so that the problem solving tests were mathematically interesting and challenging and required various levels of problem solving ability to solve, but were not beyond participants' scope mathematically. The goal was to test problem solving growth rather than growth in a particular mathematical content topic.

Each item was scored using a rubric adapted by the researcher that focused upon understanding, strategies, and accuracy on the task. The rubric was adapted from a problem solving scoring rubric that was used by the Oregon Department of Education (1999) to grade statewide open-ended problem solving assessments and benchmarks. The rubric provided a guide toward uniform evaluation of problem solving performance.

The scoring of all items took place after the conclusion of the yearlong program. The items were independently scored by three experts. After all scoring was completed, the group of scorers met to discuss any discrepancies in scoring. None of the scorers was aware of the scores that had been assigned by the others until this meeting. Where scoring differed, the scorers discussed the rationale for their scoring and a consensus was reached on the final scoring of the problem to be used in the data analysis.

The scores associated with each category of the rubric ranged from zero to seven for understanding, zero to seven for strategy, and zero to five for accuracy. Thus, each item had three numerical scores. Scores were tabulated on frequency charts. For data analysis, the scores were grouped into two categories. The scores 0-3 were grouped in each category to indicate underdeveloped understanding, ineffective strategies, or incomplete/incorrect accuracy, respectively. Scores higher than 3 were grouped to indicate expanded understanding, useful strategies, or adequate accuracy, respectively.

Frequency charts were made recorded to tally the number of each score in each category (understanding, strategy, accuracy) by item number or sophistication level (1,2,3,4) and then another frequency chart for each category again by item number that tracks the time the test was given or testing period. The average scores for each particular problem (example: 2B: Item 2 of the projectile, etc.) were also calculated and charted together for U-understanding, S-strategy and A-accuracy.

The data was modeled in terms of main effects of the factors involved. A model-smoothed estimate was obtained based on logistic regression modeling. From this, a probability estimate was calculated for scores on each problem with regard to the item number and the testing period. Because of the small sample size, the scores for understanding, strategy and accuracy were grouped as mentioned earlier to create dichotomous data in each category.

Results/Discussion

Using a model-smoothed estimate, there was a statistically significant increase in "understanding" scores for all item numbers in that testing period three and four were higher than the first testing period. Probability estimates on "strategy" scores reveal a statistically significant increase in all tasks and all item numbers in that testing period three was higher than the first testing period. Accuracy scores showed that for testing period four there was a statistically significant increase when compared to testing period one regardless of task or item number. It is to be noted that testing period three is immediately following the problem solving course and testing period four is immediately after the probability and statistics course (the last course in the sequence).

The statistically significant increase in understanding scores on all problems for all item numbers for testing periods three and four may be attributed to the ongoing nature of the professional development. A high score in the understanding category denotes competency in interpreting concepts and processes and translating them into mathematical statements. This reflects positively upon the professional development program in that participants showed an improved ability to interpret concepts and processes in problem solving after the MPS course and after the probability and statistics course compared to their initial understanding scores.

For the strategy category of the rubric, there was a statistically significant increase in strategy scores on all problems and all item numbers during the third testing period compared to the first testing period. A high score in the strategy category indicates an enhanced use of pictures, models, diagrams, and/or symbols used to solve the task. It is important to note that the most significant change took place after the course in Problem Solving. One of the central themes in the problem solving course entailed multiple representations in solving problems and the creation of rich problem solving tasks that required multiple solution strategies and approaches. This result will help focus further research about the effects of courses on mathematical problem solving.

Although the accuracy scores were not of primary interest in the data analysis, the finding that there is a statistically significant increase in the accuracy scores in testing period four when compared to period one possibly indicates that persistence and ability to justify and support mathematical conclusions were developed over the course of the professional development sequence.

Overall, the results indicate a positive influence of the professional development sequence on mathematical problem solving among the participants. With largest gains, in understanding and strategies following the MPS course. The overall vision for all the courses contributed to these outcomes in that the courses embraced ideas on models of successful problem solving classrooms (Santos-Trigo, 1998; Schoenfeld, 1991).

Although a statistically significant increase in the scores in all areas was noted, the complexity of the statistical analysis on the data collected was limited by the small sample size and the number of variables involved (problem, difficulty, and testing period). The sample size and number of variables also reduced the number of conclusions that could be reliably claimed from the data. Other factors that may have influenced outcomes are the possible subjectivity of the grading rubrics and the possibility of the variation of the grading from problem to problem. It is also important to note that to achieve a high score on a difficult problem required more mathematical translation than on less difficult problems because the translation was provided. This means that a high score on the less difficult problems could be obtained with less translation and more manipulation or mechanically solving the problem. As with any qualitative approach to scoring open-ended and open-middle problem solving tasks, Vasquez-Levy et al. (2001) note that inconsistency in grading problem solving—even when rubrics are provided and used—and granting partial credit. They also note that many times partial credit that is awarded is awarded more generously if a solution is approached a manner consistent with the grader's own approach to solving the problem.

If we are to have secondary mathematics teachers that are successful in teaching mathematical problem solving to their students, then they must first become successful problem solvers in their own right (Wilson, Fernandez, & Hadaway, 1993; Schoenfeld, 1992). To be successful in problem solving, teachers must be exposed to challenging and interesting mathematics in an environment that models a successful problem solving classroom. In this study we provide supporting evidence that professional development for secondary teachers in the form of graduate mathematics education courses with a dedicated focus on problem solving can improve teachers attitude toward mathematics as well as their problem solving ability with respect to understanding of problems that require more translation into mathematics, strategy for approaching and thinking about those problems which in turn improve the accuracy in which they solve them. Embedded in this work is the idea that teaching teachers in a manner that models successful teaching strategies for teaching K-12 students will help improve mathematical instruction for all students. Further work is needed on how professional development experiences not only increase a teacher's own problem solving knowledge but also their effectiveness at increasing problem solving knowledge for the students they teach. This research provides a foundation for exploring what specific professional development experiences are necessary to develop teachers as problem solvers, what duration is most effective, and what long lasting effects these experiences have on teachers' attitudes and beliefs about mathematics.

Endnotes

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LEARNING TO TEACH PRESERVICE MATHEMATICS TEACHERS: THE ROLE OF A DOCTORAL COURSE

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The teaching preparation of mathematics teacher educators has not been an area of programmatic or systematic attention in doctoral programs of education, which most prominently focus on the content, discourse, and practices of educational research. Such inattention to the teaching development of teacher educators propagates the very same myths schools of education tend to challenge, such as 'experience is the best teacher.' This study investigated the experiences of 8 first and second year doctoral students in a 'teaching practicum' doctoral course designed to support their teaching of future teachers of mathematics and their development as mathematics teacher educators. The analysis of these experiences shed light on the study's questions: what is involved in learning to teach future teachers of mathematics and what role does (or might) a doctoral course play in such learning?

Introduction

While challenges of learning to teach mathematics have been widely documented over the past two decades (see Ball, Mewborn, & Lubienski, 2001; Brown & Borko 1992), challenges of learning to teach future teachers have received much less attention. Challenges of teacher education have been explored broadly (e.g., Katz & Raths, 1992) and several endemic dilemmas have been identified, such as interplay between theory and practice, and programmatic coherence. Challenges of learning to teach future teachers has also been explored by individual teacher educators as they reflect on and study their own practices (e.g., Feiman-Nemser & Featherstone, 1992).

In light of such challenges, researchers (e.g., Heaton, 2000; Nicol, 1997) raise questions about the lack of attention to learning to teach prospective teachers in graduate schools of education. As a recent study of students' experiences reports, 83% of doctoral students surveyed stated that, "enjoyment of teaching made them interested in being a professor" (Golde & Dore, 2001, p. 21). Yet, respondents also indicated that support for such work - organized and sustained professional development - varies greatly across institutions and within and across departments. Furthermore, as Golde and Dore note, it is not preparation for teaching that comprises a significant portion of graduate student work. Rather, preparation to conduct research tends to receive the greatest attention in courses, guided practica, and faculty-student interactions. Despite this, it is a widespread practice for graduate students to teach undergraduate preservice courses as part of their assistantships.

Concern for the teaching preparation in doctoral programs is also represented in a recent document that compiled mathematics educators' discussions around preparation of doctoral students (Reys & Kilpatrick, 2000). In this document, Lambdin and Wilson (2000) stated that: "doctoral programs in mathematics education must ensure that students are involved in a variety of teaching experiences, both in schools and at the university level" (p. 82). The form, length, and number of such experiences, however, were not specified and were reportedly a point of disagreement among the mathematics educators at this retreat.

In response to concerns similar to those stated above, our Department of Teacher Education launched a programmatic effort to explicitly mentor graduate students into their roles as future teacher educators. Doctoral students, who are or will be teaching preservice teachers for the first time, are required to take a "practicum in teaching" course. The purpose of this study is to investigate the role such a course plays (and might play) in helping doctoral students learn to teach and to inquire into their teaching of future teachers of mathematics.

This study contributes to the scarce literature on preparation and development of beginning mathematics teacher educators. It also provides another perspective to the largely "self-study" approach to the process of learning to teach future teachers. It examines this process in the context of a graduate course aimed at supporting the development of future teacher educators. In addition, this study serves to open up the conversation on (a) what is involved in learning to teach prospective teachers of mathematics and (b) the kinds of formal and programmatic experiences that might help prepare future teacher educators to learn to teach K-12 preservice mathematics teachers.

Theoretical Perspectives

The design of the course and study draw on the perspective of learning to teach as a complex life-long process (Feiman-Nemser, 1983). Similarly, processes of learning to teach prospective teachers are considered life-long endeavors that cannot be addressed solely through course work or teaching experience. This is also consistent with constructivist views of learning as prospective teacher educators come to graduate school with past experiences, knowledge, and beliefs, which influence how each will experience the teacher education program as learner and as teacher. The study also takes the perspective that teachers' practices are shaped by their knowledge and beliefs (Borko & Putnam, 1996; Calderhead, 1996) and therefore these must be objects of inquiry and sites of learning. With these in mind, the course aimed to provide students with experiences in support of the following goals (each illustrated with an example of the kinds of activities that aimed to support them:)

- Examine perspectives on what teachers need to know and be able to do to teach mathematics in elementary and secondary schools. Supported by examinations of their own ideas through mathematics education biography and learning about others' perspectives as stated in standards, course syllabi, readings;
- Experiment with a variety of pedagogical approaches and resources in mathematics teacher education. Supported by reviewing and trying resources and developing records of practice and/or a collection of teaching resources along with a statement of teaching philosophy;
- Become familiar with the variety of contexts for teaching and learning in the teacher preparation program. Supported by creating a map of program in relation to their teaching assignment; observing in other classes; interpreting these in light of National Council of Teachers of Mathematics (NCTM) research companion and/or other relevant readings;
- Consider questions, approaches, and methods of research in mathematics teacher education. Supported by reading and discussing research articles that used different approaches and methods to investigating similar research questions; and
- Design and conduct research in the context of their teaching. Supported by discussions of various iterations of drafts of 'researchable questions'; public poster presentation of draft question and research plans; paper and presentation of research project findings.

Data Sources and Analysis

Data collected focused on class activities and participants in the "practicum in teaching" course. Instructors for the course are both junior faculty who teach in the teacher preparation program at this institution. The course meets throughout the Fall and Spring semesters and it is scheduled as a 2-hour seminar and a lab activity every other week. The data consist of class agendas, written assignments, audiotapes of selected class discussions, students' feedback and interviews regarding their experiences, and classroom observations of the participants' teaching.

Participants include first and second year doctoral students with varying degrees of K-12 teaching experience, with different nationalities, and with different teaching assistantships (secondary or elementary). Data of eight course participants collected over two years are used to construct individual cases that are later used to uncover patterns and develop themes across cases (Yin, 1989). Case studies document prior experiences, knowledge, and beliefs these novice teacher educators brought to the course, as well as how those factors interacted with their learning from course activities and from their own teaching experiences. We present preliminary insights and provide an overview of questions we are continuing to explore.

One theme that emerged through this analysis is that of identity, which led us to explore the role it plays in the ways in which participants engage with course activities. One identity-related theme is students' reluctance to call themselves "teacher educators." One participant, for instance, wrote "Sometimes when I really think about where I am and why, I am surprised. My most bizarre vision of what I might become when I finally grew up never included math teacher educator." Another stated: "While I have always wanted to be a teacher educator, I was as surprised as anyone to discover that what I wanted to focus on was math." And another confessed: "I found myself interested in teaching teachers because I needed a job in graduate school."

Another theme we examine is the value participants attributed to the "practical" and "research" aspects of the course, and the ways these activities influenced their thinking and teaching practices. Throughout the course, students had opportunities to analyze syllabi, create and discuss records of teacher educators' practices (lesson plans, cases, class videos, observations of others' teaching), and analyze preservice teachers' work. They also had opportunities to engage in more research-oriented activities, such as reading and analyzing research papers with a focus on mathematics teacher education, designing and conducting a research project within their teaching context, and sharing their work with a broader audience through presentations. Understanding our participants' perceptions of their experiences with these two kinds of activities can shed light on their development processes.

Results

We choose to report by theme rather than by individual case to ensure anonymity of our students. To that end, we are purposefully unspecific about the research project questions any one student investigated since these projects have been presented at conferences and might become publications. We use clusters of quotes and summarize similar points of views so that no one particular case is revealed. Instead we present the results as a composite case of the common issues experienced by the students. Next we report on preliminary findings from our analysis of the data. First, we report on students' experiences related to developing a mathematics teacher educator identity; and second on interactions with the teaching and research related activities of the course.

To Be or Not to Be! Challenges of Becoming a Mathematics Teacher Educator

Wenger (1998) argues that learning involves the development of identity, the changing of who we are, in the context of the communities of practice in which we participate. He states: "Because learning transforms who we are and what we can do it is an experience of identity. It is not just an accumulation of skills and information, but a process of becoming–to become a certain person or, conversely, to avoid becoming a certain person" (p. 215). Our identities, then, are shaped by our participation or non-participation in various practices, which in turn shapes our communities of practice. Developing an identity is a constant process of negotiation. "We are always simultaneously dealing with specific situations, participating in the histories of certain practices, and involved in becoming certain persons" (Wenger, 1998, p. 155).

The move from classroom teacher to graduate school is as awkward as any other life event when the old self moves out of familiar places and practices and into new ones. The graduate students came with varying degrees of experience teaching in K-12 schools (zero to ten years); three had teaching experiences beyond K-12; one taught content courses (in a different subject); another taught teacher education courses outside of the U.S.; and another was a teacher leader who provided professional development to other teachers.

The students differed in teaching assignment (elementary, secondary, not teaching). The three who were not teaching 'shadowed' another instructor throughout the duration of the course. They also differed in the extent to which they identified themselves as 'math smart' or 'math strugglers;' and in their views on their accomplishments as teachers. Most, regardless of background in mathematics (major, minor, minimal), considered themselves 'math frauds'—students who had good or hard-earned grades but had never really understood mathematics until some life changing experience with it occurred (during undergraduate studies, in teacher preparation, in professional development) that hooked them into wanting to learn more for themselves and share it with others. Some said they liked mathematics more when they started to teach than when they studied it in school. Regardless of their experiences with mathematics or teaching, each student had come to graduate school because they wanted to learn more about mathematics, research, and/or teaching.

As mentioned earlier, students hesitated to identify themselves as mathematics teacher educators. In their mathematics education biographies they wrote little about what they anticipated learning and doing in their new teaching roles even though this was an explicit prompt of the assignment. We gleaned more into their ideas through discussions and later assignments such as when we asked the students on the first day of class to find their place in a continuous line labeled math/math educator at one end and teacher/teacher educator at the other end. We also asked students to quick-write and discuss responses to questions such as: "*What do you think is the most challenging thing about learning how to teach mathematics? What do you think the most important thing is you can do to help your students learn to teach well*?

The map of the teacher/teacher educator—math/math educator revealed that our students located themselves nearer to one of either side rather than close to or at the middle. Students' responses to questions that elicited their thinking revealed the wealth of knowledge and ideas they had brought with them that were consistent with many of the views and goals explicitly stated in teacher preparation standards and our institution's program standards but that also lacked clear articulation. Their statements also revealed their troubles identifying the expertise they brought that could help their students learn to teach well. Their responses to what is challenging about learning to teach mathematics included: "move from being a student to becoming a professional," "teaching people that are different from yourself," "developing a good understanding of mathematics," "managing class discussions," "develop confidence in own ability to do math" (elementary majors), "translate content they know to make it understandable to kids" (secondary). Responses to what they could do to help their students learn to teach well included statements such as: "model good practice and being a transparent facilitator—letting them see what you're thinking," "being flexible, positive, and open-minded," "help them understand the whys in mathematics."

Students were less clear or forthcoming about the experiences and knowledge they brought to their new roles. Their statements reflected desires to share and promote in their future students a passion and enjoyment with mathematics and teaching: “My gift as a teacher educator is that I know what a mathematical awakening looks like;” “I want to provide preservice teachers with the kinds of mathematical experiences I had.” Their comments also reflected views of teachers and teacher educators in their past that they wanted to emulate and others they much rather not be. They described teachers who did or did not practice what they preached, who knew (or not) their content and could or could not teach it, and teachers who did and did not seem to care for their students.

These comments make evident students’ preoccupation with establishing themselves as knowledgeable teachers who care for their students. The students’ reluctance to claim identities that included expertise in the subject and or practices of mathematics teaching interacted in interesting ways with the course activities, teaching assignments, and their developing identities as mathematics teacher educators. Consider the double bind for them to claim an identity either as a math struggler or a math smart. Claiming either identity undermines the possibility of connecting with future teachers of mathematics who look to ‘learn from’ either an expert in the content they will be teaching or someone who understands what is like to struggle with mathematics. These identity ‘crises’ created two interrelated challenges that participants experienced throughout the course—developing credibility and integrity as mathematics teacher educators.

Issues with developing credibility (being regarded as having expertise in the content and practices of mathematics teaching) arose during the course activities when these revealed the participants’ inexperience with either mathematics and/or with teaching practice. These occurred, for example, when particular content was examined through a reading (e.g., division of fractions in the case of Ms. Daniels in Borko and colleagues, 1992), when students presented their ‘researchable questions’ to other mathematics education students and faculty across our campus in a public poster session, or when peers or instructors challenged their ideas about mathematics or teaching. In the context of their teaching, issues of credibility were more pressing when their knowledge and practices were challenged by their students, the collaborating teachers who worked with their students, or by fellow instructors. These issues were amplified by the history and reputation of the teacher preparation program in which they now worked and studied.

Issues with developing integrity in their practice (modeling the kind of mathematics teaching they wished to promote) were also constantly brought to the fore during course discussions and activities. Typical opening discussions during the seminar where students would share ‘how things were going’ in their classes were filled with stories of struggles to enact what they believed to be good teaching practices in a context that was not quite like the classrooms they had left behind. The fact that their students were adult learners who did not appreciate being treated as children (even if role playing), for example, was one such challenge to modeling the kinds of practices they wanted their students to experience and adopt. Another was facing the fact that they were now outsiders to what happens in real mathematics classrooms (they were no longer practicing teachers, had not taught in this country, or had not taught at all) so using examples of their own teaching did not always achieve the intended purpose. Another challenge was to allow their teacher education students to take risks and make mistakes—which they valued as a process for meaningful learning—when those mistakes involved real children. Resisting the temptation to give into a pedagogy of ‘showing and telling’ turned out to be much more challenging than any of them had expected. The following quote written by one of the graduate students during the second half of the course reflects issues of credibility and integrity the students experienced.

> How do we create meaningful activities, engage in powerful and reflective dialogue, and facilitate conversations if our personal contexts and those we teach in are so different and all disconnected from the reality of specific classrooms and kids? Is there a "better" way to construct teacher education? If so, how might this look?

Researching and Teaching: Looking for Connections and Balance

The challenge of developing an identity as a mathematics teacher educator is also a challenge of learning to connect and balance the worlds of research and teaching and learning and to move from the outside to the inside of (and between) these communities of practice. The students' positioning as insiders or outsiders to either practice interacted with the course requirement to conduct research in the context of their teaching. This brought to the fore tensions between research and teaching in ways that typical work in doctoral courses do not.

Tensions between researching and teaching are widely documented in educational research literature where disparate views about their relationship abound. Some say that educational research does not speak to the concerns and interests of teachers (Atkin, 1992). Others observe that educational research does not often seem to speak to academic researchers either (Eisner, 1984). There are those who see the two practices in competition with one another (Kline, 1977; Wong, 1994) and those who claim the two are essential to one another (Wilson,1994).

Issues of connectedness and balance when conducting research in the context of their teaching became explicit foci of conversations and preoccupations for the graduate students and their instructors. Questions raised throughout the course in relation to these issues included: How much emphasis and attention should be placed on teaching and how much on research projects? Where do research questions come from: theory, practice, both? How are these kinds of questions different or similar? Does research inform teaching? If so, in what ways? How can one be both a believer and a skeptic of one's teaching and what students are or are not learning?

Issues of connectedness between researching and teaching were experienced differently by those who were and were not teaching during the course. These were evident in the kinds of 'researchable questions' the two groups of students posed, how much or how little their questions changed over time, the extent to which their questions focused on exploring, assessing, or changing their students' thinking, in terms of the conclusions they reached about what the students had or not learned in teacher education courses, and the value and usefulness they attributed to teaching and research related activities of the practicum in teaching doctoral course.

Issues of balance between teaching and researching were also experienced differently, although both groups (teaching/not teaching) spoke often of not letting their teaching and/or research activities 'take over their lives.' Questions of balance were brought up for both groups when they had to make a final commitment on what to study in this context when they admittedly had many questions and wanted to learn as much as possible about teaching future teachers. Questions were also raised by both groups about the expectation that they would devote ten hours a week to their teaching responsibilities (as stated in their teaching assistantship contract) and the fact that they were (or could see themselves) working twice that amount of time.

Another issue of balance had to do with figuring out what could be learned from their research studies that was specific to their teaching. Those who were teaching seemed to have a harder time relating their research to a broader audience whereas the other seemed to struggle with drawing lessons from their studies that would help them in their future teaching. Finally another issue of balance related to how much time was spent in the course on either of these activities and the challenges of designing meaningful discussions and activities around disparate teaching contexts and research projects. The following quotes reflect issues of connection and balance in the students' writings. Both reflect insights they experienced through research activities that connected with their teaching. The latter raises questions about the feasibility of doing research while teaching.

> Previous [to this research project] my true goal, even when asking questions, had always been to change my students' teaching—to get them to expand what they had done well and reduce what had been done poorly. It was only when I gave up that agenda that I was really able to hear what my students were saying and to give them the space to reflect on their own teaching.

> This is the first time I have ever transcribed anything. I often found that I would type out what I heard and then listen to the tape again only to discover that I had unintentionally edited the transcript. Usually my mental editing maintained the meaning, but occasionally the meaning was different! I was amazed at how much information I might lose by taking notes and not audiotaping. But who has time to do all this?

Discussion

We return to the questions raised earlier to discuss the significance of these results. In terms of what is involved in learning to teach future teachers of mathematics, we find evidence in our data of Wenger's proposal that processes of learning, in this case of learning to teach future teachers of mathematics, involves not just an accumulation of skills and information, but also an experience of identity—that of becoming or avoiding becoming a certain person. In this process of becoming, students wrestled with common challenges associated with beginners—credibility and integrity. This is an interesting parallel to challenges associated with beginning and novice teachers. This suggests that these issues (and perhaps others) are indeed central to what it means to learn a new practice such as the practice of teaching. Finding these factors present and central among this new population (preservice teacher educators) serves to reinforce their generality and thus importance.

Another important result relates to differences found between the two groups of students who participated in the course—concurrent and delayed teaching. These students' experiences in the course differed in terms of their developing identity as mathematics teacher educators but most prominently in their perspectives on what was interesting, useful, feasible, and valuable about the teaching and research aspects of the course. Whether one had an authentic context in which to explore what was being learned in the course played out in some unexpected ways. It issued challenges in terms of how either group connected and balanced activities of teaching and researching. Managing these challenges afforded and constrained what students chose to explore, what they chose to experiment with, and what they could see from their research studies.

In terms of the kinds of formal and programmatic experiences that might help prepare future mathematics teacher educators, we propose that it is possible for a course to offer rich learning opportunities for those who are concurrently teaching and to those who are delaying teaching (but have a teaching site in which to explore what they are learning in the course). Although it can be challenging to design experiences that are meaningful to both sets of students, restricting the course to either group would limit the richness of their respective experiences. We also propose that such a course needs a dual and equal focus on teaching and research activities. Attention to either one alone would fail to address the development of the students' identities as mathematics teacher educators as well as their ability to see and seek connections and balance between the two practices.

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SIXTH GRADE MATHEMATICS TEACHERS IN TRANSITION: A CASE STUDY

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This empirical study reports on the evolution of two sixth grade mathematics teachers' teaching practices over two years of professional development activities. The final year of professional development consisted of a project that aimed to (1) develop a cohesive year-long sixth-grade mathematics curriculum that addresses the California state mathematics standards, and (2) facilitate and support two sixth-grade teachers in the implementation of, reflection on, and revisions of the developed curriculum. Analysis of journal entries, classroom observations, and interviews led to an emerging picture of how these teachers' knowledge, perspectives, and classroom practices changed over the course of the project, leading to several implications for professional development.

Describing the effects of professional development activities is of critical importance in designing effective means of improving teaching practices. There is precedent for investigating such activities. See Becker and Pence (2003); Farmer, Gerretson, and Lassak (2003); and Murata and Takahashi (2002). However, a report by RAND (2003) suggests that professional development practices have not received sufficient attention from mathematics education researchers, and that there is a critical need for better descriptions of successful professional development practices.

In this case study we describe two sixth grade mathematics teachers in transition, relative to the second of two years of professional development activities. These teachers initially pursued professional development opportunities out of a desire to improve their teaching in the face of new demands for accountability and increasingly complex school structures. After a year of professional development institutes focused on mathematical content, these teachers expressed a desire for more practice-oriented professional development activities that would help them actually implement some of their new-found mathematical and pedagogical content knowledge in the classroom while accounting for some of the real challenges they faced on a daily basis. This need provided the impetus for us to design and implement an intensive year-long professional development project with these two teachers.

Objectives

The goals of this professional development project were to (1) develop a cohesive year-long sixth-grade mathematics curriculum that addresses the California state mathematics standards, and (2) facilitate and support two sixth-grade teachers in the implementation of, reflection on, and revisions of the developed curriculum. At the same time, we planned to study the effects of our professional development project on these teachers' classroom practices, perspectives, and knowledge. In particular, we wanted to investigate the following questions.

(1) What changes in these teachers' classroom practices, perspectives, and knowledge emerged over the course of our year-long professional development program?

(2) What aspects of our professional development program (if any) contributed significantly to these changes?

Perspectives and Framework

Mathematics, Learning, and Teaching

Our perspectives on the nature of mathematics, learning, and teaching guide our design of professional development activities and influence what we take to be important. We believe that understanding of mathematics involves (1) the ability to solve problems (non-routine and routine, open-ended and closed-form), (2) the ability to connect mathematical ideas to one another and to real-life contexts, and (3) the ability to communicate mathematical knowledge and processes to peers. We believe that learning of mathematics is best achieved by active engagement in activities that call on and develop the abilities involved in understanding of mathematics. Thus, the teaching of mathematics should be designed to provide such opportunities to students.

A Framework for Professional Development

In our project we adopted a framework for designing professional development from Loucks-Horsley, Hewson, Love, & Stiles (1998). These authors discuss the elements that go into the design process: knowledge and beliefs, strategies, context, and critical issues, as well as the implementation process: set goals, plan, do, and reflect. This framework does not advocate strict adherence to a model, but rather thoughtful, conscious, evolving decision-making throughout a professional development program. With this framework in mind, we interviewed each teacher to acquire a sense of her needs, school context, and critical issues. This led to a collaborative setting of goals for the project. In broad strokes, our yearlong project consisted of collaborative development of curriculum materials and plans during an intensive summer workshop, followed by a year of implementation, reflection, and revision of the developed materials and curriculum.

The Workshop

The project began with an intensive professional development collaboration in summer 2003 with our two mathematics teachers. During this 20-hour, one-week workshop, we collaboratively developed and adapted mathematics activities for their sixth-grade courses. These activities were designed to address more than one concept and sought to make connections among concepts. For each of the activities, the four of us discussed the quality of the activity, the concepts it covered, its placement in their curriculum, and the types of adaptations required before using it in their sixth grade classrooms. When adaptations were required, each of the two teachers completed homework that night to make some changes for discussion the following day. When the activity was revised to the group’s satisfaction, one of the teachers created the teacher notes to accompany the activity. These teacher notes included (1) a description of the activity, (2) concepts addressed, (3) tips on how to implement the activity, (4) connections to the state and national standards, and (5) solutions.

Ongoing Support

Follow-up support focused on classroom observations and debriefings of three activities developed in the summer workshop. Debriefings took place with each teacher immediately after the activity. Joint meetings were also held to facilitate reflections and discuss necessary revisions. A wide range of teaching practices were discussed in relation to each activity: preparatory concepts, activity warm-up exercises or problems, activity extensions, access for students with special needs, homework, follow-up, assessment, and placement in the curriculum.

Mode of Inquiry

This research employed a qualitative, case-study design involving classroom observations and interactive interviews of two sixth grade teachers. Each classroom observation focused on one of three learning activities developed during a week-long summer workshop. Each interview elicited reflections and perspectives at one of six different points in the year-long study. Following an "account of practice" strategy developed by Tzur, Simon, Heinz, and Kinzel (2001), the primary data consist of teachers' written responses to structured journal prompts, written learning activities and teacher notes, oral comments made during interviews, and actions exhibited during classroom observations. These data were analyzed to identify turning points in these teachers' knowledge, perspectives, and practices.

Participants

The two sixth-grade mathematics teachers in this case study, Celia and Dee, were purposefully selected. Both researchers had worked with these teachers during a year of content-based professional development institutes. Over this period, both of these teachers demonstrated initiative to improve their teaching, a desire to learn new mathematical and pedagogical content, a willingness to try new teaching practices, and engagement in ongoing reflections on mathematics and the teaching of mathematics. On the other hand, Celia and Dee varied significantly in their mathematical knowledge and teaching experience. Dee, with over 30 years of teaching experience, had taken calculus in college, whereas Celia, with 7 years of teaching experience, had taken the required 6 hours of mathematics coursework for future elementary school teachers while in college. The researchers felt that this combination of qualities would provide sufficient variability in knowledge and teaching experience to elicit rich comparative data while minimizing the need to account for motivation.

Selected Learning Activities

Three of the learning activities developed during the summer workshop were selected for intensive study and follow-up support. It was agreed that these three activities were well-developed and well-spaced throughout the curriculum. The first two activities involve a birthday party business, called Perfect Party Place, which provides and sets up card tables for birthday parties. Each table is square and seats one child on each side. These square tables are arranged into a single rectangle at each party. Depending on the location, the tables must be set up differently each time. The first activity involves exploring the possible rectangular table arrangements for a party with 18 children and identifying patterns in table arrangements, dimensions, and areas. This scenario stipulates a fixed perimeter and requires an investigation of different areas that correspond to the perimeter of 18. The second activity involves exploring possible table arrangements for a party that can accommodate 24 tables and identifying patterns in table arrangements, dimensions, and perimeters. A connection can be made between the dimensions of the different tables and the factors of 24. The third activity, Rectangle Ratios, involves the exploration of similar rectangles. Students are given a collection of 14 rectangles and asked to sort them into "families". This is followed by exploration of ratios of length to width and how this relates to the idea of similarity of rectangles. In the results, we refer to the first and third activities.

Data Collection and Preparation

Journal Prompts

We asked each teacher to respond in writing to journal prompts at three points in time during the project: before the summer workshop, after the summer workshop, and at the conclusion of the project. In the first set of journal prompts, we asked Celia and Dee to write a math autobiography and describe some of their current teaching practices. In the second set of journal prompts, we asked Celia and Dee to comment on the effectiveness of the workshop activities. The final set of journal prompts asked for their reflections on the overall project as well as their impressions of how and why their knowledge, perspectives, and teaching practices had changed over the course of the project.

Classroom Observation Protocols

We both were present as participant-observers at each of several class sessions for each teacher, while she implemented three selected activities developed in the summer workshop. We took extensive field notes, structured as chronological records of all class activities during each observation session. After each observation, we reviewed and revised our notes, adding parenthetical remarks noting our impressions and questions to ask each teacher during post-activity reflective interviews.

Interview Protocols

Interviews were conducted with each teacher after each classroom observation to reflect on how well the activity went. In addition, joint interviews with both teachers were conducted after we had observed the same activity in both teachers' classes, to discuss common problems, activity revisions, and extensions. Interviews were loosely structured to allow free-ranging exploration of issues related to their practices, including mathematical content knowledge and pedagogical perspectives. We audio-taped each interview and took notes as well. Interview tapes were then transcribed.

Data Analysis

Two data matrices were designed to help organize the data, one for each teacher. These matrices focused on (1) mathematical knowledge and abilities, (2) perspectives, and (3) teaching practices at each of six different points during the professional development project: (a) pre-workshop, (b) post-workshop, (c-e) each of three activity observations, and (f) project conclusion. Each cell of each data matrix contained relevant evidence drawn from written responses to journal prompts, comments during interviews, and practices exhibited during classroom observation. These data matrices were analyzed for changes in each teacher's knowledge, perspectives, and practices over the course of the project.

Discussion

Over the year-long course of our professional development activities, these teachers exhibited significant changes in their knowledge, perspectives, and practices. Moreover, these three aspects of teaching seem to be interconnected in a fairly complicated way.

Knowledge

Pre-Workshop Knowledge

Over the course of getting to know Celia, she revealed some insecurity about her knowledge of mathematics. In an early journal entry, she commented, "In high school, I was a failure in math because I knew basic math well, but I could not comprehend the abstract in algebra." Celia's completion of a 40-hour professional development institute in fall 2002 and a special three-unit university course for in-service mathematics teachers in spring 2003 contributed to a significant broadening of her mathematical knowledge and abilities. In a later interview, she mentioned that these experiences resulted in greater confidence in her abilities.

Dee began her college career majoring in math. She took four quarters of calculus as well as abstract algebra and symbolic logic before switching to another major. Dee completed the same institute and university course as Celia during the 2002-2003 academic year, but in Dee's case, these experiences deepened and strengthened her mathematical knowledge and abilities. She commented in an early journal entry, "I was far better prepared to teach probability, ratio and proportion, and percent than before." In both her written and oral work, she appears to have a greater understanding of mathematics than the average middle school mathematics teacher.

Changes in Knowledge

Changes in their knowledge and abilities occurred during the professional development sessions and throughout the following school year. Their knowledge of mathematics broadened and deepened as they came to view mathematics, both concepts and processes, as a connected body of knowledge. For Celia, this growing understanding manifested itself as a desire for her students to experience mathematics as connected as well. This led her to search for ways to revise her curriculum and incorporate more algebraic reasoning and problem solving. The turning point for Dee was that, as her understanding of the connectedness of mathematics increased, she began to analyze more critically how and whether the textbook was actually addressing the state standards. She came to realize that she could not just depend on the textbook and began to rely on her own knowledge of mathematics and the standards to determine the curriculum that worked best for her.

Perspectives

Pre-Workshop Perspectives

Both Celia and Dee believed that, in order to address the state standards, they needed to teach from the textbook. They had both tried activities to supplement the book, with mixed opinions about the effectiveness. Celia did believe, however, that activities were important to include in her lessons. She commented in a journal entry, "I like to break up the routine and surprise them with some fun days. I just need more activities that support the text!" One of her goals at the time of the workshop was to develop an activity for each chapter of her text. In a pre-workshop interview, Dee conveyed great concern about the standards and the need to cover everything, but felt she didn't have a lot of time to "stray from the book." In addition, she felt that routine practice was valuable for her students, and was not convinced of the value of the regular use of activities in mathematics instruction.

Changes in Perspectives

For Celia, the intensive workshop was a catalyst for complete revision of her perspective on teaching. She began to rethink how, what, and why she was teaching, and decided to make major changes, starting with the review material in the first chapter. Instead of reviewing, she believed that a unit on problem solving would point her students in the direction she wanted to go for the rest of the year. She also came to view patterns as a central theme throughout the sixth grade mathematics curriculum. Furthermore, Celia, who was also teaching a seventh grade mathematics class, realized that the same activities she used for sixth graders could be adapted and extended for her seventh graders.

Dee, who has always been reflective about her teaching, began to reflect more upon effective ways of teaching. For example, during the Perfect Party Place activity, students created many table arrangements that were not rectangular. This unexpected outcome of the lesson led into a very nice discussion about the definition of a rectangle, whether a rectangle can have "holes," and how to find the area of the shapes that were made. This activity was an important one for Dee. She realized that, although her sixth graders were adept at applying the formulas for area and perimeter, these concepts had not been previously well developed. In a follow-up discussion of this activity, Dee commented how valuable this lesson was for her students, and although she had spent more time on the activity than originally planned, she strongly perceived the time to be well spent.

Teaching Practices

Pre-Workshop Practices

For Celia, a typical day began with warm-up exercises, followed by "giving notes, introducing an activity, or reinforcing concepts", after which homework would be discussed and students would start their work. Once each week, she gave a basic computation test that all 6^{th} – 8^{th} grade students took and once each week, she assigned a single word problem or puzzle problem for homework. Her recent completion of a special university course for in-service mathematics teachers led her to try some of the activities from that class with her own students.

For Dee, a typical day began with students checking answers from the previous day's homework, followed by students taking notes on a lesson she presented at the board, and finally working on practice problems and starting their homework. Dee gave three out-of-class projects over the course of the year. She has tried activities, but has usually reserved those for her "special projects" class, a supplementary math class for gifted students. Like Celia, Dee also felt that the university course she had just completed led to some changes in her teaching. In a journal entry, she commented, "I like giving students manipulatives now and understand what I expect them to do with them far better than before. I also liked the way we analyzed problems in Math 105, so I try to incorporate discovery teaching when time allows and it's a new topic for most students."

Changes in Practice

Three major changes in both of these teachers' practices emerged over the course of the project. First, both teachers began to focus more on problem solving, exploration, and activity-based lessons. When Celia started the school year with a unit on problem solving, she essentially set the tone for the year, and tied much of the subsequent content to problem solving. Algebraic patterns were a big addition to her teaching practice, due to her newfound confidence in her understanding of algebra. For both Dee and Celia, the inclusion of exploration and conjecture became important parts of their classroom cultures. This was evident in all three activities observed. For example, in the Rectangle Ratios activity, Celia more than once told her students, "You cannot answer this one wrong. How did *you* choose which rectangles went into each family?" while Dee asked, "Do you see patterns happening when you stack them [the rectangles] in this way? .... You need to make some decisions and write some generalizations down."

The second major change in practice that emerged was that both teachers began to seek out ways to implement these teaching practices rather than traditional lecture-and-homework practices. In more than one joint interview, both Celia and Dee commented on a desire for activities revolving around specific topics. In particular, the concept of fractions was a big concern for both teachers. This led to conversations about what other concepts in mathematics could be tied to fractions and what activities could be created to address this need. We felt this was a turning point for both teachers, because it indicated they were moving beyond the expectations of the professional development project and taking initiative to improve other aspects of their teaching not explicitly addressed in the professional development project.

The third change was that both Celia and Dee began to revise the activities in different ways to meet the needs of their students. In the Perfect Party Place activity, Celia emphasized the patterns that arose from the investigation. In fact, when reviewing this activity, she decided that she would make some of the algebraic patterns more explicit by asking her students to determine generalizations. Dee chose a different direction for this activity. As a result of the need for students to investigate areas of irregular shapes, she decided to include an extra day of exploration in this direction. We felt that this was a turning point for Dee, due to her prior convictions about lack of time and straying from the book. She was beginning to see that she could cover many topics in a single activity.

Interconnections of Knowledge, Perspectives, and Teaching Practices

For these teachers, the actual implementation of multiple learning activities they had helped to develop appeared to be the most significant factor in their changing knowledge, perspectives, and practices. Each activity presented unexpected difficulties for their students and elicited unexpected responses from their students. Coping with these issues in the classroom pushed their knowledge, perspectives, and practices beyond what they had experienced before and provided a rich set of experiences on which to reflect. Subsequent revisions to each learning activity completed the intended cycles of analytical thinking about their teaching practices.

Implications for Professional Development

We found the following professional development activities to be critical in effecting significant change for these two teachers. A vital role of professional development is to facilitate and support these activities.

1) Teachers need to engage in the entire process of design, implementation, reflection, and revision of instructional materials in their own classrooms. It is not sufficient for teachers to see this process modeled in a professional development workshop/institute. They must be involved in all phases in this process.

2) Learning activities and curricula must be designed to complement teachers' current textbook materials and standards, yet be adaptable in more than one way to meet differing needs. The key to this adaptability is to design activities that connect multiple concepts. Pedagogically, this allows for multiple entry points and multiple extensions. Mathematically, this maximizes the impact of each activity.

3) Reflections must focus on comparisons of actual experiences with different teaching practices. It is not enough for a teacher to compare a learning experience in a professional development institute/workshop to his or her current teaching experiences. It is the implementation of alternative teaching practices that sheds new light on current teaching practices, hence fueling the reflective process in a powerful way.

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EXAMINING ASYNCHRONOUS AND SYCHRONOUS COMMUNICATION IN AN ONLINE MATHEMATICS TEACHER EDUCATION COURSE

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Current trends in higher education suggest a rapidly expanding interest in the development of computer-based distance education. Lacking is empirical research on Computer-Mediated Communication to help educators develop appropriate online learning environments. This study investigated types of participation that transpired during 4 weeks of discussions in an online mathematics teacher education course, 2 weeks involving discussion boards and 2 involving chat sessions. Discussions were coded with respect to 4 substantive and 5 non-substantive categories, and comparisons were made between asynchronous and synchronous environments (i.e., discussion boards and chat sessions, respectively). Surveys of students' perceptions were also collected. Findings revealed similarities and differences between the environments that suggest the integration of both for more effective instructional use of computer-based distance education.

Introduction

Interest in distance education, particularly involving web-based learning environments, is increasing in higher education and includes such well-established institutions as Harvard, Yale, Stanford and others (Carr, 2000). The Center for Education Statistics recently reported that "distance education appears to have become a common feature of many postsecondary education institutions and that, by their own accounts, it will become only more common in the future" (Lewis, Snow, Farris, Levin, and Green, 1999, p. vi). Additionally, computer-mediated communication [CMC] is seen as having the potential for meeting the professional needs of teachers, moving the profession "from one of isolation and individualism to one of professional community" (Brown and Koc, 2003, p. 2-146). In developing an online learning community, it is arguably important to engage participants in the interchange of ideas and the development of their social presence within the environment. The exchange of ideas through CMC may occur in two formats: (1) synchronous, happening at the same time from different locations, and (2) asynchronous, occurring at different times from different locations. Developers of online learning environments often suggest that asynchronous communication is preferred over synchronous because of advantages such as providing more equal opportunity for participation among learners and allowing more time for reflection (Driscoll, 1998); however, research-based articles to support these assumptions are lacking. On the other hand, literature considering the social presence of individuals that may be promoted through asynchronous written communication characterizes this form of communication as weakest among CMC formats (Newberry, 2001). In order to more effectively design online courses for the professional development of mathematics teachers, it is important to investigate the types of communication taking place through each CMC format and participants' perceptions of these formats based on their experience.

The present study sought to compare the merits of synchronous and asynchronous CMC within a mathematics teacher multicultural education course. The focus of the investigation was on the comparison of discussions engaged in by participants in each mode and the perceptions of participants concerning each format. Both synchronous and asynchronous communications were carried out within Blackboard, a web-based instructional delivery tool designed to support the development and implementation of online distance education. Blackboard facilitates the development of course websites through which all instruction, communication, and assessment may occur. The asynchronous communications were in the form of threaded discussions on discussion boards within Blackboard and the synchronous communications were conducted through online chats facilitated primarily through Blackboard and, for one group partly through Yahoo (this group had extreme technical difficulties chatting through Blackboard).

Rationale and Perspectives

Recent literature suggests a fast expansion of interest in online education. Articles discussing best practices in CMC environments and the potential for deeper understanding of content by students proliferate the literature in this area. Additionally, studies investigating the technical features of these media have attempted to connect these features with problems of instructional design for web-based instruction. Very limited, however, are empirical studies investigating the quality of online professional communication. In particular, there is a need to understand the quality of discussions and participation that take place on discussion boards and in chat sessions; in order, to inform best use of these modes of communication for online instruction.

In traditional, face-to-face teacher education classrooms, discussion is thought to be a powerful method for learning through interaction with other prospective or inservice teachers. Instructors use discussion strategies to "draw out our students' opinions, prior knowledge and experience upon which they construct new knowledge" (Bowman, 2001, p. 2). Potentially, the same is possible for online instruction. As with face-to-face discourse, online communications may be analyzed for types of participation in order to examine patterns of communication that mediate the learning taking place. This analysis may take place for each synchronous and asynchronous discussions in order to better understand discussion patterns in each mode and the potential learning through each.

Types of participation in online discussions can be characterized as substantive and non-substantive (Davidson-Shivers, Muilenburg & Tanner, 2001). Substantive statements relate directly to the discussion topic or content and can be divided into 4 subcategories: (1), structuring, statements that initiate or focus attention on the discussion topic; (2) soliciting, questions, commands or requests attempting to solicit a response; (3) responding, a statement in direct response to a solicitation; and (4) reacting, a reaction to a structuring statement or another's comment that is not a direct response to a question. Non-substantive statements are messages that do not relate to the discussion topic or content and can be divided into 5 subcategories: (1) procedural, including scheduling information, announcements, logistics; (2) technical, including computer-related questions, suggestions; (3) chatting, including personal statements, greetings, jokes, introductions; (4) uncodable, including statements with too little information or unreadable; and (5) supportive, including statements similar to chatting but with an underlying positive reinforcement. The use of these categories to analyze synchronous and asynchronous discussions taking place within chat sessions and discussion boards, respectively, for the same group of participants will provide valuable insight on each format. This insight will help inform further developments of online courses for teachers.

Methods

Participants

Participants for this investigation were 15 students enrolled in a graduate level mathematics education course. The course focused on issues of multicultural mathematics education and ethnomathematics. Three fifths of the students were female and three-fifths of the students had experience with at least one online course. All had experience with the internet. Additionally, two-thirds had K-12 mathematics teaching experience. During the first week of the course, this information along with the participant's reactions to the statement 'Mathematics is culture-free' were used to form heterogeneous groups of three among the students. These groups remained intact throughout the 12-week course as students were engaged in asynchronous and synchronous communications. Each week the small group members were expected to engage with each other in course related discussions, as well as engaging in discussions with the broader course community.

Course structure, Data Collection and Analysis

The course was structured so that each participant was part of the large community of all class members and a small community of three expected to communicate with one another on a weekly basis. The course structure was established to foster relationships and individual's social presence within the online community, as well as to provide course and technical support among the students. On a weekly basis, students discussed course readings, videos, and assignments within a particular topic related to multicultural mathematics education. Discussions were initiated through instructor posed items on weekly discussion boards. The small groups were used to facilitate the development of each individual's social presence in the course by requiring every student to complete weekly computer-mediated communications with their group members, as well as responding to other participants of their choice from among the entire class. Also used to foster the social presence of each class member were student pictures collected and posted on the course blackboard site during the first and second weeks of the course.

Throughout the first six weeks, threaded discussions of weekly topics took place only on whole class discussion boards. Then during the next six weeks, chat sessions of weekly topics were conducted by students within their small groups, summarized by group members, and posted to the class discussion board for the week. The data collected for this investigation included transcripts of discussion board postings, transcripts of chat sessions, and an online surveys of students' perceptions of several aspects of the course. The end of the semester survey was used to provide supplementary information that would put in context each student's participation in the discussion boards and chat sessions. For example, a few groups had technical difficulties with the chat sessions. It was important to understand their perceptions of those difficulties since these might influence the participants' contributions and participation in the discussions.

Transcripts for two weeks of discussion board postings and two weeks of chat sessions were analyzed for this investigation. The data analysis involved unitizing communications and characterizing them with respect to types of participation. Complete thoughts including partial sentences, single complete sentences or several sentences corresponding to distinct types of communication were unitized and coded as one. The CMC discussions were first categorized for members of each small group. Patterns were sought across group member communications within each mode of communication (synchronous and asynchronous) and compared to individual members' and class perceptions about each mode of communication gleaned from the surveys. Second, the findings were compared across groups and between groups to arrive at patterns of participation that mediated the communication in each mode and, in turn, the potential learning. As we analyzed the data for each mode of communication, we found that many of the chatting communications during the chat sessions had to do with remarks, jokes or individual comments expressing frustrations or difficulties with the technology (e.g., "got knocked off" or "this comp is a piece of junk. I can't even scroll up and down the chat page").

We created a category, technical chat, separate from both the chatting and technical categories that would capture this type of chatting. This type of chatting arose out of technological difficulties with the chat sessions on Blackboard and would not typically be found in other course discussions using other modes of communication. By coding this communications separately, we were able to consider the extent of chatting devoid the technical difficulties specific to Blackboard chat sessions, as well as consider the extent of chatting produced by these difficulties within the Blackboard environment.

Results

Based on analysis of the transcripts, both modes of communication produced statements in all substantive and all but two non-substantive categories (see Figure 1). Procedure and uncodable communications were found in the transcripts of the chat sessions but not the discussion boards. Both modes of communication resulted in more substantive statements than non-substantive as was found by Davidson-Shivers, Muilenburg, and Tanner (2001). The highest frequency of communications for both modes were found within the reacting category and the least frequency was found in the structuring category. Within the reacting category, participants discussed points they agreed with as well as points with which they disagreed. Although there appears to be more reacting communications in the chat sessions than in the discussion boards, in the discussion board mode these communications tended to be longer and contain more complex and complete thoughts explaining why they agreed or disagreed with another's statement and how it related to their own experiences. In the chat sessions, the communications tended to be shorter with more interactions between discussants. More communications seemed to be needed during a chat session to convey a similar point to that on the discussion board.

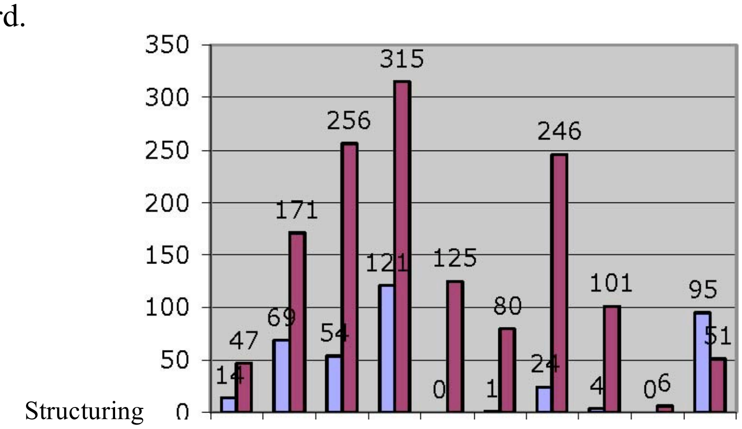

Fig. 1: Types of Communication on Discussion Boards and Chat Sessions

With respect to communicating their ideas in both asynchronous and synchronous modes, 80% of the students agreed that they felt equally comfortable expressing their thoughts, including disagreements with other's comments, in both the discussion boards and the chat sessions, with another 20% feeling neutral. This was important given that 20% of the participants felt more comfortable expressing their thoughts in the online modes of communication than in similar face-to-face classes and 40% felt they were able to share more of their thoughts in the online discussions than in similar face-to-face discussions. Both discussion boards and chat sessions provided students with environments to share their views, comments, and questions more freely than in face-to-face discussions.

The asynchronous nature of the discussion boards provided more time for students to think about and record their responses. The participants, at times, supported their statements by referencing particular passages in the readings, referencing other literature and attaching documents to their statements that others could read, or providing complete references for articles or websites others could explore through the internet related to the point being made. These features of the communications have the potential for generating alternative points of view and deepening students' understanding as they progress in developing their knowledge of a topic. With respect to the soliciting and responding categories, the data reveal that in the discussion board mode some solicitations went unanswered. Figure 1 reveals more soliciting statements than responding statements for the asynchronous mode of discussion. On the surveys, 100% of the participants indicated that they typically checked back to see if someone had responded to their postings. Additionally, 60% were disappointed if they did not receive a response or some acknowledgement on their postings and 80% felt it was important to read and respond regularly to others' postings. Unanswered questions or statements that were unacknowledged or reacted to may influence individuals' perceptions of their social presence in the community and influence their learning from the discussions. The discussion board communications regularly included supportive statements. These tended to be of the form "Once again, great comments!" or "Thank you for your insight! ☺" and seemed to be a method the participants used to acknowledge the value of others' comments and their presence in the community. The transcripts of the discussion boards revealed more supportive statements than during the chat sessions. Additionally, as would be expected, the participants appreciated the flexibility in joining the discussions provided by the asynchronous mode of communication.

In contrast to the asynchronous mode, the synchronous communications provided a format for participants to receive immediate feedback to their questions or statements. During chat sessions, all communications tended to be shorter in length and not as well developed as those in the discussion boards. However, in chat sessions the students' statements were typically responded or reacted to (or at least students thought they were heard or read) as opposed to during the discussion board communications where a comment might remain without a response and thus a student might not know if it was read and considered by others. For the chat sessions, unlike the discussion boards, the frequency of postings in the soliciting category was much less than the responding category and no questions raised during the synchronous discussions were unacknowledged. Questions raised tended to be followed by several responses and allowed for students' immediate clarification of their understanding through an on the spot give and take. The surveys revealed that 67.6% of the participants felt the discussions during the chat sessions were more valuable than on the discussion board. Lara, Howell, Dominguez, and Navarro (2001) similarly found that their participants tended to prefer the synchronous interactions to the asynchronous interactions; however, they had 100% agreement. In addition to facilitating participants' understanding of the content, the immediate responses and feedback to their questions or statements provided greater opportunity for individuals to connect with others and develop a social presence within the online community. The chat sessions, more so than the discussion boards, seemed to provide individuals with a greater sense of social presence in the course through the immediate connections to other class members. In this environment, there were more interchanges between participants; thus providing more opportunities to connect intellectually or socially with other individuals in the course. A difficulty arising from the rapid interchange of ideas was the development of multiple discussions at one time, making it difficult for participants to follow the development of each individual discussion. When this occurred for extended exchanges, members of the small group would call attention to the multiple discussions in order to redirect the group to a single discussion.

Figure 1 shows that more soliciting remarks were expressed during the chat sessions in comparison to the discussion boards. From the transcripts soliciting remarks appeared to be similar in complexity and length across both modes. Despite the technical and scheduling difficulties experienced by each of the small groups as part of the synchronous discussions, 60% of the participants felt the discussions during the chat sessions were worth overcoming any difficulties with only 13.4% disagreeing. These participants experienced extreme difficulties joining the chat sessions. Students felt that the chat sessions were most valuable in connecting with their colleagues and receiving instant feedback. During chat session communications, the participants asked each other many questions related to one another's perceptions and experiences. These sessions seemed to promote a more social presence for the students than the discussion board communications.

The chat sessions included more non-substantive statements than the discussion boards (see Figure 1). In particular, the chatting was more extensive in the chat sessions and revealed different aspects of the participants' personalities, including their sense of humor, outside interests, and aspects of their personal lives. The technical chat communications arose mainly in the chat sessions due to difficulties students were experiencing with the technology and their use of comments including humor to easy the frustration or apologies for their technology mishaps (e.g., 'Sorry, I was cut off, but I'm back," "Come back Rita…Come back," or "So we're all slow pokes together. ;-)"). Additionally, procedure and technical communications were observed primarily during the chat sessions and almost non-existent during the discussion boards. The chat sessions required the students to arrange the logistics of the discussions, including the discussion times and roles of the participants. The technical communications were concerned primarily with questions and responses related to maneuvering within the chat session environment, including recording and accessing archives of the discussions.

Concluding Remarks

The results reveal the importance of integrating both types of communications in online learning environments. The discussion boards added depth to students' understanding by providing more time for thought on others' comments before responding or reacting and allowing students to investigate and reference other resources that support their views or add ideas to the discussion. The chat sessions, on the other hand, provided students with opportunities for immediate clarification in areas they had questions or on the spot reactions to their statements that helped them develop alternate views through on the spot exchange of ideas. The chat sessions also seemed to add a more social dimension to students' participation by inclusion of less formal communications and more personal connections through the chatting that took place. Although developers of online learning environments taut the merits of asynchronous communication over synchronous (Driscoll, 1998), this study demonstrated that both environments add to student learning in a somewhat complementary fashion. Developers of online environments for the professional development of mathematics teachers should consider the use of both synchronous and asynchronous modes of communication for enhancing the richness of the learning experience for their participants.

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INCREASING THE LEADERSHIP CAPACITY OF ELEMENTARY SCHOOL TEACHERS IN MATHEMATICS

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To implement educational reforms, teacher leaders are needed to redirect conversations around student thinking, create environments of sustained professional inquiry, and offer professional development for colleagues. This study describes the development of seven teacher leaders. Data were collected over an 18-month period of time and included: monthly reflections, interviews, observations, documents, and my field notes. Using a modified ethnographic methodology, data were analyzed using constant comparative methods and matrices to discern patterns of leadership. Four patterns of teacher leadership emerged from the data analysis and suggested a model of teacher leadership capacity that describes how teachers influence their colleagues. This model can help professional developers design experiences that increase the leadership capacity of teachers.

Reports from NCTM (1980, 1989, 2000) and other influential organizations (National Commission on Excellence in Education, 1983; National Research Council, 1989; United States Department of Education, 1999) recommend that classroom teachers rethink their goals and practices of teaching mathematics. To change teaching practice requires supportive school structures, collaborative relationships between teachers, and high quality professional development programs that guide and support teachers' growth (Barth, 2001; Louckes- Horsley, Hewson, Love, N., & Stiles, 1998). High quality professional development that is designed to meet the unique goals, needs and constraints of schools requires increased leadership capacity (Frechtling, 2001; Fullan, 1994; Nesbit et al., 2001). To meet the growing need for this new kind of professional development, teacher leaders must be developed to support the growth of teachers and the implementation of mathematics reforms.

Ferrini-Mundy and Graham (1997) and Friel and Bright (1997) suggested that research is needed to understand the transition teachers make when they move from living their professional lives within the walls of their classroom to enlarging it as they assume leadership responsibilities. Research studies (Miller & O'Shea, 1992; Smylie & Denny, 1990; Wasley, 1991) on teacher leadership describe the leadership roles and characteristics of teacher leaders. Snell and Swanson (2000) and Wilson (1997) illustrated the complexities of teacher leadership as teachers increase their sphere of influence. However, researchers have not reported on the evolution of teacher leaders as they participate in professional development designed to increase their leadership capacity. This study was designed to fill that void in the research literature. It sought to describe the leadership roles that teachers assumed in their educational communities by describing the ways that teachers exhibited leadership and how they identified the influence they had upon colleagues while participating in a leadership institute.

Theoretical Framework

Bruner (1960) described learning as a process of interpreting, negotiating, and recreating ideas derived from our cultural past. As individuals work together, they share previous experiences, collaborate with each other, and learn. Learning can be thought of as a process by which individuals procure knowledge with tutelage through participation in social activities, and an individual's development of knowledge is demonstrated through his or her competence in cultural activities (Cobb, 1995). Meaningful learning intertwines culturally defined knowledge with the personal sense. As individuals with limited knowledge assimilate culturally accepted meanings and practices, they gain respect and acceptance by their community (Lave & Wenger, 1991).

Lave and Wenger (1991) depicted evidence of learning as the increase in an individual's participation in the actions of the group as they move from peripheral involvement to the center of the action; the term associated with this is legitimate peripheral participation (LPP). LPP provides a framework for examining learning as a situated activity within a community and an enactment of socially valued actions. The LPP framework implies a complex social world in which the individual's role within the community depends on his or her level of participation (Lave & Wenger). All of the members in a community are considered legitimate, but novices act as peripheral members while they learn the skills necessary for full participation. From this perspective, those who participate, learn.

This research study examined how participation in a leadership institute as a community of practice influenced the leadership roles that teachers assumed in their educational community. Teachers' learning was situated in social activities as they collaborated with each other to examine teaching practices and to create professional development presentations. LPP provided the framework for interpreting the ways teachers used recommendations from professional developers and enacted new leadership roles in their school communities. These new leadership roles were interpreted as evidence of learning and indicated an increase of teachers' influence in their educational community.

Methods

New forms of ethnography emerged as different disciplines adopt it as a research methodology (Tedlock, 2000). A modified ethnographic methodology permits a researcher to select data sources, create collection methods, and report findings to answer focused research questions without requiring full immersion in a culture for an extended period of time (Kincheloe & McLaren, 2000). Researchers using ethnographic methods respect the participants' perspectives by honoring their voice through shared responsibility for data collection and interpretation. To understand the enactment of leadership by teachers, a modified ethnographic methodology was constructed to give voice to the participants' interpretations of their influence. Teachers wrote reflections and used charts to record and interpret their leadership activities. Data were analyzed using qualitative data reduction methods (Maykut & Morehouse, 1994; Miles & Huberman, 1994), such as constant comparative methods, matrix analysis, and comparative methods. The analysis of the data was interpreted using the perspective of LPP (Lave & Wenger, 1991) to describe the changes of teachers' influence in their educational community.

Data sources

Ten elementary school teachers from different schools and grade levels joined a teacher leadership institute to develop their leadership capacity while participating in a mathematics systemic change institute. Seven teachers from this institute agreed to participate in this study. These teachers had different prior leadership experiences that ranged from no reported leadership activity to regularly providing professional development sessions for colleagues. To investigate the leadership roles assumed by each teacher participant, four major data sources were used as available. They included: (a) teachers reported their leadership roles on questionnaires, (b) interviews conducted with each teachers' principal, (c) documentation of teacher-led professional development sessions, and (d) observation of two professional development sessions. These multiple data sources were used to provide triangulation (Miles & Huberman,

1994) for developing a holistic understanding of how teachers develop their leadership capacity and were collected during an 18-month period of time.

Results and Conclusions

The perspective of LPP presumes that individuals demonstrate their learning by new actions and the new leadership roles that teachers accepted were interpreted as evidence of their learning. Two of the teachers had no prior leadership experience, three of the teachers assumed a small leadership role within their school as grade level representative or content coach, and two teachers held school leadership role and presented workshops at the district or state level. During the 18-month leadership institute, the teachers had opportunities to deepen their pedagogical content knowledge by participating in a lesson study and to explore school change through teacher leadership. They planned professional development workshops for colleagues and for state and regional conferences. I first discuss the leadership patterns that emerged from this study and then propose a model to describe the actions of teachers that indicated an increase of influence in their educational community.

Two data sources were utilized to describe teacher leadership patterns. The teachers identified their leadership roles and described how they influenced others in their educational community. The principal of each teacher described the leadership roles that the teacher assumed and her influence on other staff members. These formal and informal roles were further analyzed using a time-ordered matrix. Cross-case analysis of these roles over the 18-month period of the institute revealed four patterns of teacher leadership. These leadership patterns are described as humble, reluctant, overwhelmed, and former leaders. Based on the patterns exhibited by teachers in this study, each pattern of teacher leadership is described.

Humble leaders listen quietly to their colleagues and build professional relationships through respect. They are committed to implementing and supporting mathematics reform recommendations, take advantage of opportunities to develop professionally, and perceive leadership opportunities within their schools. *Reluctant leaders* establish themselves in hidden leadership roles with low visibility. These roles enable the reluctant leader to function independently in positions that are valued by principals but are not sought after by peers. Reluctant leaders are sensitive to the opinions of their peers and a few negative comments can cause them to reconsider their leadership roles. When reluctant leaders receive positive feedback, their confidence grows and they continue to develop their leadership capacity. Based on the pattern exhibited by one teacher in this study and phenomena reported by other researchers (Zinn, 1997) *overwhelmed leaders* assume responsibilities from a number of sources only to discover that they compete for attention and time. These sources include: (a) classroom responsibilities, (b) school committee work or extra duties, (c) family responsibilities, (d) school district responsibilities, (e) community volunteer work, and (f) graduate work. With responsibilities that extend from the classroom to school and family, overwhelmed leaders eventually withdraw from leadership responsibilities and may resume their leadership activity when demands on time are reduced. *Complacent leaders* are confident teachers with considerable knowledge, skills, and experience creating presentations to parents and teachers. They are described as accomplished teachers who can be relied upon for support, advice, and help. However, two of the teachers who were identified as leaders in their school challenged outside experts and did not take advantage of opportunities to develop their own personal growth. Both of these teachers were resistant to critical reflection and did not develop beyond their previous leadership roles.

The four leadership patterns identified in this study indicate that as teachers gain leadership capacity, they broaden their sphere of influence within their educational communities. I have concluded from these data that the characteristics of teachers' influence comprise the foundation of a model that describes teacher leadership capacity. The proposed model, presented in Figure 1, indicates the level of influence that teachers portray as evidenced by their actions.

Leadership Capacity	Characteristics of Teachers' Influence
0	• No demonstrated leadership in an educational community
1	• Informally shared ideas with colleagues • Coordinated special school activities • Joined school or district committees • Disliked making presentations and avoided opportunities
2	• Engaged colleagues in conversations about mathematical ideas • Made mini presentations at faculty meetings • Planned and presented professional development sessions with a partner • Developed a professional relationship will a colleague Informally discussed reform mathematics ideas Informally mentored a peer new to the teaching team
3	• Planned and presented professional development sessions independently • Recognized leadership opportunities and assumed responsibility for a new role School study group Scheduled on-going team meetings to discuss mathematics reform Mentor for novice teachers • Increased self-confidence

Figure 1. Model of Teacher Leadership Capacity.

Analysis of the four patterns of teacher leadership indicated that, as teachers gain leadership capacity, their influence in their educational community changes. In this study, five of the seven teachers assumed new informal or formal leadership roles and these actions indicated the learning of new skills and knowledge to support mathematics reform. These skills and knowledge included understanding the change process, developing a reflective teaching practice, and deepening pedagogical and content knowledge by participating in a lesson study. The two other teachers, described as complacent leaders, continued to provide school leadership in previously held roles but they did not assume any additional leadership roles. The absence of new leadership action was interpreted to indicate that these two teachers had not learned new skills or knowledge. The model of teacher leadership capacity depicts characteristics of teachers' influence in their educational community and suggests that many teachers who may not be initially recognized as leaders have the potential to develop their leadership capacity and influence their colleagues to implement reform mathematics recommendations.

Implications

This study has the potential to contribute to the field of professional development in several ways. The first is by informing professional developers as they work toward increasing the leadership capacity of teachers within school districts to support mathematics reform. It suggests that professional developers select teachers who engage in self-reflection about their teaching practices for leadership development. In this study, those teachers who were described as humble and reluctant leaders were not the individuals who immediately came to mind as strong leaders in the educational community but they increased their leadership capacity. Teachers who may not be initially recognized as leaders have the potential to develop their leadership capacity and influence their colleagues in implementing reform mathematics recommendations. Second, the model of leadership capacity describes patterns of leadership roles that teachers assumed as they developed their leadership capacity. It suggests leadership experiences that professional developers can provide to scaffold the development of teachers' leadership capacity.

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IMPROVING STUDENTS' MATHEMATICAL UNDERSTANDINGS: AN EXPLORATION INTO TEACHER LEARNING IN AN URBAN PROFESSIONAL DEVELOPMENT SETTING

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This theoretical examination of teacher learning within a professional development experience for elementary mathematics teachers in urban charter schools aims to continue the conversation about supporting teachers in their efforts to teach mathematics in conceptually based ways. Although research has identified components necessary to include within professional development opportunities, how these experiences should be envisioned and their impact on teacher change, and eventually student learning, have not been fully explored (Fishman, Marx, Best, & Tal, 2003). This work extends this underdeveloped area of research in two substantial ways: (1) in our effort we have implemented recommended facets of professional development through the lens of content and pedagogy, and (2) we have situated this work in an underrepresented context (urban, charter school).

This paper theoretically examines teacher learning based on our efforts to create and study a professional development experience for elementary mathematics teachers in urban charter schools. Although research has identified components necessary to include within professional development opportunities, how these experiences should be envisioned and their impact on teacher change, and eventually student learning, have not been fully explored (Fishman, Marx, Best, & Tal, 2003). Within urban charter schools empirical research related to effectiveness of professional development is glaringly missing, suggesting a rich investigation context. Charter schools are often plagued with uncertified staff, high teacher turnover, and no clear vision of how to achieve high student outcomes (Carver & Neuman, 1999). Our goal is not to enter the debate surrounding the importance of charter schools, but to explore ways to support teachers in efforts to align practice in ways that improve students' mathematical understandings.

This paper explores the conference theme, building community connections, in two ways: (1) we examine the interplay between research and practice by connecting our research-based professional development to teachers' efforts to implement learned practices, and (2) we have designed an interdisciplinary, professional development experience that is based on research related to mathematics, teacher learning, and general pedagogy. In examining these two areas, we aim to create a map that will aid others in developing on-site, long-term teacher education programs.

Theoretical Perspective

Relevant research in the areas of mathematics teaching and learning, and teacher development and change, provide a framework for our discussion of how to structure professional development experiences for elementary mathematics teachers, particularly those in inner-city settings. It is our belief, and the finding of many studies, that effective professional development can be an invaluable foundation for high-quality, reform oriented teaching that leads to improved student learning and achievement (e.g., US Dept. of Education, 2000). Furthermore, teachers serve as the primary catalyst for change in students' learning (Borko & Putnam, 1995).

National (e.g., NCTM, 2000) recommendations call for an approach to mathematics teaching that allows students to communicate, problem solve, and engage in conceptual mathematics. This shift toward inquiry-based instruction assumes teachers view mathematics as a tool for thought, rather than a set of rules and procedures to be memorized. However, teachers are unlikely to make adjustments in their thinking without intervention and deliberate support (Richardson & Anders, 1994). Given this understanding, professional development experiences must intentionally provide experiences that will assist teachers in learning new ways of thinking about mathematics and its teaching (Farmer, Gerretson, & Lassak, 2003).

It is well substantiated that teachers' knowledge and beliefs have a strong influence on classroom practices and ultimately student learning (Thompson, 1992). To help teachers develop new practices the connection between content knowledge, beliefs, and pedagogical knowledge must be considered (Stipek, Given, Salman, & MacGyvers, 2001). Knowledge of subject matter and general pedagogy, filtered by teachers' conceptions, help form pedagogical content knowledge (Marks, 1990); therefore, all types of knowledge must be included within professional development programs.

Isolated, short-term staff development has proven inadequate for effective school reform and improved student achievement (e.g., Darling-Hammond, 1997, 1999). This approach does not allow teachers to address misconceptions, construct new orientations, and learn to teach for understanding. Teachers often report these one-shot workshops to be irrelevant. They forget 90% of what they perceive they have learned (Miller, 1998). Hence, to facilitate growth in teachers' knowledge and beliefs professional development interventions must be long-term and incorporate teachers' understandings.

Research suggests the following be included in successful professional development efforts: (1) university and school collaborative partnerships, in which teacher educators play an important role in the development of teachers' thinking and independence (Little, 2002; Putman & Borko, 2000), (2) opportunity for teachers to reflect in a collaborative format (Farmer, Gerretson, & Lassak, 2003), (3) guided help with the study of curriculum, assessment, and instruction (Newmann, Secada, & Wehlage, 1995), (4) modeling of practices that promote effective student learning, and (5) opportunities to negotiate learning within the context of the teachers' own practice and classroom (Wilson & Berne, 1999). Within professional development for elementary mathematics teachers, we must therefore include challenging mathematics learning experiences complete with opportunities for teachers to reflect on practice within the context of their teaching (e.g., Farmer et. al, 2003).

Designing professional development becomes increasingly complex when layered with issues specific to inner-city settings. In urban schools, teachers often avoid teaching that requires students to use higher-order, critical thinking (Walker & Chappell, 1997). Given the focus on problem solving in reform oriented approaches to learning mathematics, this propensity towards procedural mathematics does not provide students with learning experiences that can allow them success on required, high-stakes tests. As Walker and Chappell (1997) state, "The question is not whether urban school students can or cannot achieve mathematical skills; rather, it is which *means* will elicit maximum success in mathematics" (p. 202). Our effort thus examines the interplay between the mathematical concepts taught and the views of mathematics embedded within each individual teacher's practices.

In the following section we use one urban, charter school as an exemplar case, and discuss professional development efforts that are being implemented and studied to change teaching practice and students' ability to demonstrate understandings both within the classroom and on standardized assessments.

An Exemplar Case

This charter school (School C) is located downtown in a large city. The majority of the student body is minority (99.8% African American) and considered underprivileged (according to state records). School C (grades K-5) articulates a focus on increasing student achievement, preparing students for post-secondary educational and/or workforce experiences. Our initial conversations with faculty indicated they are dedicated and concerned about their students' learning, and interested in on-going professional development with our university, particularly in the area of mathematics. Many full-time teachers have not yet completed state-level certification, including passing the required state basic mathematics examination.

Although our observations suggest a positive learning environment and student/teacher ratio (approximately 22-1), this school has been unable to make any gains in standardized mathematics assessments, with all of last year's scores being below state average and lower than in previous years. Aware of the acute high-stakes testing situation, this school has made a proactive attempt to improve scores through isolated, short-term in-service presentations, with no documented or visible results. For example, one observed presentation involved an educational consultant demonstrating for fourth and fifth grade students, songs that were to help them memorize multiplication facts. Based on research in mathematics education, this activity will not lead to changes in achievement. Although charter schools are being held accountable at the same level as their district counterparts, most professional development done in these schools is initiated by for-profit groups with little or no research supporting their effectiveness (Ascher, Jacobowitz, & McBride, 1999; DiLorenzo, 1996). What is clear in the research on mathematics teacher education is that without on-going professional development that addresses teachers' understandings of mathematics and supports their efforts to improve practice within their own classrooms, no gains can be made in students' mathematics achievement (Ball, 2000).

Our effort at School C is based on current research, as well as what we have learned about the needs of teachers and students at this particular school. Two existing mediating factors are salient: (1) High teacher turnover (61% per year) and a large percentage of the teachers are either uncertified (64%) or hold emergency credentials (40%); additionally, only 10% have graduate degrees. Thus, we designed this professional development effort to allow teachers to work toward advanced degrees and certification. (2) The majority of students are unable to pass the mathematics portion of the State's high-stakes standardized assessment (over 75% in 2003), and in the past two years the failure rate has continued to increase. Given this urgent situation all professional development now targets weak concept areas as noted on assessment results and helps teachers to modify instruction that facilitates student conceptual understanding of this mathematical content.

To assist this school with improving student understanding and achievement, we have developed an alternative, long-term approach to professional development appropriate to the needs of these teachers and students. Key components of this experience include: (1) An intensive, week-long summer institute that immediately emerges teachers into a content-focused program, engages teachers in school-based dialogue that will help build community, and allows teachers to earn continuing education credit, (2) Courses that work with teachers on their content, pedagogical content knowledge, and how to implement mathematics curricula in their own classrooms, (3) Collaboration with teachers surrounding a focus on students' improved conceptually based mathematical understandings and achievement, (4) Bi-monthly workshops that provide teachers a community that generates on-going intellectual and emotional support through conversations surrounding practice, and (5) Modeled on-site practice within teachers' classrooms. With this experience we are facilitating teacher learning in collaboration with teachers supported by current research in the areas of teacher learning and mathematics education.

Implications for Theory and Practice

In light of this school's articulated goals and what research recommends for long-term growth in mathematics teaching and learning, we have initiated experiences that work with teachers on their content, pedagogical content knowledge, and how to implement mathematics curricula in their own classrooms. As university faculty in mathematics and education, we have collaborated to create and research professional development that is long-term, done with teachers directly and students indirectly through practice-based discussions and research, targeted to specific areas of mathematics in conceptually based ways, and tailored to the needs of the articulated specific needs of teachers and students. As noted previously, university/school partnerships provide a ripe context for implementing these types of efforts.

With this work we aim to contribute to what is understood about teacher learning by using the lens of development of teachers' conceptions about mathematics and its teaching to foster change in practice. Although much has been learned over the past two decades related to teachers and teaching, there is a lack of research that explicitly constructs empirically-based theories of teacher learning. This work extends this underdeveloped area of research in two substantial ways: (1) in our effort we have implemented recommended facets of professional development through the lens of content and pedagogy, and (2) we have situated this work in an underrepresented context (urban, charter school). This work is part of our endeavor to connect theoretical knowledge with practice. By situating this discussion at the intersection of mathematics education, teacher learning, and professional development literature, we have generated a working model for facilitating inner-city student achievement in mathematics, thus advancing what is understood the conversation about elementary mathematics professional development.

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POSITIONING TEACHERS TO ENACT STANDARDS-BASED INSTRUCTION

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This study takes a reflective look on a professional development program in which grades 4-8 mathematics teachers from nine rural schools were introduced to the PSSM. The ways in which the teachers received and reacted to the PSSM and changed their beliefs and classroom practices are addressed by measuring teachers' progress along three Indicators of Influence, namely Calculator Technology Indicator, Mathematics Learning Indicator, *and* Problem Solving Indicator. *Results indicate some statistically significant changes in the beliefs of the teachers along each indicator and changes in classroom practice with regard to the calculator technology indicator.*

A reason often postulated for the lack of success in some efforts to implement standards-based mathematics curriculum programs is that teachers generally have not been adequately prepared for this endeavor (Hiebert, 2003). Especially for inservice teachers, such preparation requires that one is aware of many critical variables including teachers' beliefs and conceptions (Thompson, 1992) and teachers' overall thinking about mathematics as a discipline, mathematics teaching and learning, and their individual mathematical growth. In fact, Mewborn (2003) asserts that "teachers' thinking needs to be at the center of professional development sessions just as children's thinking needs to be at the center of mathematics instruction" (p. 49). Thus, the more providers of professional development know about the beliefs and conceptions of teachers, the better they can craft a program that places teachers in a position to effectively carry out standards-based practices in their mathematics classrooms.

Purpose and Framework

This report takes a reflective look on a professional development project designed to help lead grades 4-8 teachers to more standards-based mathematics teaching. A framework developed by the National Research Council [NRC] (2002) provides a structure for this reflection in that it offers critical questions that lead to understanding the influence of mathematics education standards on school mathematics programs and teachers' classroom practices. The overriding question of interest for the reflective examination is: How should professional development be crafted in order to position mathematics teachers to begin the ongoing process of standards-based instruction? The teachers who participated in this study were introduced and, beyond a year, immersed in the *Principles and Standards* [PSSM] (NCTM, 2000). In the NRC's Framework context, the more specific questions guiding our reflection are: (1) How did the teachers receive and interpret the PSSM? (2) What changes in the teachers' classroom practices were brought about as a result of their exposure to the PSSM? Project and Participants

The project under discussion was an Eisenhower-funded teacher enhancement project that established a partnership between three Tennessee county school systems and an institution of higher education. Incorporating best practices of professional development, the 18-month project served approximately 50 of the grades 4-8 mathematics teachers in nine schools in three rural counties of the state. These best practices included long-term professional development with follow-up support; active participation of school principals (Loucks-Horsley et al., 2003); integration of content and pedagogy, and a collaborative environment in which the teachers could address issues they deemed important (Cooney and Krainer, 1996). The professional development activities conducted fall into four major categories—namely, analyzing teaching practice; improving content knowledge; modeling classroom investigations; and exploring resources. Development leaders were careful that each activity entailed many of the pedagogical characteristics for standards-based teaching asserted by Cramer (2004).

Two facts are worth mentioning regarding the participants. First, the teachers did not self-select into the project. Having the goal of effecting change at the school-wide level, all teachers who taught mathematics at the participating schools were involved as a collaborative team along with their principal. Second, prior to the project, the majority of the teachers was unfamiliar with the PSSM or had only a cursory knowledge of it. In combination, these factors provided the researchers an ideal group of teachers with whom to investigate the research questions.

Method and Data Sources

This study examines the professional development project as a case of teachers advancing through a process of understanding the PSSM with the intent of implementing standards-based instruction in their schools. In the project, our approach emphasized that this process of understanding is—as Mewborn (2003) recognizes—the beginning of a "continual journey" (p. 48). Hence, our overall goal was to identify where different teachers entered the journey, how far they could progress, and how much they could take from the journey to their classroom in order to enhance their students' learning.

A variety of data sources was used as a basis for this reflective study. These include: (1) teacher journals collected for a year throughout the 18-month project; (2) participant/instructor dialogue forms collected after each professional development session; (3) pre-, mid-, and post-project self-reports by teachers; (4) pre-and post-project responses to the Standards Beliefs Instrument (SBI) (Zollman and Mason, 1992); (5) pre-and post-project attitude surveys with regard to the nature of mathematics; (6) pre-and post-project school practice surveys; (7) pre-and post-project mathematics content knowledge exams; and (8) teacher observations and interviews with a subset of the teachers. Some teachers participated in only certain phases of the project resulting in pre- and post-project data from 26 teachers. Results on data sources 1, 3, and 4 from these 26 teachers make up the remaining discussion.

In order to gauge the teachers' reactions to the PSSM and shed light on our guiding questions, we identified three indicators of influence: the *Calculator Technology Indicator*, the *Mathematics Learning Indicator*, and the *Problem Solving Indicator*. These indicators were chosen post-project because they not only reflected major themes of the PSSM, but covered the scope of the professional development activities and the data generated from the teachers. Both quantitative and qualitative analyses were applied to the aforementioned data sources to measure progress of the teacher participants with regard to each indicator.

Results and Discussion

Upon examining the pre-project data on the teachers' self-reported beliefs and practices, it became apparent that teachers entered the journey towards standards-based mathematics instruction at different points. To illustrate, teachers were asked to respond to a series of questions about their perceived success in (i) using calculators and manipulatives in the classroom and (ii) engaging students in mathematical thinking. Two contrasting sets of responses are representative of the pre-project extremes. In response to (i) *Teacher A* wrote, "I choose not to allow the use of calculators in class. I prefer they [the students] get the basic concept first." … "Our curriculum currently doesn't support the use of manipulatives." *Teacher B* stated, "At the 7th and 8th grade level, I believe that calculators should be used to keep small computational accidents and long calculations from getting in the way of success for students. I especially like the graphing calculator to bring abstract concepts to the concrete." …"Number lines are about all that we have been able to do! I want to do more!" Commenting on item (ii), *Teacher A* seemed not to understand replying, "Vague question. What are you asking?" However *Teacher B* wrote, "I try to model mathematical practice and advise against common pitfalls and misconceptions while fostering creativity and process in math."

As one way to determine teachers' entry points, appropriate correlations (Spearman's Rho) between variables of interest were investigated. One finding was that correlations between the teachers' pre-project rating of their familiarity with the PSSM and the various items on the SBI were negligible to small with results ranging from -.11 to .21. These results are consistent with the finding of Zollman and Mason (1992) that teachers' familiarity with the *Standards* does not necessarily mean that they incorporate them into their belief structures. Items from the SBI constitute the belief aspects of the three indicators of influence.

Calculator Technology Indicator

With regard to this indicator, the professional development engaged teachers in activities in which they used calculators for problem solving and mathematical discovery. The teachers were separated into grade level groups (4-6 and 7-8); the grades 4-6 teachers used scientific calculators and the grades 7-8 teachers used middle school graphing calculators.

Items were administered pre- and post-project at the end of successive school years: *Appropriate calculators should be available to all students at all times (Belief Item, CB1)*; *To what extent do you feel you are successful in engaging your students in the use of calculators? Explain/provide an example. (Answer about your classroom practices this school year.) (Practice Item, CP1)*. Table 1 indicates movement from non-belief toward belief. Table 2 shows that actual teacher practice moved in the direction of more calculator use, though still not frequent use. Changes on both items were statistically significant (CB1: $p = .001$; CP1 $p = .003$) using one-tailed Wilcoxon signed rank tests with level of significance of .05.

Table 1: Pre- and Post-Project Responses to Calculator Belief Item (n=26)

	1–SA	2–A	2.5–N[a]	3–D	4–SD	Mean
Pre	2 (7.7%)	2 (7.7%)		12 (46.2%)	10 (38.5%)	3.15
Post	6 (23.1%)	7 (26.9%)	2 (7.7%)	10 (38.5%)	1 (3.8%)	2.27

[a]Two individuals created the category 2.5.

Table 2: Pre- and Post-Project Responses to Calculator Practice Item (n=26 unless noted)

	1–not at all	2	3	4	5–to a great extent	Mean
Pre (n=24)	12 (50%)	5 (20.8%)	3 (12.5%)	3(12.5%)	1 (4.2%)	2
Post	3 (11.5%)	9 (34.6%)	7 (26.9%)	4(15.4%)	3 (11.5%)	2.81

The relationship between teachers' beliefs and their classroom practice was examined. Initially 16 of 24 teachers (66.7%) reported disbelief in and little use of calculators. However, this decreased to 19% post-project representing movement in the desired direction. Similarly, three of 24 teachers (12.5%) reported belief in and frequent use of calculators. This increased to 19.2% post-project, also representing movement in the desired direction.

As cited in Franke et al. (1997), Fennema and her colleagues observed the occurrence of all possible sequences of the evolution of teachers' beliefs and their practice. For example, they report two behavior patterns: (1) teachers may change their beliefs prior to making a change in their practice; and (2) teachers may change their practice and then subsequently change their beliefs. Unlike the work of Fennema et al. which captured periodic snapshots of the teachers' beliefs and practices as they evolved, this data presents pre-and post - project findings only. However, the fact that three teachers held the same beliefs from pre- to post-project but increased in practice, while four teachers increased belief but stayed the same with regard to practice confirms the existence of the two patterns of behavior. This non-concurrent advancement of the teachers regarding their beliefs and practice is a plausible explanation for the finding that the correlation between belief and practice decreased from .33 to .17, pre- to post-project.

Teachers' explanations were requested for item CP1, revealing their views of the roles that calculators can play in teaching and learning mathematics; those which actually were revealing fell into these categories. Pre-project: using calculators to prevent computation errors from getting in the way of students' success; using calculators to check and show students' work. Pre-and post-project: using calculators only with students who have mastered certain basic skills; using calculators only after administrating achievement tests. All except the first category reveal a very limited view of the usefulness of calculators.

However, evidence of a shift in thinking about appropriate calculator use can be found in a subset of the teachers' journals. The following is an example of such a shift evidenced by seven teachers in their journal writings. Mid-project, one teacher wrote: "I have been guilty of the 'old school of thinking' that students need to be able to do mental math and paper and pencil math before they are allowed to use a calculator. I see that if the right tasks are set up, a calculator will free the students to do some higher level thinking without getting weighted down in mathematical operations."

Mathematics Learning Indicator

The second indicator of influence relates to teachers' perceptions of what it means to learn mathematics and how students learn mathematics. Four of the 16 items on the SBI pertained to this indicator: *Learning mathematics is a process in which students ABSORB INFORMATION, storing it in easily retrievable fragments as a result of repeated practice and reinforcement (MLB2); Learning mathematics must be an ACTIVE PROCESS (MLB3); Children ENTER KINDERGARTEN with considerable mathematical experience, a partial understanding of many mathematics concepts, and some more important mathematical skills (MLB4); Mathematics can be thought of as a language that must be MEANINGFUL if students are to communicate and apply mathematics productively (MLB1).*

Table 3 shows statistically significant movement toward standards-based beliefs from pre- to post-project with regard to MLB1 ($p = .015$) and MLB3 ($p = .015$) and small non-statistically significant movement on MLB4. One-tailed Wilcoxon signed rank tests with level of significance of .05 were used. An interesting observation is that on average teachers originally held standards-based beliefs with regard to MLB3, while originally not holding standards-based beliefs toward MLB4. The slight movement in the desired direction on MLB4 left the teachers lacking still a standards-based perspective on this item. Also, the teachers originally did not hold standards-based beliefs with regard to MLB2, and there was a slight statistically insignificant move toward less standards-based beliefs in this area.

Table 3: Pre- and Post-Project Responses to Math Learning Belief Items (n=26 unless noted)

	1–SA	2–A	2.5–N[a]	3–D	4–SD	Mean
MLB2 Pre	6 (23.1%)	13 (50%)		6 (23.1%)	1 (3.8%)	2.08
MLB2 Post	4 (15.4%)	19 (73.1%)		3 (11.5%)	0	1.96
MLB3 Pre	14 (53.8%)	12 (46.2%)		0	0	1.46
MLB3 Post	23 (88.5%)	3 (11.5%)		0	0	1.12

MLB4 Pre (n=25)	2 (8%)	4 (16%)	0	15 (60%)	4 (16%)	2.84
MLB4 Post	5 (19.2%)	3 (11.5%)	1 (3.9%)	11 (42.3%)	6 (23.1%)	2.71
MLB1 Pre	10 (38.5%)	14 (53.9%)		2 (7.7%)	0	1.69
MLB1 Post	19 (73.1%)	6 (23.1%)		1 (3.9%)		1.31

[a]One individual created the category 2.5.

Comments from four different teachers at mid-project may reveal why these changes in the respective directions occurred for the MLB items. "For all of these years I thought I had been teaching mathematics, but in reality I was mostly teaching memorization and procedures without connections." "In the past I would never have given a student a task for which I did not know the answer. Now I realize that the thinking process is more important than the answer" *(grade 4)*. "When we described patterns in blocks (referring to a specific activity), this showed me how we need to give children more activities where they have to do some thinking for themselves" *(grade 4)*. "As long as students do well in math, I am open to their thinking for themselves" *(grade 5)*. "…they (students with whom the teacher was working) did understand some of the more difficult patterns better than I anticipated. It is really important to let the students discover and investigate on their own. When I did this, I was really impressed by their comments and observations" *(grade 5)*.

Problem Solving Indicator

The three problem solving SBI items were: Problem solving should be a SEPARATE, DISTINCT part of the mathematics curriculum (PSB1); In grades 4-8 mathematics, skill in computation should PRECEDE word problems (PSB2); A demonstration of good reasoning should be regarded EVEN MORE THAN students' ability to find correct answers (PSB3).

Table 4 shows that on average beliefs on each item changed in the desired direction. On two items, PSB1 and PSB3, on average the teachers originally held standards-based views although much more so on PSB1 than on PSB3. On PSB2 the teachers originally lacked standards-based views and the movement in the desired direction was so slight that the post-project disposition remained non-standards based. The changes on PSB1 and PSB3 were statistically significant (PSB1: $p = .006$; PSB3: $p = .001$) using one-tailed Wilcoxon signed rank tests with level of significance of .05. The increase on PSB2 was not statistically significant.

Table 4: Pre- and Post-Project Responses to Problem Solving Belief Items (n=26 unless noted)

	1–SA	2–A	2.5–N[a]	3–D	4–SD	Mean
PSB1 Pre	1 (3.9%)	5 (19.2%)		12 (46.2%)	8 (30.8%)	3.04
PSB1 Post	1 (3.9%)	1 (3.9%)		3 (11.5%)	21 (80.8%)	3.69
PSB2 Pre	8 (30.8%)	13 (50%)		5 (19.2%)	0	1.89
PSB2 Post	8 (30.8%)	12 (46.2%)		5 (19.2%)	1 (3.9%)	1.96
PSB3 Pre (n=25)	1 (4%)	12 (48%)	3 (12%)	8 (32%)	1 (4%)	2.42
PSB3 Post (n=23)	5 (21.7%)	15 (65.2%)	0	3 (13.0%)	0	1.91

[a]Three individuals created the category 2.5.

Noticing the slight amount of change that was evident in PSB2 in Table 4, the participants' responses were examined. Seven teachers indicated a change toward a more standards-based view of PSB2, while seven others indicated a change away from such.

Relationships Between Indicators

Findings that teachers holding a rule-based view of mathematics tend to believe that calculators do not enhance instruction (Tharp et al., 1997) warrant an examination of the correlations between the calculator belief item and the mathematics learning and problem solving belief items. In Table 5 before correlations were computed, the responses for the negative valence items were re-aligned so that a positive correlation between two items tends to mean simultaneous agreement or disagreement with the *Standards* on those items.

Table 5: Pre- and Post-Project Correlations Between Belief Items (Post-Project in Bold)

	CB1	MLB1	MLB2	MLB3	MLB4	PSB1	PSB2	PSB3
CB1	□□	**-.28**	**-.21**	**.25**	**.16**	**-.35**	**-.37**	**.21**[c]
MLB1	.17	□□	**.16**	**.04**	**-.27**	**-.05**	***.39**	**.10**[c]
MLB2	-.03	-.01	□□	**-.26**	**.10**	**-.04**	**.06**	**-.40**[c]
MLB3	.26	*.60	.07	□□	**-.02**	**-.18**	**-.05**	***.49**[c]
MLB4	*.46[a]	.04[a]	.25[a]	.15[a]	□□	**-.07**	**-.07**	**-.02**[c]
PSB1	*.40	.23	.32	.29	.26[a]	□□	**.35**	**.05**[c]
PSB2	.27	-.04	.19	-.07	*.46[a]	.18	□□	**.21**[c]
PSB3	.29[a]	.17[a]	-.22[a]	.01[a]	.18[b]	*.48[a]	*.46[a]	□□

Note. n = 26 unless noted. [a] n=25 [b] n =24 [c] n=23

*Statistically significant difference from 0 (two-tailed test, level of significance .05)

Two of these results merit further investigation. Not surprisingly prior to the project, teachers' belief in using calculators correlated moderately (.40) with their belief that problem solving should not be a separate part of the curriculum and, to a lesser extent (.27), with their belief that skill in computation should not precede word problems. However post-project, these correlations changed directions, resulting in -.35 and -.37 respectively.

Interesting correlations are evident in Table 5 concerning pre-project relationships within and between the problem solving and mathematics learning items. The correlations between the problem solving belief items were .18, .46, and .48 with the correlation between PSB1 and PSB2 being the less strong (.18). While MLB1 and MLB3 had a strong correlation of .60, other correlations between mathematics learning belief items were less strong varying between -.01 and .25. The 12 pre-project correlations between mathematics learning and problem solving items varied between -.22 and .46. It is interesting to note the pairs of items with the strongest correlations, the change in direction of several of the correlations from pre to post, and the change in strength of 8 of the 21 correlations within and between problem solving and mathematics learning items by at least .30 from pre to post.

Conclusion and Implications

This study's overriding question concerns how professional development is crafted so that teachers are in position to begin the process of standards-based instruction in mathematics classrooms. The data confirm that a tailored professional development program including various types of activities (e.g., analyzing practice; improving content; modeling investigations; and exploring resources) can indeed better position teachers to enact standards-based instruction. Specifically, evidence supporting the three indicators of influence reveals some significant changes in teachers' beliefs with respect to calculator use, mathematics learning, and problem solving. That is, the manner in which many teachers received and interpreted the PSSM throughout the duration of the 18-month project resulted in a fundamental change in their belief structure. As the professional development unfolded, changes in practice were initiated—in particular with regard to calculator use. Given the rise in popularity of standards-based curricula, these results provide promise to leaders that teachers can become prepared to implement them.

We offer however the following caveats. Long-term, sustained professional development provides more opportunities for teachers to make fundamental changes in their beliefs and practices. It also allows for classroom implementation to occur simultaneously resulting in more meaningful professional development. As evidenced in some of the reversed patterns in our correlations from pre- to post-project, it is observed that teachers do not always make these changes in a linear fashion. Collectively different teachers may progress at varying rates and, individually, one might advance along one dimension and simultaneously digress along another. Another important component is getting the teachers to reflect often through writing which can serve two functions: facilitating their metacognitive growth and providing leaders with evidence for formative and summative evaluation. Lastly as Thompson (1992) asserts, leaders must recognize limitations on potential inferences made from data on beliefs due to teachers' broadening interpretations of language in related instruments as they progress through professional development. Hence, as with growth in any aspect of life and in any occupation, the more teachers learn the more they realize they need to learn.

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INVESTIGATING THE DEVELOPMENT OF A PROFESSIONAL MATHEMATICS TEACHING COMMUNITY

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This paper documents the development of group of middle school mathematics teachers into a professional mathematics teaching community. This study is significant because it not only provides an analysis of the evolution of the teachers into a community of practice, but also the means by which that evolution was supported.

The purpose of this paper is to provide an analysis that documents the development of a professional teaching community from its initial formation and the means of supporting its emergence. Gamoran, et al. (2003) call for such an analysis when they note that there are currently no longitudinal analyses that report on the development of professional teaching communities from inception. The importance of professional teaching communities is explicated by Secada and Adajian (1997) who assert that mathematics teachers' professional communities provide an important context for understanding the nature of teachers' practices, change, and learning. Mathematics educators therefore need to better understand the nature of professional teaching communities in order to better understand the process of mathematics teachers' learning and how best to support it.

Professional Teaching Community Characteristics

What differentiates a professional teaching community from a group of teachers? Grossman, Wineburg, and Woolworth (2001) argue for the importance of distinguishing between a professional teaching community and a group of teachers:

Even a cursory review of the literature reveals the tendency to bring community into being by linguistic fiat. Groups of people become communities, or so it would seem, by the flourish of a researcher's pen. Researchers have yet to formulate criteria that would allow them to distinguish between a *community of teachers* and a *group of teachers* sitting in a room for a meeting. (p. 943)

Researchers who have collaborated with groups of teachers to establish professional teaching communities (cf. Franke & Kazemi, 2001; Grossman et al., 2001; Lehrer & Schauble, 1998; Rosebery & Warren, 1998; Stein, Silver, & Smith, 1998; Warren & Rosebery, 1995) make clear that a group of teachers who collaborate with each other in some way does not necessarily constitute a community. An important first step is therefore to clarify what distinguishes a group from a community. Wenger (1998) discusses three interrelated dimensions that clarify what distinguishes a community of practice from a group: joint enterprise, mutual engagement, and a shared repertoire. I will expand on these three dimensions and discuss how they relate to a professional teaching community.

A joint enterprise is a negotiated venture "produced by participants within the resources and constraints of their situations" (Wenger, 1998, p. 79). More than merely a stated goal, the joint enterprise creates a sense of mutual accountability that becomes an integral aspect of the practice of the community. Secada and Adajian (1997) introduce a similar notion when they describe community as a group of people who have organized themselves for a *shared purpose*. "They adopt or are assigned formal and informal roles, they organize additional structures (such as times for meeting and planning) as needed, and they take actions—all in order to achieve their purposes" (p. 194). For a professional teaching community this shared enterprise might be teaching mathematics for understanding where there is a focus on students' learning of significant mathematical ideas.

Mutual engagement includes the social complexities and relationships that are developed in pursuit of a shared enterprise, as well as the norms of participation that are specific to the community. Bellah, Madsen, Sullivan, Swidler, and Tipton (1985) speak in similar terms when they define community as "a group of people who are socially interdependent, who participate together in discussion and decision making, and who share certain practices that both define the community and are nurtured by it" (p. 333). For a professional teaching community this would include both social norms of participation as well as norms that are specific to mathematics teaching such as the standards to which the members of the professional teaching community hold each other accountable when they justify pedagogical decisions and judgments (Cobb, McClain, Lamberg, & Dean, 2003).

A shared repertoire includes historical events, tools, styles, discourses, actions, stories, artifacts, and concepts. These have been produced or appropriated by the community in the course of its existence and have become a part of its practice. The elements of a repertoire "gain their coherence not in and of themselves as specific activities, symbols, or artifacts, but from the fact that they belong to the practice of a community pursuing an enterprise" (Wenger, 1998, p. 82). Since this communal repertoire is developed during the process of the collaborative, coordinated effort to pursue a shared purpose, it is specific to the community and this shared purpose. For a professional teaching community, this shared repertoire includes normative ways of reasoning with resources when planning for instruction and making students reasoning visible.

Although the notion of a community of practice as developed by Lave and Wenger (1991), Rogoff (1995), and Wenger (1998), has been used relatively widely to characterize professional teaching communities (Franke & Kazemi, 2001; Lehrer & Schauble, 1998; Stein, et al., 1998; Warren & Rosebery, 1995), other researchers have contributed to additional criteria for what constitutes a professional teaching community. Following Newmann and Associates (1996), Gamoran, Anderson, Quiroz, Secada, Williams, and Ashmann (2003) identify the elements of a professional teaching community as follows: (1) exhibiting a *shared sense of purpose* in their attention to student thinking, (2) *focusing collectively on student learning*, as opposed to teachers' more common conversations about administrative details and managing student behavior, (3) *collaborating* on ways to improve their students' understanding of mathematics, in contrast to teachers' usual practice of working in isolation, (4) engaging in *reflective dialog*, a conversation about the nature and practice of teaching, and (5) *making their own teaching practices public*, instead of keeping their practice private and confined within the classroom. The first, second, and fourth characteristics are variations on Wenger's notions of joint enterprise and shared repertoire previously discussed. However, the third and fifth characteristics bring in an important aspect of Wenger's dimension of mutual engagement that has not been explicitly addressed—the de-privatization of teachers' instructional practices. Teachers working in isolation keep aspects of their instruction such as decisions made and tools used during planning, facilitation, assessment, and reflection private. Conversely, as teachers are mutually engaged in the pursuit of a shared purpose, they develop norms of participation, which bring about the necessary consequence of teachers' instructional practices becoming public. This in turn cultivates a mutual accountability of justifying and critiquing pedagogical justifications within a professional teaching community. Secada and Adajian (1997) operationalize a professional teaching community specific to mathematics teaching along four dimensions: (1) a shared sense of purpose, which they describe as the nature and extent of the school staff's shared values and goals; (2) a coordinated effort to improve students' mathematics learning including teachers working together and setting aside personal prerogatives in favor of shared goals; (3) collaborative professional learning, meaning how well and closely the teachers work together to learn about and to improve their practice as related to mathematics; and (4) collective control over important decisions affecting the school's mathematics program or in other words, whether teachers have the power as a group, to focus the direction of their program. Although similar to Wenger's (1998) and Gamoran, et al.'s (2003) descriptions, Secada and Adajian's dimensions raise two important issues when clarifying what constitutes a professional teaching community. First, like much of the discussion about professional teaching communities in the research literature (Franke & Kazemi, 2001; Grodsky & Gamoran, 1998; Grossman, et al., 2001; Newmann & Associates, 1996; Stein, et al., 1998), Secada and Adajian focus on the school as the location of the community. This obviously hints at the fact that the professional teaching community is institutionally situated. However, it is also important to note that a professional teaching community would not have to be confined to the boundaries of a school nor involve all the teachers in a school, but could include a group of teachers who come from different schools within the same school district. Secondly, I believe Secada and Adajian's fourth dimension may need to be modified when taking the teachers' institutional context into consideration. Obviously, members of a professional teaching community should come to see themselves as the professionals most capable of making decisions that affect the mathematics teaching within their schools. However, whether or not they have the power as a group to do so does not make them any less a professional teaching community. This autonomy merely reflects the location of the community within the institutional context in which they work.

Synthesizing the literature presented on communities of practice, professional communities, and professional teaching communities, I will use the following criteria to determine when the group of teachers that are the subject of this study evolved into a professional teaching community:

- *A shared purpose or enterprise* such as ensuring that students come to understand central mathematical ideas while simultaneously performing more than adequately on high stakes assessments of mathematics achievement
- *A shared repertoire of ways of reasoning with tools and artifacts* that is specific to the community and the shared purpose including normative ways of reasoning with instructional materials and other resources when planning for instruction or using tasks and other resources to make students' mathematical reasoning visible
- *Norms of mutual engagement* encompassing both general norms of participation as well as norms that are specific to mathematics teaching such as the standards to which the members of the community hold each other accountable when they justify pedagogical decisions and judgments

Data collection involved a focus on the work sessions and summer seminars conducted in collaboration with the professional teaching community during the first two years of the project. Data sources include videotape of each work session, field notes, and copies of the teachers' work during the session.

The approach that I took when analyzing the emergence of the professional teaching community involved a method described by Cobb and Whitenack (1996) that was developed for analyzing longitudinal data sets of the type generated during design experiments. This method is a variant of Glaser and Strauss' (1967) constant comparative method and is specifically tailored to the *systematic* analysis of longitudinal data sets in mathematics education. The first phase of the analysis involved working through the entire data corpus in chronological order. In this phase, conjectures made while analyzing particular episodes about norms that had been established by the teaching community were tested and, as necessary, revised while analyzing subsequent episodes. In the second phase of the analysis, the resulting chain of conjectures and refutations were analyzed to produce empirically-grounded accounts of the learning of the teaching community that consisted of a network of mutually reinforcing assertions that span the entire data set.

In order to analyze the emergence of the group into a professional teaching community, I identify the successive norms that became established in the community. It is important to clarify that norms are identified by discerning patterns or regularities in the ongoing interactions of the members of the professional teaching community. A norm is therefore not an individualistic notion but is instead a joint or collective accomplishment of the members of a community (Voigt, 1995). A primary consideration when conducting analyses of this type is therefore to be explicit about the types of evidence used when determining that a norm has been established so that other researchers can monitor the analysis. A first, relatively robust type of evidence occurs when a particular way of reasoning or acting that initially has to be justified is itself later used to justify other ways of reasoning or acting. In such cases, the shift in the role of the way of reasoning or acting within an argument structure from a claim that requires a warrant, to a warrant for a subsequent claim provides direct evidence that it has become normative and beyond justification. A second, more robust type of evidence is indicated by Sfard's (2000) observation that normative ways of acting are not mere arbitrary conventions for members of a community that can be modified at will. Instead, these ways of acting are value-laden in that they are constituted within the community as legitimate or acceptable ways of acting. This observation indicates the importance of searching for instances where a teacher appears to violate a proposed communal norm in order to check whether his or her activity is constituted as legitimate or illegitimate. In the former case, it would be necessary to revise the conjecture whereas, in the latter case, the observation that the teachers' activity was constituted as a breach of a norm provides evidence in support of the conjecture (cf. Cobb et al., 2001). Finally, a third and even more direct type of evidence occurs when the members of a professional teaching community talk explicitly about their respective obligations and expectations. Such exchanges typically occur when one or more of the members perceive that a norm has been violated.

The successive norms that became established in the professional teaching community included norms for: 1) general participation, 2) pedagogical reasoning, 3) mathematical reasoning, and 4) strategic reasoning. The analysis of norms for general participation documents the evolving participation structure of the community (Lampert, 1990; Shulman, 1986). As an illustration, this analysis documents whether it became an established norm in the community for the teachers to question and critique each others' reasoning or whether norms involved what Grossman, et al. (2001) term pseudo-agreement in which the teachers refrained from confronting issues that relate to their instructional practices. The analysis of norms for pedagogical reasoning documents the norms that became established as the teachers both reflected on their instruction and planned for instruction. In focusing on the key norm of what counts as an acceptable pedagogical argument, for example, I documented the extent to which the teachers in the community became obliged to justify their pedagogical judgments in terms of analyses of students' mathematical reasoning. The analysis of norms for mathematical reasoning document both the norms for mathematical argumentation and the norms for reasoning that became established as the teachers explored particular mathematical domains. When the teachers engaged in activities that involve analyzing data, for example, I documented whether the norms that became established for statistical reasoning involved additive or multiplicative reasoning. The analysis of strategic norms documents the evolution of the teachers' understanding of the institutional setting and its influence on their teaching of mathematics. I choose these four norms a priori based on specific conjectures. Grossman, et al.'s (2000) claim that there must be an essential tension and change of norms of interaction in order to break down a pseudocommunity and allow a group to evolve into a professional teaching community is one of the foundations for my conjecture that norms of general participation must be documented. Another rationale for documenting norms of general participation also gives background for documenting norms of pedagogical reasoning. This being the third characteristic of what constitutes a professional teaching community: mutual engagement. As stated previously, this includes both general norms of participation as well as norms that are specific to mathematics teaching such as the standards to which the members of the community hold each other accountable when they justify pedagogical decisions and judgments. Pedagogical reasoning is also important to analyze given our overarching goal of supporting the eventual development of teachers developing instructional practices in which teaching is a generative, knowledge-building activity with students' reasoning at the center of instructional decision making. The decision to document the norms for mathematical reasoning comes from the literature that claims that in order for teachers to understand student thinking and the best way to support student learning of significant mathematical ideas they must have a deep understanding of the mathematics they teach (Bransford, Brown, & Cocking, 2000; Ma, 1999; Shulman, 1986). Although Talbert and McLaughlin (1994) and Gamoran, et al. (2003) point to the importance of understanding the institutional context, it is not obvious from the literature that documenting strategic reasoning norms is important. However, based on our log of on-going conjectures, it became evident that the teachers' view of the institutional context and how it supported or constrained their instructional practices changed. Thus, I believe it is important to document these changes and how they were supported.

Limited space presents the necessity to limit the results of my analysis to only one of the four norms presented above. As norms of general participation are part of the third criteria for what constitutes a professional teaching community, I will choose this type of norm to document in this paper.

At the beginning of our collaboration, not unexpectedly, teachers were anticipating our collaboration to be like other professional development sessions they had experienced, in that they were treating the researchers as experts there to disperse information for the teachers to take back to their classrooms and use with their students. Thus the general norms of participation included the researchers presenting information and teachers' discussion occurring only after researchers' questions. The teachers' answers were always aimed directly back to the researcher and not towards each other.

The first noticeable shift in the general participation norms occurred during our fifth session with the teachers. Although the nature of discussion was still in the spirit of turn sharing among the teachers, there were a few notable instances when the teachers questioned each other for clarification of statements and conjectures. This shift was supported through a discussion of student work generated by an assessment task teachers had done with their students prior to attending the session. However, the group would still be termed what Grossman, et al. (2003) call a pseudocommunity. Meaning, teachers would suppress conflict and not challenge each other on conjectures made during discussions and thus, present a false sense of unity.

The second shift in general participation occurred after one year of working with the group of teachers during our seventh session when a video was interrupted by one of the researchers. The purpose for the interruption was to redirect the teachers' focus to the students' reasoning as they were attending solely to student behavior. One of the teachers explained that the teachers naturally focus on student behavior because that is how they are evaluated by administrators. She explained that they do not have the opportunity to observe other teachers' classrooms. This event led to the teachers being more open with their responses and no longer presenting information as if they were being evaluated. Researchers were still viewed as specialist experts, but the teachers started viewing themselves as authorities on what teaching entailed in their specific district and started requesting certain activities and topics to cover during sessions.

Another major shift occurred during our 11th session with the teachers. The catalyst for this shift was having the teachers plan and teach a lesson together during the session using students from one of the participants' classes. As teachers prepared for the class activity, they debated with each other the goals of the lesson and how best to achieve them. Teachers' questions were directed to the group instead of to the researchers. The teacher whose students were used led the activity. Afterwards, the teachers constructively critiqued the lesson. The pseudocommunity was starting to break down at this point.

During the next two sessions, teachers challenged each others conjectures and pushed for justifications in both mathematical and pedagogical reasoning. Disagreements were no longer seen as uncomfortable, but expected. The relationship between researchers and teachers became more collegial, with the teachers even challenging statements made by the researchers. At this point we would claim the group of teachers had emerged into a professional teaching community.

Although the reported analysis is limited in scope by focusing on the changes only in general norms of participation, the group's evolution is still evident. What is of most importance is to note the specific means that supported these shifts. This is by no means a prescription for how every group will evolved. However, the results of this analysis can generalize to other cases in that it can enable researchers and teacher educators to adapt the means by which the learning of the professional teaching community was supported in a conjecture-driven manner.

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USING SPATIAL TASKS TO ASSESS NUMERACY

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What can children's responses to spatial tasks teach us? This paper explores children's spatial thinking on one particular task, teacher's responses to the children's thinking, and proposes three possible aspects of performance that can inform the teaching of early numeracy. These aspects are the use of imagery, dispositions or habits of mind, and strategic thinking.

Over the past decade there has been increased interest in the role of imagery in developing mathematical understanding (Brown & Wheatley, 1997; Owens & Clements, 1998; Battista, 1999). Though imagery is recognized as important to spatial thinking, the connection between imagery and numerical thinking has not been clearly established. Battista (1999) for example has explored how elementary students structure spatial environments and the connection of this structuring to the development of multiplicative thinking. Wheatley and Cobb (1990) found a strong relationship between young children's responses on imagery tasks with their number development classification, and proposed that spatial patterns and imagery may play an equally important role in number development as well as geometry. Brown and Wheatley (1997) report a case analysis of one student who had difficulty forming a useful image and therefore had difficulty representing or transforming that image in order to help her solve mathematical non-routine problems. Cruz, Febles, and Díaz (2000) explore the role of visual imagery through the case of a 'visualiser" student who correctly solved problems in a creative, connected, visual way yet failed the mathematics test because he did not use more algebraic methods. These studies explore the use of visual imagery in solving mathematical problems, both geometric and numeric, and challenge learning models that do not adequately account for mathematical conceptions that are more spatially based. This research further highlights how, as Cruz, Febles, and Díaz (2000) state, "visual education is often a forgotten area in educational practice, in relation to the importance that the numerical and algebraic content have" (p. 35).

Teaching number, geometry, or algebra so that students learn it meaningfully requires an understanding of how students construct their knowledge in various contexts, including more visual spatial situations. For teachers it requires drawing upon this understanding to chose and adapt tasks, assess students' work, and respond to student thinking. Previous studies have focused on teachers learning more about their students' thinking and how this learning can generate and sustain changes in practice (e.g. Fennema, et al., 1996; Clarke, 2001). Much of the work in this area is based on teachers' interpretation of students' strategies for solving number operation problems. Our study contributes to this work by focusing on how young children solve spatial tasks, and the impact this has on teacher learning. In particular we pose two questions: 1) What can we learn from using spatial tasks for assessment purposes? 2) What aspects of spatial task performance do teachers find most useful for better understanding children's mathematical thinking?

Theoretical Considerations

Presmeg (1986) refers to individuals who prefer to use visual methods for solving problems as *visualisers*. In her study with students in their final year of high school she categorized various kinds of images used by visualisers into five types: concrete pictorial imagery; pattern imagery; memory images of formulae; kineasthetic imagery; and dynamic imagery. Wheatley and Cobb (1990) present another framework for considering young children's responses to a visual spatial puzzle task. Wheatley and Cobb presented young children with 5 geometric shapes and asked them to construct a square with the pieces. A pattern for completing the square using three of the pieces was quickly shown to the children and then removed. Children's capacity to capture and use this visual pattern in completing the puzzle was described in five levels: 1) image of linear objects (shapes construed as "lines"); 2) global covering (shapes used to "fill up space"); 3) structuring an unfilled space (sees unfilled space as a region); 4) partial image construction (uses the offered pattern to begin placing pieces); and 5) relational image construction (uses the offered image to quickly place all 3 pieces). In levels one through three, children did not access the visual information presented to them, while in levels four and five, children used this image to guide their solution of the puzzle task.

These frameworks for observing students as they access and apply imagery present interesting questions for educational researchers. What impact does awareness of children's use of imagery have for teachers and their practice? How do teachers make sense of what they observe while children work? What other aspects of spatial/mathematical thinking can teachers observe in their students' responses?

Study Context and Methods

For the past several years, we have worked with teachers to learn more about how to best assess and support the development of early numeracy in young children (Kindergarten and Grade 1). Our main project, the Early Numeracy Project [ENP], involved sixteen teachers and their students from five school districts across the province, and included both rural and urban schools. Project goals included working with teachers to: 1) create and use performance-based tasks most appropriate for assessing numeracy in young learners; 2) create and refine instructional strategies to support numeracy development at school and home; and 3) develop reference standards on key assessment items that provide a portrait of young students' mathematical thinking. The assessment developed by the ENP team focused on four aspects of early numeracy: mathematical disposition, number skills, number concepts, and spatial thinking. Project teachers received some release time to extensively field-test the assessment items with their students. Project teachers also received release time to meet as a group three to four times a year over a three-year period.

Data collected include results for approximately 200 kindergarten students on 17 tasks from an individually administered performance assessment. Twenty-one of these students provided permission to have their performance assessment video-taped. Researcher field notes collected during project meetings provide data on teachers' questions and concerns as the project progressed. Audio taped interviews with project teachers conducted throughout the project, and written and oral feedback from practicing teachers using the assessment items were also used. For this paper we focus our attention on students' and teachers' response to one of these ENP tasks that highlights children's visual spatial thinking. This task, the Squares Task (adapted from Clarke (2001) and Wheatley and Cobb (1990)) asks students to select three shapes from a set of six geometric pieces (a rectangle, 2 isosceles right angle triangles, 1 large and 2 smaller right angle triangles) and to use these to form a square. If students were unable to make the square, a hint was given for how to create the larger right angle triangle from the two smaller ones.

Results and Conclusions

Both Wheatley and Cobb's (1990) levels for image re-presentation and construction, and Presmeg's (1986) delineation of 5 types of images used by visualisers in solving mathematical problems were helpful in shaping a framework for examining student responses to the Squares Task. However, it was not imagery levels alone that provided information for teachers. What emerged from the task was a vivid picture of the child's strategic thinking (using a logical strategy, learning from mistakes, evaluating possible moves before placing pieces) and habits of mind (such as perseverance, flexibility, curiosity, etc.) As a result, the continuum of responses used to analyze the task features 4 levels that describe the use of imagery, the strategic approach, and the habits of mind typical of any one level for solving the Squares Task. First we share typical student responses for each of the levels then we share teacher responses to their students' work on this task.

Analyzing Children's Spatial Responses

The continuum of responses to the Squares Task features 4 stages, which describe the behaviours demonstrated by the children while working to solve the puzzle. There are some important components to consider in these stages. First, in Stage One – Pictorial, and Stage Two - Random, children did not appear to understand the task as one of "selecting and fitting pieces into the frame to form a square with no overlap", while children operating at Stages Three and Four were very clear on the task and approached it with purpose. As children progressed from Stage Three – Static, through Stage Four – Dynamic, they appeared to connect their prior learning to new situations, learning from their mistakes and sometimes commenting on their "ah-ha" moments. Each stage is described below, and includes student performance examples.

Pictorial

Children at this stage were preoccupied by the construction of shapes and objects in the environment, like houses and flowers. They made connections to real objects, but not to the task. Clues given by adults were not always helpful in supporting students. For example, Christie (6 years, 1 month) began building her square off the template page. She did not understand that the task involved positioning her shapes inside the square template. When prompted to do so, she used her shapes to outline the template – seeing the forms as lines rather than intact shapes – and commented on their appearance: "That's a funny square. It's like a dragon's tail. Weird" (Figure 1. Stages 13). Although the teacher offered support in a variety of ways, Christie struggled to position the shapes within the template. Nonetheless, Christie remained engaged and appeared confident with the task, talking and smiling throughout.

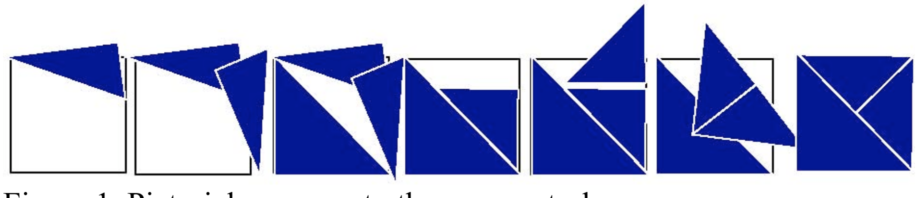

Figure 1. Pictorial response to the squares task

Random

Children at this stage were random in their selection of pieces to fill the square form. They saw the task as one of "covering" or "smothering" the square form – and would overlap pieces and extend beyond the edges of the square without being bothered by it. Pieces that fit within the form were seen as "lucky"; children did not seem to connect to or learn from their experiences of success at this stage. Clues given by adults were sometimes helpful in supporting students. Ray (5 years, 11 months) began to solve the puzzle by building off the page as well, and when directed to do so, began to place the pieces on the template. He paid more attention to how the pieces fit within the frame than Christie, but insisted on stacking and layering shapes to cover unfilled space (Figure 2. stages 2, 3, 4). Ray used self-talk while he worked, commenting on the task "I need more small ones," and persisted to complete it.

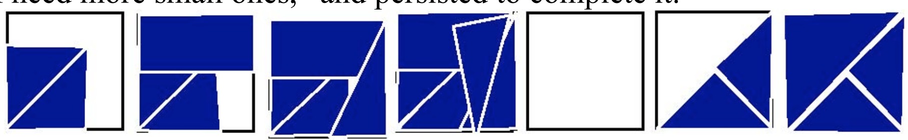

Figure 2. Random response to the squares task

Static Imagery

Children at this stage understood the task as "making a square" or "filling in the form". They selected shapes from the available pieces and rotated them to make a fit, but were not able to predict which pieces would match without testing them first. Children at this stage tended not to flip pieces, but rather to turn or slide them. Clues given by adults were often helpful in supporting students. Maria's (6 years 5 months) flexibility in approaching this task was evident in the different pieces she selected and in the ways she tried to orient them to make them fit. She did not get stuck in one position, but substituted pieces and shifted them readily. Maria paid attention to the angles and the ways the pieces fit together. Notably, in her transforming of the pieces she did not flip shapes, but only rotated and slid them (Figure 3. stages 2, 3, 4, 5). When given the clue (stage 6), Maria grinned and said, "Thank you for giving me an idea!", then quickly added the missing piece to the page. She could visualize the placement of the pieces from the brief teacher demonstration, and was able to re-create that image to solve the problem.

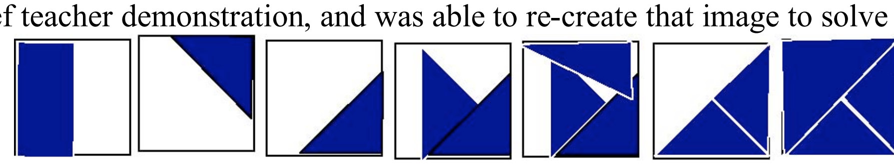

Figure 3. Static imagery response to the squares task

Dynamic Imagery

Children at this stage were able to predict which pieces would fit from among a jumble of available shapes. These children could mentally turn, flip and move pieces in making their predictions. Often, these children would perform the transformation of the required piece in the air before laying their shape on the form, all done in a fluid motion. Children operating at this stage tended not to need clues to support their solution finding. Rather, they evaluated each piece and its placement before placing it within the form. Charlotte (6 years, 6 months) used dynamic imagery to select, rotate, flip, match angles and place the pieces in order to form the square (Figure 4).

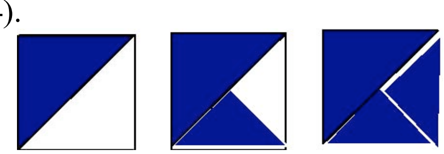

Figure 4. Dynamic imagery response to the squares task

Impressed with Charlotte's ease in completing this task, the project teacher encouraged her to share her thinking

T: How did you know what pieces to choose?

C: Because I did it in my brain that these points (pointing to the right angles of the large right angle triangle) go over here (referring to the right angle of the square). I don't think the small–these points (pointing to the acute angles of the triangle) go there. Because, it's like when you put it together in your imagining then it makes a square.

From Charlotte's explanation and actions we can see that she was able to mentally move the shapes in her head before she placed them on the square. She has an understanding of the connections between the various shapes and the properties they share. Few students solved or articulated their thinking as quickly and clearly as Charlotte. In fact, few project teachers solved the problem with such ease.

The continuum of responses to the Squares Task features 4 stages, which describe the behaviours demonstrated by the children while working to solve the puzzle. There are some important components to consider in these stages. First, in Stage One – Pictorial, and Stage Two - Random, children did not appear to understand the task as one of "selecting and fitting pieces into the frame to form a square with no overlap", while children operating at Stages Three and Four were very clear on the task and approached it with purpose. As children progressed from Stage Three – Static, through Stage Four – Dynamic, they appeared to connect their prior learning to new situations, learning from their mistakes and sometimes commenting on their "ah-ha" moments. Each stage is described below, and includes student performance examples.

What Teachers Learned From Children's Spatial Responses

In sharing and discussing interpretations of responses, such as Charlotte's, teachers began to appreciate how students were using visual images, representations, and mental transformations to solve problems. This teacher's comments are representative of others when she stated:

> Well, I didn't give much attention to the significance of spatial awareness before. … I'm certainly more aware of it now…. And I'm in the search for students who have strong spatial skills. (T1, interview)
>
> This task is excellent for highlighting students' habits of mind and general spatial awareness... how students can do mental transformations and how they stick with the task or give up. (T4, interview)

Teachers spoke about how the spatial task provided them with not only insight into the mathematical strengths and thinking of their students but also new ways for them to think about their teaching.

> I believe spatial visual tasks like the spatial task are essential to math understanding but I had never taught it so specifically as I do now. My increased awareness about this area has directly affected my teaching practice. (T8, interview)

Although now more aware, teachers also questioned how to bring or highlight visual spatial aspects of number and geometry into their practice. Having limited experience themselves with visual spatial tasks, teachers lacked resources and possibilities for how they might emphasize and help students develop visual imagery for solving mathematics problems. Analysis of our data indicate that although all project teachers expressed interest in developing students' visual representations and valued learning more about their students' visual representations, only a third of the teachers actively pursued this in their teaching. These teachers spoke about how the task provided new ways to support their students.

> Whenever I notice a child who has a strength on this task I often see this strong ability present in other visual spatial tasks. The task highlights the strength for me and I try to build on that child's strength throughout the year. I use this strength to build the child's confidence. It also helps me to support that child by trying to present more visual strategies to understand a concept but I'm still struggling with ways to do this well. (T2, writing)

Other teachers, however, were more cautious about how visual spatial tasks such as the square task could be used to inform instruction. As one teacher stated when asked if and how she is using the task in her practice:

> I have been puzzled by the results of this task since we started using it. Some people (adults and children) are much better at it than others, but I have not been able to see any correlation between that ability and any other abilities. As it stands the squares task is fairly low on my ranking of the ENP tasks. I would love to see how a consistent practice of such tasks would influence the performance of a group on other activities. (T14, writing)

These results contribute to what we know in terms of using the analysis of student thinking as a focus for teacher learning/inquiry and the challenges some teachers face in sustaining that inquiry in their practice. Teachers as part of the Early Numeracy Project had opportunities to try the squares task with various students in their own classrooms and in colleagues' classrooms. Teachers came together in regular ENP meetings to discuss what they noticed in students' strategies with this spatial task, how to interpret students' responses, and the extent to which the task was useful in terms of informing their understanding of students and their practice. The opportunities to discuss and share student thinking related to this and other ENP tasks were noted as key for all teachers in the project. This collaborative inquiry was quite different from the typical workshop type professional development most teachers had experienced. Ball and Cohen (1999) argue for moving beyond traditional workshop-type professional development programs by considering how the practice of teaching can be a powerful context for teacher learning.

Some professional development literature indicates that teachers' individual and collective analysis of children's thinking is a promising context for learning in and from teaching practice (Fennema et al, 1996; Schorr & Alston, 1999; Clarke, 2001). Our study contributes to and extends this research by focusing on teachers' opportunities to analyze and discuss the actions and imagery created by students in representing spatial problems. Although some teachers questioned what the task offered, most teachers found the task highlighted students' use of imagery, habits of mind, and strategic thinking. In addition, the task often challenged teacher's assumptions about the mathematical strengths and weaknesses students bring with them to class. It served to heighten teacher's curiosity about connections between spatial and numerical thinking.

Our work with ENP teachers and children on this spatial task has lead us to experiment with how the video clips of students working on this task might be used to engage other teachers, those who did not have the opportunity to participate in the ENP over the past few years, in an investigation of students' spatial thinking. What do teachers notice as they play and re-play students' spatial responses? What do they notice about the kinds of teacher hints and interventions provided students as they work on the task? How does analyzing students' responses to spatial tasks inform teachers in their design of problems and activities to develop students' numerical thinking? These questions are interesting and important as they extend our thinking, not only about characteristics of professional development that involve teachers in inquiry, but also in the connection between spatial and numerical thinking. This connection is a challenge for educators and an area that warrants continued research.

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HOW EFFECTIVE ONLINE COURSE ACTIVITIES CHANGED A TEACHER'S PRACTICE

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Literature on teacher conceptual change focuses on the mathematics classroom teacher and aspects of their conceptual change in the teaching and learning of mathematics (see Shaw & Jakubowski, 1991). Additionally, Garet et. al. (2001) identifies both structural and core features of effective professional development for teachers. This study examines the conceptual change of one practicing mathematics teacher, Andrew, who has been part of an online learning community. It observes the relationship between the core features of effective professional development (Garet, et al., 2001) and the changes that manifested themselves in Andrew's knowledge, skills, and classroom teaching practices. The results show that effective professional development can be obtained via meaningful and carefully selected online course activities.

Introduction

Research in the area of teacher change has identified several characteristics of the nature of conceptual change (see Shaw, & Jakubowski, 1991; Shaw, Davis, Sidani-Tabbaa, & McCarty 1990; Pajares, 1992; Posner, Strike, Hewson, Gertzog, 1982). Some of the research in this area has focused on the mathematics classroom teacher and aspects of their conceptual change in teaching and learning of mathematics (see Shaw, & Jakubowski, 1991). Studies have found that what may influence one teacher's desire and motivation to change may have no effect or the opposite effect on another. Furthermore, some teachers may posses this desire for conceptual change but are not able to make the next step in actually changing the way they learn and teach mathematics. Others still are able to make the transition to significant change (Shaw & Jakubowski, 1991). What fosters meaningful change in some mathematics teachers with respect to both their classroom practice and acquisition of new knowledge in mathematics? This study focuses on the conceptual changes of a single online student who is currently enrolled in a graduate program in mathematic education and a practicing classroom mathematics teacher. This study represents one case of a larger project involving several other members of the same online learning community.

Statement of the Problem

The purpose of this study was to identify what conceptual changes, if any, in learning and teaching mathematics have occurred for one practicing mathematics teacher who is a member of a graduate level online learning community (part of an online degree). Online activities were created for the purpose of causing a perturbation with practicing teachers' beliefs about the way they learn and teach mathematics. For the purpose of this study four activities were analyzed. Using a case study design the following research questions were examined. First, how do online courses that use core features of effective professional development cause perturbations in a teacher's thinking? Second, what conceptual change is evidenced in the knowledge and skills of the teacher?

Theoretical Framework

Many teachers are not prepared to implement instructional strategies that are grounded in high curriculum standards (Cohen, 1990; Elmore & Burney, 1996; Grant, Peterson, & Shojgreen-Downer, 1996). The shift in emphasis on mathematical understanding (as compared to memorization facts) means that teachers must learn more about mathematics as well as how students learn this mathematics. Within any profession (and teaching is not an exception) an integral part is the continual deepening of knowledge and skills (National Board for Professional Teaching Standards, 1989).

Garet, Porter, Desimone, Birman, & Yoon (2001) describe structural and core features of effective professional development. Structural features focus on the types of activities (e.g., workshops, institutions, courses); duration; and collective participation. Core features are focusing on content, promoting active learning, and fostering coherence. These two types of features are based on evidence from research studies that examined professional development.

The inclusion of a focus on content in professional development activities is seen to vary across four dimensions. These are the degree of emphasis on subject matter and teaching methods, the specificity of change in teaching practice encouraged, the degree of emphasis on goals for student learning, and the degree of emphasis on the ways students learn. Garet, et.al, (2001) describes the second core feature of promoting active learning as one that "concerns the opportunities provided by the professional development activity for teachers to become actively engaged in meaningful discussion, planning, and practice" (p. 925). The third core feature of fostering coherence is concerned with "the extent to which professional development activities are perceived by teachers to be a part of a coherent program of teacher learning (p. 927).

Using these dimensions, 1027 mathematics and science teachers (representing a national probability sample) were surveyed to determine the degree of influence each feature had on teachers' self-reported increases in knowledge and skills and the resulting changes in classroom practice. Garet, et.al. (2001) found that all three of the core features identified had a positive influence on teacher knowledge and skills.

Research Design

This case study is on one student, Andrew. The focus on one student, who has been part of this online learning community for four semesters and is in his last semester, has allowed the researchers to make assertions that are being used in the evaluation of the online program. Course activities used as data sources represent the first three semesters of courses taken by Andrew. The study was designed to enable the researchers to analyze the relationship between core features of effective professional development (Garet, et al., 2001) and the changes evident in Andrew's knowledge, skills, and classroom teaching practices.

Data from course assignments, interviews, and discussion groups were used. The researchers (who had been the instructors for the courses) initially identified three activities that would be used as primary data sources. From interviews with Andrew, additional activities were identified that the student felt had caused him a perturbation leading to significant change in his teaching practices. Using the core features stated by Garet, et al. (2001), the activities and Andrew's work were analyzed to identify the extent to which they first corresponded to the core features and second to how Andrew's thinking and descriptions of teaching practices reflected a change.

Results

Andrew teaches ninth and tenth graders on the east coast of Florida. Beginning spring 2003 in the program, he graduates in summer 2004. Each semester Andrew enrolled in the courses offered (usually six semester hours). He has indicated a high degree of satisfaction with the online program thus far as evidenced by his comment in an interview in January 2004 (start of his fourth semester in the program). "I am very pleased with the quality of the selection of the courses. I have learned a tremendous amount about math education so far" and in a discussion board forum setup to introduce yourself to the rest of the class he wrote "This is my 3rd semester in the online program and I absolutely love it! I am convinced that online learning is the best way to go." Course alignment by the program faculty has been purposeful. For example, Using History in the Teaching of Mathematics and Number Systems are offered concurrently as are Research Methods and Analysis of Student Learning. Similarly, the choice of assignments for each course has been carefully considered usually by not only the instructor, but in collaboration with other faculty and the course mentor. Andrew, in an introduction of himself to others in a course, told his classmates that "The majority of my coursework has been professored by Dr. Corey and I attribute a great deal of my learning to his careful selection of learning activities. Every time I finished one of his courses I felt that my knowledge base had increased greatly." Thus, Andrew's responses would indicate that the design and implementation of course offerings respond to the goal of fostering coherence (Garet, et al, 2001) whereby a level of positive effectiveness has been established.

A primary goal of the online program has been the fostering of teacher professional growth for the practicing professional. This is grounded in the premise that "if we want schools to produce more powerful learning on the part of students, we have to offer more powerful learning opportunities to teachers" (Feiman-Nemser, 2001). Thus, the selections for course assignments, discussion topics, and readings have been carefully done in order to support a coherent program of teacher learning.

Recent research (e.g. Cohen & Hill, 1998) suggests that effective professional development incorporates both a focus on content knowledge and an understanding of how students learn the content (core feature of focusing on content). Through two complementary assignments in courses teachers reflect on their own process of learning mathematics content (e.g., algebra, geometry, number theory) and reading research articles on student learning in these areas. After completing the Problem Solving course, Andrew indicated a change in his own practice by incorporating more in his own class. Andrew wrote, "the Problem Solving course really changed the course of direction I take in math instruction. I have used several problems in my classes. Primarily, I learned that the student's interest is most peaked when a concept is discovered and taught THROUGH problem solving. Now, when planning, I always look for a problem to match my topic of instruction." This course also caused him to reconsider how mathematics is taught in the classroom. In a final paper for the Problem Solving course he stated, "My view of problem solving and its role in the mathematics classroom has certainly changed since beginning this course. Learning to do math using the traditional drill/practice method is not learning to understand. Instead, it is learning to repeat (such as on tests). Exercising problem solving strategies teaches students how to think independently and at a higher level."

The final core feature of effective professional development concerns the opportunities provided for teachers to become actively involved. The online courses provided multiple venues for engagement in meaningful discussions (e.g., group discussions on readings, debates), reviewing student work, reading and writing, and planning classroom implementation. There is evidence that course design was such that active engagement caused perturbations in the teacher's thinking whereby changes were made in teaching practices. For example, one assignment in the School Mathematics Curriculum course was to debate an educational issue. Students were assigned either pro or con, had to research their position, develop a 350 word position statement, post it to the discussion board, and respond to other group members. Andrew commented that:

> The issue I was assigned to take was: High stakes testing is necessary for determining yearly mathematical growth for students. Before the debate, I was opposed to high stakes

> testing. However, because of researching the issue and debating others, I actually convinced myself that standardized testing, when used appropriately, are among the most sound and objective knowledge and performance measures available. My point is that debating educational issues can enlighten and open a participant's mind to another perspective (interview January 2004).

He still maintained that "traditional paper and pencil assessments do not necessarily prove a student's understanding of a math concept. Students typically learn and repeat an algorithm without grasping the conceptual underpinnings and without making necessary connections… Alternative assessment measures that can be used by teachers include nontraditional paper-and-pencil tasks, use of open-ended questions, journals, portfolios, focused observation, diagnostic interviews, and performance-based assessments. Most teachers typically use closed-ended questions as means of assessing student performance on quizzes and tests. However, student answers to open-ended alternative assessment questions are usually a better indicator of student understanding about a math concept or skill." However, the effect of considering alternative views provoked Andrew to become more cognizant of testing results and the information that could be obtained from student scores. He is now using analysis of student work to inform his planning of instruction.

Changes in Andrew's knowledge and skills are subtle but nonetheless evident. These are more evident when he completed assignments that engaged him in doing mathematics and those that required an examination of student learning. During the Problem Solving course he wrote "Consequently, after carefully analyzing problems and the mathematics involved, the teacher should enter each problem-solving lesson with a clear agenda for the student's learning. The teacher should create a detailed plan for guiding both the student's thinking about the problems and their reflections on the mathematical ideas that arise. The teacher should strive to balance his role as someone who facilitates instead of dictates. He wants his students to discover important concepts while guiding them to learn what he intends." The learning experience he had not only perturbed him but also provided examples of how mathematics might be taught that he in turn used in his own teaching.

Conclusions

In conclusion, effective professional development can be obtained through meaningful activities that are part of online courses. Through meaningful activities teachers were provided a perturbation that, in this case, lead to change in views on teaching. While this may seem to be a simplistic conclusion, the findings in this case study have lead one of the researchers to reexamine the types of activities provided in face-to-face courses so that more students are presented with perturbations that lead to change in teaching practices. Careful alignment of courses and activities is a powerful way of offering a coherent program of professional development that may lead to fostering teacher growth whereby instructional practices are changed. Further, an inclusion of content knowledge that perturbs teacher learning coupled with an examination of student learning of the content promotes teacher reflection with changes in teaching practice. Finally, active learning among practicing teachers can be an integral component of online courses in order to engage teachers in reading, discussing and writing on important educational issues.

Core features of effective professional development are useful in analyzing how an online degree could cause perturbations within teachers whereby meaningful changes in practices may occur. A limitation of this study is the use of self-reported information as a data point in triangulation. However, Andrew's interactions with other classmates, primarily through discussion board assignments lead the researchers to accept his descriptions of how he has changed as being reflective of his teaching practices.

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MATH-ANXIOUS, ELEMENTARY TEACHERS' CHANGE PROCESSES IN A GRADUATE COURSE AIMED AT BUILDING MATH CONFIDENCE AND REDUCING ANXIETY

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Some of the findings of a qualitative study that took place in a new graduate course especially designed for elementary teachers with math anxiety are presented. The course focused on developing the teachers' conceptual knowledge, mathematical thinking and problem solving skills, while at the same time helping them reflect on their math experiences and deal with their anxiety. Based on teachers' written reflections and researcher notes as main data sources, the paper describes the teachers' experiences during the course, including their learning processes, affective responses and changes in attitudes, self- perceptions and confidence.

Introduction and Related Research Literature

Nearly all adults and many children in our society have been taught math in a traditional manner utilizing rote learning and memorization, which focused mainly on developing their procedural knowledge but left many of them deficient in conceptual knowledge (Battista, 1994). A low level of conceptual knowledge reduces mathematical power and increases anxiety. It is no wonder then that math anxiety is so widespread in our society (Hembrees, 1990; Evans, 2000).

Math anxiety is commonly defined as a feeling of tension, fear and helplessness that interferes with math performance. There is a rather extensive literature on math anxiety and its educational and personal consequences (e.g. Hembree 1990; Ashcraft 2002; Tobias, 1993). Math anxiety includes both emotional and cognitive components (Ho, 2000) where the cognitive component consists of worry and other cognitive manifestations of anxiety. Highly math-anxious individuals hold negative self perceptions about their math ability, espouse negative attitudes toward math and tend to avoid math. They usually take fewer elective math courses in their secondary and higher education which ultimately undercuts their math competence. As can be expected, math anxiety has been shown to be inversely correlated with math achievement (Hembree, 1990). Furthermore, psychological studies have shown that math anxiety can seriously interfere with cognitive functioning during math activity by compromising ongoing activity in working memory, the system for conscious, effortful mental processing which is required for most math tasks. (Ashcraft, 2002).

Researchers have extended early work on math-related anxiety and attitudes to include the broader perspective of the 'affective domain' (or 'affect') in math. The latter was initially defined as consisting of three facets of affective states: emotional states, beliefs and attitudes (McLeod, 1992; 1993). Later on values and morals were added as a fourth facet of affect (deBellis & Goldin, 1997). Much of this research dealt with student affect during non-routine mathematical problem solving (McLeod, 1992; 1993).

Math anxiety and negative attitudes toward math are also commonly found among pre-service and in-service elementary teachers (Hembree, 1990; Bush, 1989; McCulloch Vinson, 2001)). Teachers with higher levels of math anxiety often lack mastery of fundamental math concepts or problems solving skills (Cohen & Green, 2002). There have been some claims as well as limited evidence that math anxious teachers may unintentionally pass on their negative feelings and attitudes to their students (Jackson & Leffingwell, 1999; Karp, 1991, Martinez, 1987). However, this claim has been debated by other researchers (e.g. Bush, 1989, Swetman, 1994).

Recent math reforms have added new dimensions to teachers' math-related anxiety (Battista, 1994). A reform-based curriculum includes not only unfamiliar content, but also educational philosophies and instructional approaches that teachers may have never been exposed to. It is therefore of utmost importance that we learn more about math-anxious elementary teachers and develop ways of helping them build a solid foundation in math and reduce their anxiety.

This paper reports findings from the second phase (Phase II) of an ongoing study of math-anxious, elementary teachers. The study examines the effects of math anxiety on teachers and their math teaching, and focuses on the development of a holistic approach to empowering math-anxious teachers in rebuilding their math knowledge and reducing their anxieties. Phase I of the study involved a group of twelve elementary teachers who participated in a series of eight Math Empowerment Workshops (Cohen & Green, 2002).

Data collection for the current study (Phase II) took place in a new graduate course for math-anxious teachers. The course provided a learning environment where teachers with math anxiety could build conceptual knowledge, develop their mathematical thinking, gain confidence and deal with their anxiety. The one term course utilized a holistic approach combining group problem solving and hands-on math explorations with group reflections, journal writing and guided visualization activities. This paper provides an overview of the course and then describes participant experiences, focusing on their learning processes, affective responses and change processes that course members went through as they advanced through the course.

Methodology

The study was a qualitative, action research-like study (McKiernan, 1991) in which the first author acted as a course instructor/researcher, with the second author as her teaching/research assistant in this new graduate course. The course, titled: *Gaining Confidence In Mathematics: A Holistic Approach to Overcoming Mathematics Anxiety*, included eleven classes during a six-week summer (July-August) term. Each class was three to three-and-a-half hours long. The class included eighteen members in total, thirteen of whom were in-service teachers. Four were pre-service teachers in a two-year master's program and one was a college math instructor. All but three of the course members had varying degrees of math anxiety. The three who were not math anxious and had enrolled in the course out of research interest, acted as "coaches" (or mentors) for their math anxious classmates during math work, as did the course facilitators.

Participants

The study focused on nine female course members who identified themselves as highly math anxious at the beginning of the course. However, the citations in this paper are only based on data from five of these teachers. One of them was a pre-service teacher and the other four were practicing teachers with a range of prior teaching experience.

Course Structure and Activities

The course was taught using reform-based principles. A highly supportive and 'safe' learning environment was created where the math-anxious teachers were more likely to take risks. The math component of the course consisted of hands-on explorations, mental math and problem solving, oral and written communication activities, math games and class discussions. Math work was done in groups of 2-4, where group members were instructed to first attempt the activity on their own, and only then share with their group members. The content covered included whole numbers and rational numbers in their various forms and representations. The confidence building component included journal writing, group reflection, two guided visualizations, strategies for dealing with anxiety and inner criticism, and discussions on various affective issues.

One of the course assignments consisted of keeping a learning journal documenting personal learning experiences, insights and reactions to readings assigned. Most journal entries were initiated by the participants and written at home, while others were based on probes offered in class, or consisted of responses to assigned readings. The instructor responded to each individual journal entry, offering feedback, praise and encouragement as appropriate. There were also four weekly math homework assignments including a few non-standard math problems each. Course members were also required to write a final reflection paper. Course members' in-class math work and weekly math assignments were never evaluated in this course. Participants got credit for their math work based only on their efforts, willingness to ask for help and perseverance.

Findings reported here are based on only on participants' journal entries written in class or at home, participants' final reflection papers and final questionnaires, and researcher field notes.

Findings

This section describes participants' experiences, learning processes, affective responses and change processes as they moved through the course. The main focus is on how teachers' math knowledge, emotions, behaviour, self perceptions and beliefs in relation to math evolved during the course. We will make use of some of the terms defined in Hannula's theoretical framework as described in the section on theoretical frameworks above.

Teachers' Initial Anxiety and Expectations

When course participants gathered for the first class, a number of them seemed nervous as they hesitantly sat down in their chairs, as if ready to get up and leave at any moment. At the end of the class, the teachers were asked to write their first journal entry describing how they felt about taking this course. Many of them wrote about their initial expectations, worries and doubts; for example: "Since I can remember, I have been extremely anxious about Math. I can recall night after night of crying over math homework, upcoming math exams and math teachers. When I saw the course in the course calendar, I breathed a sigh of relief to know that some help was available…". But once she signed up, "I had my doubts, worrying that this course would turn out to be like every other math experience I have had in the past".

Another teacher explained: "I was interested in taking a course that would help me overcome my own intense math anxiety, but I was afraid that I would be expected to actually 'do' math." Like many of her classmates, she was afraid of doing math in class so everybody would see how 'stupid' she was. After learning that math work would indeed be required, she "..felt like a deer who had wandered unknowingly into a trap. I was here to work on my math anxiety not to learn math, I had already enough difficulty with that the first time around". Yet she decided to stay in the course for at least the first two weeks and give it a try, and proceeded to become one of the most successful math learners in the course.

Another participant wrote: "I came to class today highly anxious about the difficulty level of this course and also about the competence of other students in the class". However, once she learned more about the course and her classmates, her anxiety subsided:

> " I feel relieved that most people in this class are as anxious as me… I was so happy to see that my mathematical abilities will not be assessed in this class. I think that this aspect will greatly increase my willingness to try, at least. I also feel proud of myself that I came to class despite my instincts telling me to run. It sounds silly but I perceive this as any other

self-help class – that the first step to getting help is showing up".

The above citations exemplify the various affective responses that arose during the first class. Some of the teachers' negative emotional reactions, as well as their worries and negative expectations, were a direct result of their prior conditioning during their school years which caused their math anxiety in the first place. However, they were much relieved to find themselves among like-minded classmates. When discussing their goals in taking the course, all math-anxious teachers stated that they wanted to overcome their anxiety and improve their math knowledge and attitudes in order to become better math teachers.

Teachers' Learning, Affective Responses and Change Processes During the Course

In this section we discuss and demonstrate through examples some of the teachers' learning processes, insights, affective responses and changes in self perceptions, beliefs and expectations that occurred as the course progressed.

Conceptual Knowledge Building and Mental Math

As mentioned above, in this course emphasis was placed on the building of conceptual knowledge, while procedural knowledge was deliberately played down. Group mental math, problem solving and work with manipulatives were the main tools used to facilitate teachers' construction of conceptual knowledge. Mental math sessions accompanied the study of most course topics. During these sessions, teachers were asked to first try to solve each problem on their own, *without* utilizing any standard algorithms or memorized procedures. They were to come up with their own ways of solving each problem, relying only on their conceptual knowledge, common sense, or on strategies they or their classmates had previously invented. Group sharing of strategies then followed.

The first few mental math sessions were quite stressful for many who were unable to solve the problems without the standard algorithms. One teacher wrote: "The mental math that we did during this course was probably the part that gave me the most anxiety." She linked her anxiety to "very bad memories" from her childhood when she was forced to play competitive games based on the speed of her recall of math facts. This time around, though, she found a lot of support doing the mental math. She writes: "The best thing about the mental math that we did was hearing the holistic strategies that other people used – I was very surprised to learn how many different ways there are to tackle a question (not even including the algorithmic way!). She then referred to one of the course readings (Mcleod, 1993) which discussed group problem solving strategies for increasing confidence, and added: "There were many times when I was presented with a question that I knew I could never solve in my head! But I used strategies as suggested by McLeod, and I was able to solve things without paper and pencil, something I never thought I would be able to do!" The use of group mental math in this way unlocked the door to the teachers' own mathematical thinking and creativity as they finally started inventing their own strategies.

Reconnecting the "Fractured Schemas"

One of the most influential readings in the course was Richard Skemp's *Schematic Learning,* one of the earliest articles on constructivism in math (Skemp, 1972). Skemp defines *schematic learning* as learning which uses existing knowledge schemas as tools for the acquisition of new knowledge. Utilizing Piaget's terminology, he explains how newly constructed schemas are connected to existing schemas through the processes of assimilation or accommodation. Upon reflecting on this article, one of the teachers noted in her reflections that "If students have gaps in their knowledge or are unable to connect their schemas, then learning difficulties arise." She then went on to discuss possible reasons for such knowledge gaps and what teachers can do to help students fill in the gaps. She then proceeded to discuss her own learning:

> "I am pleased that the holistic techniques used in this course helped me unlearn concepts that I found confusing, and reconnected my *fractured schemas* [emphasis added]. I was supported by working in groups, and because I had manipulatives which helped me see the relationships between concepts."

Skemp's article has provided this teacher and her classmates with a deeper understanding of the process of knowledge construction they were going through in this course.

Affective Responses During Problem Solving

The teachers experienced various affective responses during problem solving, similar to those reported in the literature (McLeod, 1992; 1993). They often experienced changes in mood based on their experiences, as seen in the following example from one teacher's journal after the third class:

> "Today was a very interesting class for us because I had a breakthrough concerning my math anxiety, as I worked through the fractions problems that were provided. … I found myself becoming more confident as I realized that I could do a lot of the problems listed! In some ways I became almost cocky because I was so surprised that I could do the work… "

This was indeed a great breakthrough for her. However, a little later when she encountered a more difficult problem, she could not understand what the problem was asking her to do. So "Immediately my confidence became anxiety, I became confused and all of a sudden I felt too tired to do any more work, or try to figure out this question." She then sat back and waited until her group members solved the question and explained it to her. However, she could not understand their explanation. When the course instructor came over to offer help, she was unable to understand her explanation either.

The above example is interesting because of the sudden change from feeling confident and becoming "almost cocky" to a state of anxiety and helplessness. Indeed anybody encountering difficulty in understanding a math problem may experience some frustration and a drop in confidence. However, this teacher's inability to benefit from other people's help indicates that she probably over-reacted to the situation and got into a state of debilitating anxiety. About three weeks later she described a similar situation in her journal: "…The anxiety I feel clouds my mind and I am usually unable to figure out what I have to do to solve the problem". The 'cloudy mind' she described here and in the above example was probably caused by a serious interference with her mental functioning caused by her anxiety, as described by Ashcraft (2002).

Learning to Ask for Help

As the teachers reflected on their experiences during math work they often discovered their own limiting beliefs and behaviours which originated from their prior conditioning. We now present a follow up to the example of the last section. The same teacher who was unable to understand the explanations given by her peers and the instructor, later in the same class had an important realization. It all started when the instructor "said something that made me think. She stated I shouldn't be afraid to keep asking for explanation until I understood it." She then started reflecting on this statement. Her first thought was: "there is no way I am going to keep asking until I understand people will think I am stupid! I was usually willing to ask once and even if I didn't understand I would simply nod and act as if I did… ". She then realized that her fear of feeling stupid was linked back to her own schooling "where asking even one question, or admitting confusion could lead to ridicule from the teacher." She also realized how quickly "any confusion or inability to answer a math question can send me from confident to anxious. ….".

Following these realizations, this teacher subsequently started "allowing myself to rely on others, without feeling stupid or ashamed, for assistance when I was unable to understand, or complete, math related work…". She no longer was afraid to ask for help.

Changes in Teachers' Attitudes, Self Perceptions and Confidence Level

As the course progressed, the teachers started feeling more empowered and resourceful when solving math problems. The teacher described in the last two sub-sections finally learned how to stay "clear headed" during problem solving. In the fourth week she wrote: "When I received this assignment I was able to calmly read it over and try to figure out exactly what I had to do to solve the problem. I think I am able to look at math problems more calmly and not immediately assume that I will not be able to solve them." Apparently she has been successful in overcoming the negative effects of her prior conditioning which used to "cloud her mind" by staying "clear headed". Her beliefs about her ability to solve math problems have also changed.

Another highly math anxious teacher became empowered in a different way. On the fourth week she wrote: "I was also afraid to take risks, something I am working on in this class, by answering questions and trying new methods." Indeed she started volunteering to explain her mental math strategies to the class, something she would have never considered previously.

Four weeks into the course another teacher wrote: "When I began this course, my math anxiety was so high that I literally felt sick when thinking about math related things…" She then proceeded to describe the amazing changes she was going through: "I can't believe the transformation that has already occurred in my mind throughout the last few weeks. I'm so excited about math these days that I keep asking Tom (my fiancé) to ask me math questions! I just can't believe that I can figure out how to do percentages and decimals! Even writing this, I'm emotional (a bit teary) because I can't believe that at 28 years old, I finally get it!" In her final reflection, she writes: "I think the most revealing part of this experience has been the realization that I have always had mathematical abilities but I just didn't recognize them as such. I always thought that my mathematical reasoning was a way of avoiding the "right way" to do it. I think that the biggest part of this experience has been figuring out different ways of calculating mathematical answers that are more conducive to my way of thinking". She then proceeded to discuss her plans for taking more math courses in the near future.

Another teacher wrote: "I started to find the problem solving fun; that's how I know that I am ready to get back onto the journey of learning to enjoy math again. Armed with holistic strategies to approach problems logically, I feel ready to put away my calculator, and start challenging myself in ways that I wouldn't have dreamed of a few months ago." However, like many of her classmates, she admitted that she still had a long way to go on her math learning journey.

Conclusion

As shown in the examples above, the teachers went through significant positive changes in their math related affect during the course. Referring to the three components of affect as defined by McLeod (1992, 1993), we now briefly examine the changes in the teachers' emotions, attitudes and beliefs in relation to math. The teachers started to feel more confident in math situations. By being exposed to assigned readings such as Ashcraft's (2002) article, they gained a better understanding of how their anxiety interfered with their mental functioning during math activities, which in turn helped them learn how to stay focussed and relaxed while solving problems. While their math anxiety was significantly reduced, they had still not fully overcome it, as one teacher wrote in her final questionnaire: "I am still anxious with certain types of problems or math concepts, however, it doesn't prevent me from attempting math questions." The teachers' attitudes toward math have also greatly improved, as can be seen from the citations above. Having been exposed to reform-based teaching that emphasized conceptual understanding and mathematical thinking, the teachers' beliefs about and conceptions of math have also significantly changed. But probably the deepest change was witnessed in the teachers' self perceptions. As one of them wrote: "In just six weeks, my perception of my math skills has changed from me as a dud to me as a mathematician." While not all of them perceived themselves as mathematicians, they did perceive themselves as capable math learners and problem solvers.

Endnote

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IMPACT OF ALTERNATIVELY PREPARED TEACHERS IN URBAN MATHEMATICS CLASSROOMS

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This study utilizes storytelling to analyze the impact of alternatively prepared secondary mathematics teachers in urban classrooms. The critical shortages of mathematics teachers in our nation's classrooms have prompted a move toward non-traditional pathways to teacher certification whereby fully certified teachers can enter the classroom with less time in teacher education programs. As such, a debate has emerged over what determines quality teachers for our nation's students. For many colleges of education, an ongoing challenge is: can we prepare high quality teachers through non-traditional pathways—especially for urban classrooms? This study investigates the connections and disconnects between the realities of urban mathematics classrooms and an alternative preparation program for secondary mathematics teachers.

Introduction

Coupled with the influx of alternatively prepared teachers into our nation's classrooms for grades kindergarten through twelve is the issue of teacher quality. As we explore and define teacher quality with respect to urban school environments, we are faced with the fact that a high percentage of alternatively certified teachers are teaching in urban settings (Wilson, Floden & Ferrini-Mundy, 2001). The impact of these teachers on the education of students in urban schools is of concern at both the local and national levels (Ingersoll, 2001). Therefore, there is a need to understand how these teachers' experiences in the alternative teacher preparation program prepare them for urban classrooms and to understand the impact of these teachers on student achievement. This study examines alternatively prepared teachers' perceptions of their impact on urban student achievement in secondary mathematics.

With respect to the alternative certification program, we were aware of the reputation of our program and were knowledgeable of our graduates who had become leaders in their schools and school districts. However, we had not, at that point, conducted the research to determine the program's strengths and weaknesses in producing high quality mathematics teachers who remained in teaching and continued to grow professionally. In particular, we did not have the research-based data to determine how well prepared the graduates were for urban schools, and the impact they have on students and student achievement. For that reason, we have taken the challenge to investigate the perceptions of teachers who graduated from an alternative teacher preparation program and are serving urban mathematics classrooms. Specifically, the purpose of the study was to understand teachers' perceptions of their experiences that depicts the connections and disconnects of an alternative teacher preparation program and the realities of their urban mathematics classrooms, and the characteristics of a highly qualified mathematics teacher. The study seeks to reveal the meaning and essence of the human experience through the lens of two high school mathematics teachers. The following research questions served the purpose of this study:

> What are alternatively prepared high school mathematics teachers' perceptions of the connections and disconnects of their program experiences and the reality of the urban classrooms?
>
> In the alternatively prepared mathematics teachers' opinions, what are the characteristics of

a highly qualified mathematics teacher?

The Alternative Teacher Preparation Program

This nontraditional approach to certification in secondary mathematics has been in existence for the past ten years at our university. Applicants to the program must hold at least an undergraduate degree in an area that includes a background in mathematics. Usually students entering this program hold a bachelor's degree in applied mathematics, science, engineering or an equivalent field. The program of study is 45 semester hours with 15 hours in graduate mathematics, 12 hours in mathematics education and 18 hours across Instructional Technology, Educational Psychology, Research and Measurement and Social Foundations.

This program is built upon the ten principles of the Interstate New Teacher Assessment and Support Consortium (INTASC), standards of the National Council of Teachers of Mathematics (NCTM), and the National Educational Technology Standards (NETS). The design of the program is based on constructivism (von Glaserfeld, 1995) and the work of Shulman on the Stanford Knowledge Growth in Teaching Project (Shulman, 1986). The environment is interactive, reflective, and based on inquiry. Shulman (1986) suggested that the transformation of subject matter for teaching occurs as the teacher *critically reflects* on and *interprets* the subject matter; finds multiple ways to *represent* the ideas; *adapts* the material to students' abilities, gender, and prior knowledge; and *tailors* the material so that students can be successful.

Through collaborative groups and reflective activities, students experience mathematics pedagogy and instructional planning through microteaching experiences under the guidance of university professors and participate in school-based internships under the guidance of middle and secondary school mentor teachers. Based on the required content background, the admission procedures, the strong integration of technology, the program of study with respect to course content and experiences and alignment with standards, the program is aligned with the six dimensions for high quality teacher preparation programs set forth by the National Commission on Teaching and America's Future (2003). These six dimensions are: (1) careful recruitment and selection of teacher candidates; (2) strong academic preparation for teaching, including deep knowledge of the subjects to be taught, and a firm understanding of how children learn; (3) extensive clinical practice to develop effective teaching skills, including an ability to teach specific content effectively, at specific grade levels, to diverse students; (4) entry level teaching support through residencies and mentored induction; (5) modern learning technologies that are embedded in academic preparation, clinical practice, induction, and ongoing professional development; and (6) assessment of teacher preparation program effectiveness. With respect to the sixth critical dimension, ongoing research studies are in progress including this current study.

Since the inception of this program, over 200 graduates have taken jobs in suburban and urban school districts. The program has earned respect among the school districts as a program that produces high-quality teachers. This honor is based in the exemplary teaching practices of the graduates having the support of an induction program that follows them through the first three years of teaching. They have earned the reputation of being well prepared educators who remain in teaching, continue to grow professionally, and become instructional leaders in their schools.

Conceptual Framework

The phenomenon under study is the teachers' perceptions of a relationship between their alternative preparation and their teaching in the urban environment. Specifically, we aim to identify and describe the subjective experiences of the graduates of the alternative teacher preparation program. Phenomenology as a research design was chosen to study the deep human experiences in the alternative teacher preparation program and the urban classrooms (Schwandt, 2001; Blodgett-McDeavitt, 1997; Husserl, 1970). This design provided the flexibility to take the rich descriptions of the teachers' experiences and reduced them to underlying common themes that resulted in short *focused* descriptions in which every word accurately depicts the phenomenon under investigation (Blodgett-McDeavitt, 1997; Moustakas, 1994). Through a research process in phenomenology, known as epoche, it was recommended that researchers set aside their taken-for-granted orientation to the phenomenon under investigation (Holstein & Gubrium in Denzin & Lincoln, 1994). Researchers must be aware of and engaged to minimize/remove any prejudices, viewpoints or assumptions regarding the phenomenon under investigation (Merriam, 1998). Then, they are able to see data from new, naïve perspective from which fuller, richer, and more authentic descriptions can be rendered (Blodgett-McDeavitt, 1997).

Phenomenologists focus on how we put the phenomena we experience in such a way as to make sense of the world and, in so doing, develop a worldview. There is no separate (or objective) reality for people. There is only what they know their experience is and means. The subjective experience incorporates the objective thing and becomes a person's reality, thus the focus is on meaning making as the essence of human experience. (Patton, 2002, p.106)

Methodology

In a phenomenological study, "the investigator writes research questions that explore the meaning of that experience for individuals and asks individuals to describe their everyday lived experiences" (Creswell, 1998, p.54). Additionally, data are collected from individuals who have experienced the phenomenon under investigation. Data for this study were collected through an interview process known as storytelling. Stories are defined as socially constructed accounts of past events that are important to members of an organization (Hansen & Kahnweller, 1993). These accounts are seldom factual, however, they reflect what people believe should be true. They differ from gossip because they have a moral. Stories permit researchers to examine perceptions that are often filtered, denied, or not in subjects' consciousness during traditional interviews. "Stories happen naturally as a way of telling one's perceptions of past events, problems, or people … They are easy to follow, generally entertaining and are more likely to be remembered than other forms of written or oral communications" (Hansen & Kahnweiler, 1993).

A storytelling approach was employed in the interviews to capture the perceptions of the teachers' experiences. This approach allowed them to convey their understanding of their relationships between their alternative preparation and their teaching in the urban environment (Eisner, 1998; Mertens, 1998). A storytelling script was used as an interview guide. Utilization of storytelling allows a unique aspect of analysis with respect to story components. Stories can have main characters, motivating difficulties, heroes, villains, turning points, and morals. After the teachers told their stories, the following questions were asked: (1) Who is the main character in your story? (2) What is the motivating difficulty? (3) Who or what is the hero(s) in your story?

(4) Who or what is the villain in your story? (5) What is the turning point in your story? (6) What is the story's moral? (7) Is there anything else you would like to add?

The participants were two high school mathematics teachers who were graduates of the alternative teacher preparation program. The participants taught within the same school district but at different high schools. Both teachers participating in the study are female. We gave them the pseudonyms Michelle and Noreen. Michelle is an African American teacher who taught two years on a provisional licensure prior to becoming a TEEMS student. While in the TEEMS program Michelle continued to teach full-time. She completed the TEEMS program in 2003 and has worked one year on a full certification with a Master's Degree. The school demographics for Michelle are 99% African-American and 1% other. During 2003, Michelle taught Algebra II regular and honors classes. Average class size was 20 students.

Noreen is a Caucasian teacher who completed the program in the year 2000 and is in her fourth year teaching since she completed the program. While in the program, Noreen was a full time student. Noreen has taught in the same school since completing the program. The demographics of Noreen's school was 70% African-American, 28% Caucasian and 2% other. Noreen taught Geometry honors, Geometry regular, and Advanced Placement Economics. Average class size was 22 students.

We informed the teachers that the goals of this study were to understand alternatively prepared high school mathematics teachers' perceptions of: (1) the connections and disconnects of their program experiences with respect to the reality of their urban mathematics classrooms, and (2) the characteristics of highly qualified mathematics teachers. The teachers were asked to tell a story about mathematics teaching and learning, and informed the story should include their ideas about their impact on their students' achievement. This story could be about any event that occurred within the last six months. It could be simply relating an incident that was interesting. The story should have a hero, a villain, a turning point, a moral, and anything else the teachers may want to add.

The research process, known as epoche, was maintained in the study (Holstein & Gubrium in Denzin & Lincoln, 1994). We bracketed the transcribed teachers' stories. Through this process we were able to dissect the stories in searching for essential structures. Data was extracted from each teacher's story about her experiences in the alternative teacher preparation program and the realities of their urban classrooms. Experiences that highlighted the content, pedagogy, professional preparation and the clinical practices were the themes used to demonstrate the connections and disconnects between the program and urban classrooms. The characteristics of a high quality mathematics teacher were also extracted from their stories.

Findings

Phenomenological analysis led to rich descriptions of the teachers' experiences that we were able to reduced to underlying common themes. In each of the stories we found evidence of the teachers' views on classroom culture, management of students, teaching content, cognitive aspects of student learning, and affective aspects of student learning. Below we first provide a brief description of the essence of each teacher's story followed by excerpts from the stories with respect to each common theme.

Michelle – African-American teacher

Michelle's story focused on a young man in her regular Algebra II class who was having problems in school. His problems were directly related to verbal abuse and ridicule from his peers. Michelle tells a story that describes how the student was able to overcome his problem through mathematics and with caring and concerned teachers.

Culture of the Classroom

The student, who is the main character of her story, was often teased by his peers. The student's attitude in class caused him to be labeled as a troublemaker who had lots of problems with many of his teachers and peers. Michelle described Ricky as the motivating difficulty in her story. "He was very smart, but would not succeed in class because of his attitude, personality and the way he was interacting with other people in the classroom."

Classroom Management

Michelle decided to challenge Ricky's mathematical ability while simultaneously using mathematics as a way to have a positive impact on his behavior. She placed Ricky in honors Algebra II. Prior to placing Ricky in the honor's class, Michelle viewed Ricky as not a sociable person, but a very intelligent student. During this time, Ricky's conflicts in class were severe enough that he was referred to the Individual Education Program (IEP) for behavior disorders. However, with the support and encouragement of Michelle, Ricky was mainstreamed into the honors class as opposed to being placed in a self-contained classroom environment. Michelle stated that the turning point in her story came when Ricky began to like the benefits of the honors class. He liked the people to think that he was smart; he liked being in a class thinking that of him-- number one ranked. He liked the feeling of being sharp, when he got feedback like "Wow! Ricky, Ricky you are going to do this?"

Content Focus

Michelle identified ways in which she could use the teaching of the content to engage a student in doing mathematics. An interesting point in this story is that mathematics was used to turn a student's behavior around. This is an uncommon approach with respect to mathematics teaching and learning--especially when the student is in an urban school and is on the verge of being placed on an IEP. Michelle recognized the student's ability to do mathematics as well as his capacity to pursue and advanced level of mathematics.

Michelle stated, "… Amazingly, because it became a way of life, he did not act so badly in class, he listened to what the teacher had to say, he respected the decisions that she made as far as his education. He tried his best to be the top in that class and he started to get along with people in that classroom, so as far as math social skills he was doing really well, to the point where his math teacher could go to other classes and get him to calm down in those classes." The content focus was strategically used to raise the cognitive level of learning, which had a positive effect on the student's self-esteem and behavior.

Cognitive Aspect of Learning

When focusing on student learning, Michelle stated, "… he is not the type of person he thought he was. He was better than what he thought he was. He also liked the acclaimed 'he got it!' He was honored at school assembly and the fact that people knew he was smart in math. The other students in the class also made him feel that way when they would question, 'What else is he smart in', 'what else can he do?', 'What else is he good at, that he's hiding?' Those are the things worth it for him."

Affective Aspect of Learning

Michelle's story includes an overwhelming focus on the affective dimension of learning. She described the student's attitude and personality as impediments in his learning. Michelle described the villain in the story as;"whatever caused Ricky to just feel that way." Michelle emphasized Ricky's change in behavior after being placed in the honors class, "amazingly it became a way of life, he did not act so badly in class, he listened to what the teacher had to say, he respected the decisions that she made as far as his education."

Noreen -Caucasian teacher

Noreen told a story about her regular geometry class. This was a class in which Noreen had many challenges during the school year. Noreen's description of the class was, "I think these kids have a lot of issues and are used to being yelled at a lot and there is not a lot of positive feedback. Their educational experience is not very positive. Some of them are in geometry again!"

Culture of the Classroom

This class was the main character of Noreen's story. Noreen stated that absence was rampant in this class. She emphasized that this class was one of the hardest things she had to deal with. In the description of the class, Noreen spoke of the students' negative attitude towards mathematics. "The students thought that mathematics was torturous". In her story, Noreen described how she worked diligently to develop an environment of trust among the students.

Classroom Management

When it came to managing this class, Noreen pointed out, "I got the yelling phase out of the way early, because the yelling didn't work for me and it wouldn't work for these kids. They are yelled at all the time, so yelling is like music to their ears, it does nothing and you wouldn't get their desired response."

Content Focus

When speaking of the content, Noreen spoke of specific topics in geometry. She focused on the teaching of transformations, triangles, quadrilaterals and circles. "So first semester we spent our journey to get through the basic geometry terms and we had logics, I laugh every time we did some logics because they don't really think that way. So we did logics and then we did triangles which are a huge, huge, a huge part of geometry tons of stuff that go on with triangles, there is so much stuff and it is, there is so many fascinating things to do with it that I could have done with them if I had the time."

Cognitive Aspect of Learning

Noreen described these students as hands-on visual learners, initially not logical in their way of thinking, and lacking in ability to apply abstract thought. Noreen was aware that these students needed to be taught in ways that would actively engage them but was not confident that she was effective in this type of teaching. "And I think there are some tasks and there is information out there that I am not using that I should do more hands-on with these guys because… But there are so many neat things that geometry lends itself to; I would say that of algebra but especially geometry so many visual things and these kids are very hands-on visuals all that kind of stuff that we could have done had I had more time."

Affective Aspect of Learning

Noreen focused on several aspect of the affective domain in her story. She was concerned about establishing a classroom climate of trust and impacting students' disposition toward mathematics. She was aware of the negative experiences these students had with mathematics and was determined make a difference in the students' dispositions toward mathematics. Noreen concluded her story with, "…our final journey was a project, I had done this before, was a project on models, scale models and we used surface areas and volumes to build our castles. They had to use solids, prisms and that kind of thing and I had an accident and so I was out of school for three days and that's when the projects were due. And I came back in my room and it was like Christmas, it was beautiful. I had castles all over my room and it was wonderful it was really neat and these kids, I also feel like I taught their class like I taught the honors class, so there were some things I changed."

Results

After telling their stories, the teachers were asked reflective questions that facilitated analysis of the stories. The teachers were asked to point out connections and disconnects between their classrooms and the alternative certification program based upon the story they told. Next the teachers were asked to define the characteristics of a high quality mathematics teacher based upon their stories. Through our analysis of the stories and the teachers' responses to the reflection questions, we addressed the research questions:

> What are alternatively prepared high school mathematics teachers' perceptions of the connections and disconnects of their program experiences and the reality of the urban classrooms?
>
> In the alternatively prepared mathematics teachers' opinions, what are the characteristics of a highly qualified mathematics teacher?

The teachers' stories provided a repertoire of their perceptions of their teaching practices. In our analysis of the teachers' stories we found common experiences in the alternative preparation program that the teachers cited as crucial in sustaining them in the classroom**.** Common themes included: (1) teaching mathematics via a hands-on approach with secondary students, (2) knowing how to integrate technology in ways that engaged students in thinking about mathematics, (3) using a project-based approach to engage students in learning, (4) possessing the skills to plan lessons that include probing questions and (5) going into the classroom as a new teacher with field experiences that allowed them to grasp ways in which to manage students, time and materials.

In their stories on the characteristics of high quality teachers, the teachers repeated many of the common themes stated above. However, it was found that the teachers mostly focused on students in this part of their stories. The teachers' main focus was on their abilities to impact student achievement as an indicator of teacher quality. Additionally, we found they made connections between students' mathematical dispositions and teacher quality. An interesting finding was that both teachers connected student engagement and student love for mathematics to teacher quality.

When asked to tell their stories about least useful components of the program in their teaching, the teachers pointed out that certain qualities needed for effective teaching could only be developed with experience in teaching overtime. For example, Noreen emphasized the only way one comes to learn to manage records for 150 students is by doing. Michelle pointed out that the technology used at the university is not available in her school and she cannot teach mathematics as technologically rich as she learned in her program.

Given the impact that alternatively prepared teachers will have on the mathematics achievement of urban students, this study is extremely important for mathematics teacher educators and school districts. Further, this study makes contributions with respect to preparing and sustaining high-quality mathematics teachers in urban schools.

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TEACHERS GUIDING INQUIRY/ARGUMENT CULTURES: TWO CONTRASTING ORIENTATIONS

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A three-year longitudinal study of elementary mathematics teachers had as its focus describing trajectories of teacher change. In particular, the focus of this paper is on two case study teachers who over time changed to successfully support classroom social interaction that would be described as inquiry/argument discussion contexts (Wood & McNeal, 2003). In the analysis, we identified two distinct trajectories of teacher change that we attribute to two different orientations for mathematics teaching—one of 'scaffolding students' and one of 'scaffolding mathematics'. An awareness of the trajectories of teacher change can inform professional development efforts.

Reform documents describe mathematics classrooms in which students develop deep understanding of mathematics content and in which the teacher supports student learning rather than acting as a dispenser of knowledge (NCTM, 1991; 2000). This vision of effective mathematical teaching demands that teachers have a deep understanding of mathematics content and knowledge of how this understanding might develop in order to guide students toward learning goals. Likewise, in order to support practicing teachers' continued learning, teacher educators need to understand the challenges and complexities of learning to teach in a reform-minded way, what the teachers learn from professional development efforts, and how they learn it (Putnam & Borko, 2000; Franke, Carpenter, Fennema, Ansell, & Behrend, 1998; Jaworski, 1998).

Successful future work of teacher educators will rely on the generation of research regarding the process of change for teachers. Analysis of the process of teacher change can help researchers develop theoretical trajectories of teacher change. These trajectories of change can be useful in identifying sites for teacher intervention so that we may identify what kinds of support we can provide teachers (Simon, 1997).

In our research, we analyzed six case studies of teachers' changing instructional practice to describe trajectories of teacher change. Closer analysis revealed that differences existed among reform classrooms in the teachers' proactive instructional decisions and their trajectories of change. I discuss two teachers who were successful at supporting a classroom culture with increased student responsibility for thinking and increased student responsibility for participation. The teachers provide an example of one of the differences evident in teachers' trajectories of change. It is challenging for teachers to develop instructional practice and the change often spans years (Fennema & Nelson, 1997). An understanding of these differences can help teacher educators support teacher development.

Theoretical Perspective

Our approach to analyzing instructional practice and changes in instructional practice reflects a view that teaching encompasses knowledge of content and students, but also involves supporting social interaction. Our analysis of the case studies focused on the proactive nature of a teacher's role in guiding inquiry in mathematics classrooms. We acknowledge the critical nature of the teachers' role in managing student learning, orchestrating discourse, and in the negotiation of the classroom norms (Simmt, Calvert, & Towers, 2002). Wood & McNeal's

(2003) framework describes three distinct social interactions or discussion contexts: Conventional, Strategy Reporting, and Inquiry/Argument. In a conventional classroom culture, discussions are characterized by a lack of demand for student responsibility for thinking and participation. Students recall correct answers and prescribed procedures; teachers evaluate answers offered by the students. In contrast, the norms in an Inquiry/Argument discussion context are that students justify answers and ask for clarification if they don't understand something; teachers ask for justification, challenge students, and ask questions to probe students' thinking. Wood & McNeal's (2003) analysis revealed that the frequency and complexity of teacher questions and demands on student thinking increased in the Inquiry/Argument culture.

Though our approach to analysis of changes in instructional practice of teacher's changing practice included interactions with students, further analysis also included teachers' learning goals, teachers' plans for learning activities, and teachers' selection of tasks. Simon (1995) described the mathematics teaching cycle as the relationships among teacher knowledge, the Hypothetical Learning Trajectory (HLT), and interactions with students. The Hypothetical Learning Trajectory consists of a teacher's learning goals, planning for learning activities, and a hypothesis of how the learning might proceed. Simon & Tzur (2004) elaborated on the hypothetical learning trajectory to include the selection of tasks and the role of the tasks in the learning process.

Setting & Participants

The six case studies of teachers' changing instructional practice were embedded in a larger study in which the focus was on deepening teachers' mathematical content knowledge and providing support for developing instructional practice (Nickerson & Moriarty, in press). Administrators of a large urban school district developed a plan to increase student achievement by improving instructional practice. The efforts at improving instructional practice in mathematics began with a focus on teachers in eight low-performing, high-poverty elementary schools. The students were 57-91% English Language Learners. The school district administrators made the decision to hire 32 additional teachers as mathematics specialists that taught only mathematics to students in grades 4–6. As part of the plan to improve instructional practice, these teachers took 6 units of university coursework in upper-division mathematics focused on relearning the mathematics they teach. They also took 6 units of graduate education courses and had site-based support from expert teachers on loan from the school district. Finally, they were also afforded the opportunity for shared daily professional development time.

Researchers collected data on six case study teachers over the three-year period to examine changing instructional practice. The focus of this paper is on two case study teachers that routinely facilitate inquiry/argument discussion contexts. We call the teachers Chris and Anne. Chris had 9 years teaching experience; Anne had 4.5 years teaching experience. They were both 5th grade teachers during the time of the study. Prior to this assignment, Chris had taught Kindergarten and first grade. Anne had taught second and third grade. They both were bilingual and had credentials to teach bilingual children. They completed credential programs from the same institution. They taught in schools with similar demographics. They used the same textbook. In terms of education, Anne had a few unconnected mathematics courses prior to beginning as an elementary mathematics teacher. At the time of the research, Chris had completed and Anne was completing a M. A. in Education.

Data Corpus

The data consisted of a set of formal classroom observations by the author at the beginning, middle, and end of each year. These observations were audio-recorded or videotaped with accompanying fieldnotes. I used a summary observation form based on an earlier research project that focused on mathematical ideas, tools, and representations as well as teachers' management of classroom discourse (Sowder, Philipp, Armstrong, & Schappelle, 1998). In addition, the classroom observation data included a scripted record of activities of the teacher and detailed activities of the students during visits to classrooms by the peer coach teachers and university instructors. The observers always conducted post-observation interviews with teachers. Teachers were asked a core set of post-observation questions. The core interview questions included but were not limited to the following questions:

What were your instructional goals?

Were your instructional goals met?

What evidence do you have that your goals were met?

What, if anything, would you change in today's lesson if you were to teach it again? Why?

In addition to asking a set of core interview questions, the interviewer pursued open-ended questions specific to what had been observed. Finally, we asked teachers to complete a survey at the end of the academic year in which they reflected on aspects of their changing practice.

Methodology

The classroom observations were transcribed. Using Wood & McNeal's (2003) framework, we coded transcripts to identify different discussion contexts. Having identified teachers who successfully challenged students to think and participate in Inquiry/Argument discussion contexts, we did a cross-case comparison of teachers. The focus of this analysis shifted from identifying classroom interaction patterns to examining teacher decisions that encompassed more than classroom interaction. Simon's (1997) hypothetical learning trajectory brought more aspects of teacher's practice into focus. In order to understand teacher change and a teacher's learning trajectory, qualitative methods were adopted for this interpretive inquiry. Transcripts of post-observation interviews were the initial focus of inquiry. Codes were developed and other data sources were then analyzed to confirm or disconfirm themes. The initial coding was guided by what the emphasis of the lesson seemed to be, the kinds of tasks the teacher selected and the manner in which the activity was guided, the representations utilized, and the questions posed by the teacher. Our codes were refined through cycles (Strauss & Corbin, 1990). These themes were examined as they related to elements of the teacher's Hypothetical Learning Trajectory. The Hypothetical Learning Trajectory encompasses (1) teacher's goal for student learning, (2) teacher's plan for student learning activities, and (3) teacher's hypothesis of student learning processes.

Results

Two case study teachers who were successful in implementing key aspects of the reform still differed along important dimensions. In our cross-case analysis, we found that not all teaching in inquiry-based classrooms could be uniformly described. As we examined themes of the teachers' development over the three years, we determined patterns that I describe as a teacher's orientation toward 'scaffolding students,' or 'scaffolding mathematics.'

With regard to a teacher's goal for student learning, Chris clearly demonstrated from her learning goals that she was oriented toward scaffolding students. Her articulated goals for student learning encompassed social as well as mathematical goals. She had a goal of students independent problem solving. She worked to embed problems in concrete tasks that she perceived to be connected to students' lives. She often constructed concrete models to support students' understanding of word problem contexts.

In contrast, Anne demonstrated from her learning goals that she was oriented toward scaffolding mathematics. She frequently cited a rationale for selecting learning activities that were geared toward preparing students for the study of algebra. Anne's longitudinal view of goals for learning had her push toward the 'algebraic part'.

Chris' planning for learning activities was in response to the perceived immediate needs of students. She frequently used her assessment of recently completed or shared student work and her assessment of student understanding as a starting place for the next lesson. For example, in one lesson students were asked to partition a drawing of a large square into pieces and then label these pieces with their fractional names. The next day she had created overheads of individual student work and other students were asked to label and justify their labeling of the partitions. Student work would often become the object of class discussion.

In our observations, Anne's students' work never became the focal point of a class discussion. Anne's plans for student learning activities frequently referred to the students making connections to mathematics already learned or preparation for other mathematical ideas. Anne used student work in looking for evidence of student understanding or lack of student understanding. When Anne analyzed students' activity, she focused on an analysis of the task she had presented. For example, in a post-observation interview, Anne reflected on a disappointing lesson in factoring. She expected students to make connections in this arena to a mathematical representation with which they had substantial experience. She said,

> They had difficulty, I think, because it was really disconnected from what they have been doing. I was really disappointed that there wasn't more discussion. (pause) But when I think about it, there wasn't much to talk about, was there? It wasn't a rich task.

Lastly, the teacher's differed on their hypotheses of the student learning process. Chris expressed her sense that students learn mathematics by making connections to their prior learning and experiences in a general way. Anne expressed her sense that students learn mathematics by making connections to their understanding of the structure of mathematics. She sought to connect one aspect of mathematics to another.

Discussion

The teachers interpreted and utilized the professional development activities in different ways. The measure of teachers' increased mathematical pedagogical content knowledge from the larger study would indicate they both learned a great deal of mathematics. Anne expressed that her learning enabled her to connect the mathematics knowledge she gained to a larger structure and landscape. In contrast, the mathematics that Chris learned enabled her to understand what her students were thinking and expressing. She talked about moments, when listening to her students' thinking, she understood something she had not understood before. By her admission, this was empowering to her.

The teachers' trajectory of development was different for the two teachers and the explanation of why they appear so different lies in the orientation the teacher brings to the hypothetical learning trajectory of the mathematics teaching cycle. In both classrooms, students participate in inquiry/argument cultures. In both classrooms, students' thinking and understanding is taken seriously in the planning and implementation of a lesson. But the students' understanding is analyzed through different lenses and orientations–one with a view toward scaffolding students and one with a view toward scaffolding mathematics.

An understanding of teachers' differing development trajectories will enable teacher educators to support practicing teachers' continued learning. In this case, teacher educators can develop an awareness of the need to support teachers' development of both a landmarks perspective and a perspective responsive to children's immediate instructional needs.

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A MULTI-DIMENSIONAL APPROACH TO MATHEMATICS IN-SERVICE[1]

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Ten grade 7-9 teachers participated in a relatively short (7 weeks) in-service program that included five elements: a rubric for standards-based mathematics teaching, self-assessment tools, peer conferencing, information about standards-based teaching, and feedback from researchers. Sources of data for measuring the effects of the inservice on teachers' beliefs and practices were five classroom observations of each teacher, interviews before and after each observation, self-assessments, and participant responses to individual case study reports. The study found that eight of the ten teachers incrementally improved. Lessons learned in the study were that changes in beliefs and practices must move in concert if substantive professional change is to develop, how teachers interpreted the rubric for mathematics teaching influenced their response to the in-service, and that the five elements of the in-service constituted an integrated set.

Objectives:

Despite substantial evidence of positive effects of standards-based mathematics teaching (reviewed in Ross, McDougall, & Hogaboam-Gray, 2002), many contemporary mathematics classrooms are no different than those of the past. Our purpose was to increase implementation of standards-based mathematics teaching through in-service.

Research Perspectives:

The theoretical framework for the study was social cognition theory (Bandura, 1997). We emphasized capacity building at two levels: self-beliefs and instructional skill, with the former prerequisite to changes in the latter. In our conception, teachers develop over time a stable repertoire of manageable instructional practices. These practices are the residue of experience, representing teachers' perceptions of what works for them. Changing their perception of what could work for them involved the integrated application of five in-service strategies:

Rubric for standards-based mathematics teaching

Our first step was to influence teacher perceptions of what constitutes excellence in mathematics teaching. To do this we provided a multi-dimensional rubric of instructional practice. Previously we conducted observations and interviews with mathematics teachers who ranged from traditional teaching to high fidelity implementation of standards-based approaches (McDougall et al., 2000; Ross, Hogaboam-Gray, McDougall, & Bruce, 2001; Ross, Hogaboam-Gray, McDougall, & Le Sage, 2003). We used these data, governing curriculum documents, and feedback from expert math teachers, to construct a rubric for standards-based teaching. For each of our 10 dimensions, we described four levels of implementation, arranged in a hierarchy of increasing fidelity to NCTM Standards (shown as levels 1 to 4). District math consultants provided evidence of the face validity of the rubric (Ross & McDougall, 2003). We anticipated that use of this rubric would focus teachers' self-assessments, peer observations, and their selection of improvement goals. The rubric describes finer distinctions in teaching than are usually available, increasing teachers' ability to generate goals associated with persistence; i.e., goals of moderately difficulty, achievable in the near future, with unambiguous outcomes (Schunk, 1981).

For example, dimension 8 of the rubric addresses how the teacher provides opportunities for student-student interaction—a key determinant of the frequency and quality of mathematical talk in the classroom. We represented this dimension of practice as containing three aspects: explicit instruction provided by the teacher about how to work in small groups, the task assigned to students in groups, and communication norms. In the rubric, the level 1 descriptions approximate traditional practice. Level 4 represents our conception of ideal practice, in the sense that it maximizes opportunities for rich talk about mathematical ideas among students. The first aspect concerns the directions that teachers give to students about how they should work in the classroom. The levels are:

1. The teacher provides instruction on expected classroom behaviours, focusing on whole class management without reference to student interaction.
2. The teacher provides instruction on expected classroom behaviours, focusing on small group management.
3. The teacher provides instruction and models expected small group behaviours, focusing on general cooperative learning skills and shared group leadership.
4. The teacher provides instruction and models expected small group behaviours, focusing on cooperative learning skills, shared leadership and effective math communication.

The second aspect of the student-student interaction dimension concerns the tasks assigned. The levels are:

1. The teacher assigns tasks that require students to work independently at their desks.
2. The teacher assigns tasks that require students to work independently within small groups.
3. The teacher assigns tasks that require students to work independently and share their solutions with their peers to check for accuracy.
4. The teacher assigns tasks that require students to work together within groups to develop joint solutions and strategies.

The final aspect of the student-student interaction dimension of the rubric concerns the norms about communication established by the teacher. The levels are:

1. The teacher controls question and answer discussions by providing opportunities for students to recite their answers to the whole class.
2. The teacher allows students to describe their answers to peers, either as a whole class or within small groups.
3. The teacher allows students to explain and defend their answers to peers, either as a whole class or within small groups. Students are encouraged to challenge the validity of their classmates' solutions.
4. The teacher allows students to explain and compare their solutions and solution strategies with their peers. They are encouraged to discuss the mathematical concepts within the problems and to be both supportive and challenging to their peers.

Self-assessment tools

Self-assessment provided a mechanism for participants to access the rubric. Our self-assessment tool consisted of four teaching descriptions for each of the 10 dimensions of our rubric for standards-based mathematics teaching. It is accessed through an interactive website http://www.solidcs.net/mathtls.htm that provides immediate feedback in the form of an overall score and individual dimension scores based on four categories ranging from "procedures focus" (traditional instruction), to “constructivist focus” (standards-based mathematics teaching). . In previous research with children we found that teaching students how to evaluate their work increased the accuracy of their self-appraisals and contributed to higher student achievement (Ross, Rolheiser, & Hogaboam-Gray 1999; Ross, Hogaboam-Gray, & Rolheiser, 2002; Ross & Starling, in press).

Peer conferencing

Teacher beliefs about their capacities develop from their interpretations of the effects of their efforts, considering the difficulty of the task and contextual features that might influence outcomes (Bandura, 1997). These private interpretations can be influenced by peers. For example, peer input can direct teacher attention to particular dimensions of practice, can moderate self-assessments with information about outcomes achieved by other teachers, can heighten the salience of specific experiences by praising success, and may weaken the negative effects of failure by suggesting ways to be successful in the future. We focused on peer coaching as a strategy for enabling teachers to access positive peer influences. In peer coaching, pairs of teachers of equal experience and competence observe each other teach, negotiate improvement goals, devise strategies to implement goals, observe the improved teaching, and provide mutual feedback. Positive effects are obtained when a climate of mutual trust, voluntarism, encouragement of reflective thinking, and principal support (McLymont & da Costa, 1998) is developed. Peer coaching increases teacher implementation of sought-after teaching practices and contributes to higher teacher efficacy (Edwards, Green, Lyons, Rogers, & Swords, 1998; Kohler, Ezell, & Paluselli, 1999; Licklider, 1995; Wineburg, 1995). In this study we re-labeled the technique as peer conferencing because two of the pairs involved teachers of unequal experience. However, we asked all participants to treat their partner as an equal, which would not be the case in a mentoring program.

Information about Standards-based teaching

The standards, self-assessment, and peer support were designed to increase teacher aspirations. To help teachers realize these aspirations for instructional change, we provided three half-day interactive in-service sessions containing information about how to implement the Standards. Knowledge input followed Garet, Porter, Desimone, Birman, and Yoon (2001) principles; i.e., ample supply of student learning materials, input from subject experts, collegial interaction to explore classroom applications, attention to beliefs about mathematics, and alignment with Standards.

Feedback from Researchers

We saw ourselves (a team of mathematics education faculty and graduate students) offering constructive feedback on teachers’ entry positions, attempted changes, and observed practices. We observed all teachers in the study teaching mathematics on five occasions and interviewed each about what they were trying to accomplish. Each teacher received feedback on our observations on several occasions, including an individual 5 000 word case report that summarized our interpretations of that teacher’s change trajectory over the course of the project. All feedback sessions were interactive and were designed to be non-evaluative.

The events of the in-service were: 1) self-assessment using an interactive website; 2) in-service on peer observation skills, setting observation priorities, and Standards application; 3) peer observation of teaching, 4) inservice on using peer observation data and input on Standards application; 5) classroom experimentation over 4 weeks; 6) peer observation, and 7) in-service on Standards application.

Methods of Inquiry & Data Sources

The study was an explanatory case study (Yin, 2003) involving ten grade 7-9 teachers, recruited to maximize variation in gender, teaching experience, and commitment to standards-based teaching. Site visits followed procedures established by Simon and Tzur (1999). We visited each classroom in September on two consecutive days during their math period (75-80 minutes per day). We interviewed teachers before, during, and after each math lesson to elicit the teacher's intentions and reflections on the lessons we observed. We recorded key events and contextual detail. The purposes of the second day of observations were: a) to ensure that as many dimensions of teaching as possible were observed; b) to determine the consistency of the teacher; c) to guard against demonstration lessons. Observations of mathematics teaching (self, peer, and external observers) were coded using the categories of the rubric (4 levels X 10 dimensions of teaching), using a template. We observed each teacher on five occasions (observations 1 and 5 were two day events; the remainder were one day).

The data consisted of the self-assessment, our observations, peer observations, individual and collective interviews, teacher responses to our 10-page case reports, and field notes of in-service sessions. Analysis was guided by three questions: In what ways did teachers think they had changed? In what ways did our observations indicate that teachers had changed? What factors contributed to or impeded teacher change. We used NUD*IST to organize the data. Themes were developed through constant comparison. Credibility of the findings was enhanced by 1) triangulating among data collection times and interpreters; 2) maintaining an audit trail by creating charts of relationships and counting instances; 3) searching for negative instances; 4) member checks.

Results

All teachers believed they had improved substantially on the two dimensions of mathematics teaching that each chose to work on. For example, "Victoria" decided to work on building student confidence and student assessment strategies. Her key strategy was to implement a tracking sheet to help students become aware of their progress. Previously she gave her students "praise in the form of lollipops, stickers", encouraging words, and other extrinsic reward strategies based on teacher perceptions of student progress. Victoria believed her students were "not aware that they are doing well or not doing well" and they often asked her about their progress, seemingly unaware of their own mathematical abilities. Her improvement plan focused on helping students develop a sound foundation for confidence in their mathematical ability. When implementing her plan, Victoria asked students "to write something positive [they learned] from their estimation activity on their tracking sheet" so they could identify their areas of strengths. In addition, she continued with previously established strategies such as identifying students who made "unique" or "out of the box" solutions; giving praise for correct responses, while for incorrect responses she would scaffold questions to ensure an entry point for students to provide an answer; and she continued to build students' confidence by reviewing and explaining questions.

Victoria created a problem-solving rubric with her class to make the assessment more transparent to students. Previously she had decided without student input how student work would be evaluated. However, only a few students were involved in the development of the rubric and the instrument was not used until after the in-service ended. Victoria continued using her regular assessment tools, such as quizzes, to assess students' understanding of concepts. What was new was a self-assessment activity in which students marked their own integer homework. In addition, while reviewing an estimation activity, Victoria categorized each question as rounding questions, percent questions, and percent from fraction questions so students could see what general areas they were good at and where they needed improvement. By the end of the project, Victoria felt that she had "moved up a level for Student Confidence and Student assessment".

Our claims about the progress of participants were more modest. For example, our review of Victoria's case indicated that she had made some progress toward the use of standards-based mathematics teaching but the amount of change was small and had yet to be internalized into her practice. We found that the in-service had an uneven effect on participants: for some it amplified an existing commitment to standards-based teaching; for others it enabled them to experiment with new ways of teaching (e.g., Victoria); and for one pair it consolidated their commitment to traditional instruction. We identified three impediments to change and six enablers.

Impediments to Change

Time

Participants believed time was insufficient to reflect on the rubric and incorporate it into their planning; to plan their lessons using new methods; to engage with their peers in conversations about instructional change. The effects of the short timeline were exacerbated by the fact that participants were trying to change two dimensions of teaching at the same time and the amount of mathematics teaching knowledge they received was relatively small. The short time line encouraged the adoption of short term goals and all participants reported difficulty in keeping to the schedule of events.

Person Factors

Although all participants volunteered, three were encouraged to do so by their supervisors, with negative effects on teacher "buy-in". These three also believed their current practices were superior to the methods demonstrated at the in-service by their peers and by the researchers. For two participants, the in-service provided an opportunity to clarify, consolidate, and strengthen their opposition to mathematics education reform. This pair appraised their practice at high levels on entry to the program, denigrated the suggested alternatives, and believed that their practice had improved through interaction with their peer. Teachers' confidence in their ability to bring about student learning, i.e., teacher efficacy in social cognition theory (Bandura, 1997), is usually associated with greater willingness to implement new strategies (evidence reviewed in Ross, 1998). But over-confidence is an impediment to professional learning (Lindsley, Brass, & Thomas, 1995).

Structure of the Self-Assessment Instrument

Although each set of response options in the self-assessment is a continuum, the feedback given by the website implied that the underlying metric is an ordinal scale. Teachers felt their practice overlapped categories, reducing the utility of the self-assessment in ways that we had not foreseen.

Enablers of Change

Climate of the Project

All teachers attributed their improvement to the risk-free opportunities to explore new ways of teaching; i.e., the project gave them descriptive rather than evaluative feedback. Teachers operate in professional isolation, physically (in closed classrooms) and psychologically (through norms of privatism and individualism). Access to the classrooms of other teachers is a rare event, particularly when combined with encouragement to talk about mathematics teaching in a specific course context.

Person Reasons

The four teacher pairs who displayed positive change were those who shared similar views about the nature of math teaching, who had worked together in the past or were seeking opportunities to do so. For example, “Barry” had a history of engaging in personal professional development and his conception of mathematics was that of "an ever changing dynamic construct that allowed him to explore mathematics teaching in new ways continuously". His teaching partner (“Mark”) was a very reflective teacher who was committed to continuously improving his practices to better meet the needs of his students because of his own love for the subject area. These two teachers shared similar attitudes towards mathematics teaching and learning, which enabled their relationship to flourish. Another teacher pair that just began collaborating together during the current school year had different background experiences but both teachers were enthusiastic about working towards changing their teaching practices to reflect the demands of the curriculum. In these teacher pairs, regardless of whether the relationship between the teachers was a "community of practice" developed over past associations or had just been developed over the current school year, the complementary personalities enhanced the teachers' ability to make changes to their teaching practices.

Self-Assessment Tool

The self-assessment helped teachers select improvement goals by providing a menu of options, a mechanism to find gaps between desired and actual practices, a ruler for measuring changes in practice, confirmation of teaching achievements, and a language for talking about teaching. The self-assessment had these effects as a vehicle for selfapplication of the rubric for standards-based mathematics teaching, changing the rubric from a theoretical construct into a practical tool. The aspirations of teachers varied as did the amount of change we observed. But all teachers, regardless of their commitment to the project, used the rubric for standards-based teaching to gauge change in their teaching.

In-Service Workshops

Four of the five pairs believed that the in-service workshops provided worthwhile teaching strategies and resources they could use in their classrooms that were not readily accessible through other means. The instructional techniques came from the classrooms of the graduate student members of the research team, all recently (or currently) classroom teachers. The utility of the instructional methods lay in their specificity to the course contexts of study participants and to the verbalization by the demonstrators of how the specific activities instantiated the theories of constructivist mathematics teaching. Each of the sessions was tailored to the specific dimensions that teachers selected for their improvement goals.

An important corollary of the in-service is that they freed teachers from their regular duties, providing a forum for sharing professional experiences with knowledgeable peers. The good will that motivates collaboration within-schools is frequently frustrated by the exigencies of daily commitments. The in-service provided new time for professional talk.

Observations and Peer-Conferencing

Observation of other teachers enabled participants to formulate new approaches to topics they were teaching. Observing their partners concretized discussions of teaching in peer-conferencing sessions. After these sessions, partners designed activities they could both use; they discussed how their students performed and determined if any changes were needed to make the activity better. Although one teacher felt uncomfortable being watched by adults, he grew more relaxed as the project continued. The other participants reported that they were comfortable being observed by both insiders (peers) and outsiders (researchers). We observed that peers made constructive comments about what they saw when they visited their partner's classroom, suggested ways to improve lesson plans, and offered thoughtful reasons for their instructional choices.

School-Related Issues

Teachers (four of five pairs) who had administrative support felt validated and motivated to change. One pair was ridiculed by immediate supervisors for participating in the project but was able to balance departmental opposition with principal support.

Conclusion

The effect of the treatment was modest. None of the teachers experienced paradigmatic change, nor would we expect a dramatic shift given the limited duration of the project. All of the participants changed in some way, four of the pairs in intended directions.

An important lesson that we learned from the study is that beliefs and practices must move in concert. Receptivity to the project was largely determined by the alignment of project goals with teachers' prior beliefs about mathematics and mathematics teaching. As the project unfolded we observed a pair (Mark and Barry) that used the project to accelerate change in the direction they were already going. These two teachers were committed to the Standards, recognized connections among the rubric, their existing practice, and their aspirations. Both realized improvements in their teaching during the project as they refined their beliefs about mathematics teaching and devised strategies to implement these beliefs in the classroom. We anticipate that this pair will continue their positive career trajectories.

A second pair saw discrepancies and struggled to resolve them. "Cheryl" reported that her classroom practices did not change very much but that her awareness of how she was teaching and assessing students increased as a result of her participation in the project. She felt she had reduced the uncertainty surrounding the new provincial guidelines and believed she was implementing those program expectations to a greater degree by developing performance rubrics to guide her student assessments. We did not see these rubrics in use when we observed Cheryl's teaching and when we asked her about them she said that they were still in development. Cheryl stated that they were "at the back of her head" when she was thinking about assessment issues and that formal classroom use would take further development. Cheryl had started to change her beliefs but not her practice. For "Don", the opposite was the case. He implemented new practices (graphing calculators and Geometer sketchpad) but his beliefs about mathematics teaching and learning had not changed. The result was that his new use of technology did not allow for student discovery of the programs and their functions. His focus on correct operation and achievement of a single correct answer inhibited the use of technology to solve rich problems and stifled the production of rich talk about mathematical ideas. Neither Cheryl nor Don had internalized new ways of teaching. Our expectation is that neither will until progress is made on both the belief and practice fronts, stabilizing in a higher instructional plane.

A second lesson learned in the project is the importance of how teachers interpreted the mathematics teaching rubric that guided the project. There were few overt challenges to the rubric, either to the dimensions selected as core features of mathematics teaching or to the specific descriptions in the levels. It was accepted as a valid representation of what research says about excellence in mathematics teaching. Yet some of the participants disagreed with the sequence of the levels and/or claimed they had achieved a particular level when their practice and their talk about that practice indicated that this was not the case. For these participants the rubric appeared to have authority sufficient to encourage recasting their descriptions of practice to "score" higher on its dimensions. Some participants emphasized superficial similarities between what they did and words or phrases that appeared in the upper levels of the rubric. In doing so they assigned iconic status to the rubric but this status did not increase its meaningfulness. Participants with belief systems that had already moved in the direction of the Standards had a clearer understanding than other participants of what the higher levels might mean in classroom application. We may have overgeneralized from our interactions with the individuals who agreed with us to the sample as a whole. In retrospect we should have spent more time ensuring that all teachers were interpreting the rubric dimensions and levels in the way that we intended.

The third lesson learned from the project is that the five treatment elements constituted an integrated set. The rubric for mathematics teaching was the foundation on which the in-service built. Willingness to move in directions set out in the in-service objectives was contingent upon agreement and understanding of the change described in the hierarchy of levels and dimensions. But without a mechanism for accessing the rubric (i.e., the self-assessment) it is unlikely that participants would have applied the rubric to their own practice. The self-assessment provided a tool for teachers to place themselves on a graduated scale, set priorities for professional change, and a language for talking about their practice with peers and researchers. But self-assessment without support for changing that assessment is likely to lead to discouragement. By demonstrating specific activities contextualized to the courses taught by participants, the in-service provided teachers with concrete illustrations, to be adapted or adopted, to implement new levels of teaching. Peer conferencing strengthened the learning process by confirming the self-assessments, helping teachers apply the models to lesson plans suitable for their own classrooms, and providing feedback on how well teachers were meeting their improvement goals. The final element, feedback from the researchers, was the least influential. There were few references to our feedback in the interviews and there were very few disagreements with our case descriptions of each teacher, even when we explicitly stated our view that one pair had regressed during the project. The influence of the researchers appeared to be indirect, through the design of the in-service activities, especially the rubric.

In summary, the in-service contributed to incremental improvements in eight of the ten cases. We also found the treatment created unintended impediments to professional growth, the most important being our inability to influence the belief systems of participants who disagreed with the Standards. The contribution of the study is the demonstration that multi-dimensional treatments can contribute to implementation of standards-based mathematics teaching.

Endnotes

1. Paper presented at the conference of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA), October, 2004. The research was funded by the Ontario Ministry of Education and Training. This report does not necessarily represent the views of the Ministry. Send comments to the corresponding author: Dr. John A. Ross, Professor of Curriculum, Teaching, & Learning, OISE/UT, Box 719, Peterborough, ON K9J 7A1 Canada.

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LONGITUDINAL STUDY OF PROFESSIONAL DEVELOPMENT TO BUILD PRIMARY TEACHER EXPERTISE IN TEACHING MATHEMATICS

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This research examines a province-wide professional development initiative designed to assist elementary teachers in their teaching of mathematics. The paper discusses the preliminary stages of a large-scale research project developed to study the process of implementing the initiative. A close examination of the complete research design suggests some ways in which research plays an important role to further support and stimulate professional development.

Context of the Study

The initiative in mathematics education is part of a larger project undertaken by the Ontario Ministry of Education beginning in 2002 to assist elementary teachers in increasing their expertise in effective reading and mathematics teaching. The aim of this province-wide professional development initiative is to improve both reading and mathematics in the primary grades across Ontario. The initiative began with the establishment of two expert panels, one for Reading and one for Mathematics which served as the basis for the professional development project known as *The Early Reading and Early Math Strategies*. The research team studying *The Early Reading and Early Math Strategies* is a bilingual team of seven researchers with a broad range of expertise in mathematics and reading education, early elementary education, survey and classroom research, and an interest in the development of professional knowledge[1]. The team also brings both Francophone and Anglophone perspectives to the research to address both French-language and English-language educational systems in the province. In addition, the research will include researchers and research assistants from other universities across the province.

Identification of the Problem

The focus of this paper is on *The Early Math Strategy* which is the professional development project to enhance elementary teachers' expertise in effective mathematics teaching in order to improve achievement in mathematics among children from Junior Kindergarten to Grade 3. There is growing evidence that children learn more mathematics when instruction is based on learners' ways of thinking when they are engaged in meaningful problem solving (e.g., Yackel & Cobb, 1996; Yackel, 1997; Graves & Zack, 1997), and when teachers assist learners in seeing the connections among various mathematical ideas (Gearhart, et. al., 1999; Lampert, 1990). However, mathematics education poses substantial challenges for elementary teachers, who often have insufficient knowledge of the mathematics required to effectively implement reform-oriented mathematics programs (Ball, 1988, 1990; Ball, Lubienski, & Mewborn, 2001; Ma, 1999). Many elementary classroom teachers have difficulty providing rich mathematical experiences, recognizing the mathematical connections that children are making, seeing children's mathematical inventions, and connecting them to mathematical norms (Lampert, 1990). There is increasing evidence that teachers' weak understanding of math may prevent them from recognizing and furthering the important concepts that are inherent in mathematical activity (Ball, 1999; Kahan, Cooper, & Bethea, 2003). A variety of initiatives have been used to address the problem. This particular provincial professional development program is one such attempt to improve the quality of mathematics teaching in learning.

The Early Math Expert Panel Report

The Early Math Expert Panel Report was the starting point for this initiative and played a pivotal role in sharing the research and best practices of teaching mathematics in the early grades. The Expert Panel in its report made recommendations to inform the professional development initiative (Suurtamm & Dawson, 2003) based on the available research. There were three important areas on which they based their report.

Mathematical and pedagogical content knowledge

The panel report identified increasing teachers' pedagogical content knowledge (Schulman, 1987) as central for improving the teaching and learning of mathematics. In addition, if teachers are going to engage their learners in meaningful mathematical problem solving, they need to develop a deeper understanding of the mathematics that may arise as students explore different types of solutions. A more complete understanding of mathematical content will allow them to listen more effectively to their students as well as choose mathematical activities which support mathematical thinking (Ball, 2000). In summary, teachers need to know both the mathematics that they teach as well as the reasons for teaching it in order to create effective learning environments for mathematics.

Opportunities to collaborate and reflect

Professional development should help to build positive beliefs and attitudes towards mathematics, beliefs about learners and learning, teachers and teaching, the nature of mathematics, professional development, and the process of change (Loucks-Horsley, 1998). This can be enhanced by opportunities for teachers to examine their own teaching, discuss student learning, and share their reflective insights with colleagues. Teachers need opportunities for analysis and reflection that include time, space, and encouragement. This may take several forms such as talking with others, keeping a journal, or engaging in action research (Darling-Hammond & Ball, 2000).

The role of the school principal

The principal is central in creating the conditions for the continuous professional development of teachers and thus, of classroom and school improvement (Fullan, 1992). The principal and other administrators need to be actively involved in the professional development process and make informed decisions about professional development at the school level (Payne & Wolfson, 2000; Burch & Spillane, 2001). An effective professional development program should include professional development for principals in order to build awareness and support for early mathematics initiatives. As school leaders, principals must provide support for effective mathematics teaching and learning by ensuring that appropriate resources are available, by creating and maintaining a collaborative school culture, and through the creative use of time. Principals also need professional training in what sound early mathematics experiences should look like and should consistently improve their own understanding of good mathematics instruction.

In summary, the findings and recommendations of the Expert Panel suggested that effective professional development in mathematics teaching requires:

- a focus on developing teachers' pedagogical content knowledge;
- a recognition and valuing of teachers' prior knowledge;
- opportunities for teachers to connect with other teachers;

- opportunities for teachers to connect new knowledge with work in their own classrooms; and
- an active role for the school principal as mathematics learner and instructional leader.

Implementation of the initiative

In response to these recommendations, the Ontario Ministry's implementation of the Early Math Strategy involved an extensive training program for 4000 principals and 4000 teachers throughout the province. At the same time, the Ministry devoted substantial time and money to develop resources and materials to support the training. An important focus of the professional development initiative was to build teachers' understanding of the mathematics they teach. Therefore the initial stage focused on the Number Sense and Numeration Strand of the curriculum to support teachers in their understanding of important concepts in this strand. Such concepts include understanding of counting, numerical representations, quantity, and operational sense. There was also a focus on a variety of teaching strategies and activities to help teachers develop their learners' understanding of number concepts. Finally the implementation involved principals and administrators connected with every elementary school in the province. The Ministry's implementation of the training may be described as occurring in three phases.

Phase 1 (May, 2003). A team of 5 trainers mainly from the expert panels provided training for about 45 specialists in mathematics. These subject specialist teams became the regional trainers for the next phase.

Phase 2 (Provincial regions, June, 2003). Each team of regional trainers returned to their own region and provided training in math to teams from each school board in the area. These teams became the trainers for each of the school boards.

Phase 3 (School Boards, School Year, 2003-04). Each team of board trainers was responsible for the training of one teacher in each elementary school in the board's jurisdiction. The board trainers also trained one principal from each of the elementary schools. The teachers who received the training, referred to as Lead Teachers, were then expected to incorporate the training into their classroom practice. Following this, each board set aside three additional training days for Lead Teachers to share their experiences and consolidate their understanding.

The Research Study

Theoretical framework

The research to examine the Early Math Strategy is a longitudinal study designed to gather data over two years to capture the developmental nature of the implementation process. Theoretically, we are drawing on a social-constructivist framework (Confrey, 1990; Davis, Mayer & Noddings, 1990; Foreman, 2003; Yackel & Cobb, 1996) in order to examine the connections between all of the components of the initiative. From this perspective, it is understood that we construct knowledge in relational networks which emerge from the interactions of people and activity contexts. More specifically, to understand the developmental changes of the Lead Teachers, the Principals, and the classroom implementations, we are drawing on Activity Theory (Cole 1996; Engeström 1994; Leont'ev, 1981; Vygotsky, 1978, 1986) a conceptual approach that provides a framework for describing the contexts of actions and processes while focusing on the mediating role of language and artifacts both material and symbolic. Investigating the ways in which these mediating resources are created and transformed within the context of the activity will assist us in understanding the changes in teacher knowledge, attitudes, and classroom practice, as well as changes in principal leadership and growth.

Rationale for the research design

The research plan is multifaceted and focuses on three aspects: 1) understanding current practices; 2) examining changes in teacher knowledge, classroom practice, and school leadership as a result of participation in this initiative; and 3) examining the effect this has on achievement. the research project includes the gathering of both quantitative and qualitative data to provide a comprehensive view of the implementation of the Early Math Strategy across the province. The project consists of data gathering and analysis at the provincial level as well as case studies of nine representative schools across Ontario.

The research design has several different components to address the complexity of the Early Math Strategy.

- Questionnaires for Principals and Lead Teachers
- Analysis of training
- Case studies

Questionnaires

Questionnaires are being administered to 4000 Lead Teachers in mathematics and to 4000 elementary school principals three times throughout the two-year study. Although a sample of the population could have been used, the research team decided to specifically invite all of the participants in the Early Math Strategy to take part in the research. The use of the full population of 8000 allows everyone to be engaged in both the initiative and the research of the initiative. In this way, all participants have a recognized voice.

Principal questionnaires examine school leadership, improvement planning, and support for teachers and students in mathematics. The questionnaires for teachers investigate the ways in which Lead Teachers understand teaching mathematics, the instructional strategies they use, the types of professional development they take part in, and the resources that they would find useful in their practice. The first set of questionnaires to Lead Teachers and Principals were administrated in spring 2004.

Analysis of Training

In order to investigate the training and resources that Lead Teachers and Principals receive, the research examined four aspects of the training process. These include:

- Interviews with the trainers at multiple levels to determine: i) the important messages they wanted to convey ii)the areas that the participants found difficult, and iii) their perception of the effectiveness of the training.
- Analysis of agendas of training of board training of Lead Teachers to further clarify the nature of the training in several contexts.
- Analysis of the training materials to provide an overview of the resources provided to teachers and principals to support their learning and school-based practice.
- Examination of professional development models used for Phase Three of the training to determine how the Lead Teachers in each school board consolidate their learning.

Case Studies

It is anticipated that the Teacher and Principal questionnaires will provide an indication of the level of implementation of the Early Math Strategy. However, these self-reports will be strengthened through observational case studies that can provide a more complete description of implementation (Ross, McDougall, Hogoaboam-Gray, & LeSage, 2003). In order to capture the diversity of education in the province, schools and boards will be selected according to their size, language of instruction, geographical location, and urban or rural context. Nine schools are being selected for case study sites from 6 school boards in Ontario. Researchers will use these schools to interview the Principal and the Lead Teacher in mathematics. As well, observations of the Lead Teacher classrooms over time will allow for the collection of data concerning classroom practice. Video capture will be used in classroom settings as it helps in the description and retention of observed events so that they can be analyzed in detail on a variety of different levels. The Report of the Expert Panel along with the findings of the first questionnaire will help to frame the analysis of the classroom data. Specifically we are interested in examining the interactions of problem-solving settings including the discursive practices in instructional activities.

Preliminary Findings and Discussion

The research team has worked steadily to implement a research design which is both complex and extensive. Even at this early stage in the research process, it is clear that the numerous types of data and the range of variables available for analysis are rich resources for further study. There are a number of substantial challenges posed by a large scale study of this kind. Choosing to survey the entire population of participants in this initiative rather than a sample posed several logistical problems. In addition, including several constructed response items on the questionnaires posed a challenge for data analysis. Organizing nine case study schools in several different locations and contexts requires extensive training of various research assistants. As well, coordinating the various components of the study and the connections between those components is complex. Setting the stage in a large-scale research project also takes time. In addition to the phases of the research that generate the data, an early part of the research focused on ethical issues and becoming familiar with the extensive training documentation and the complex series of implementation components associated with the initiative. At the same time, we established positive working relationships with the multiple partners in the field. These partnerships have been considered essential for the effective collection of data for all phases of the research. As this project is in its initial stages, very preliminary results are being realized. However, there are several initial outcomes that are evident. Nearly 11, 000 questionnaires were distributed in the first round of gathering questionnaire data and over 6, 000 questionnaires have been completed and returned. This high response rate suggests that the desire to give all participants in the initiative a voice was well-received. In addition, the questionnaire included a number of constructed response items where respondents were invited to provide additional details and clarifications. Participants took the opportunity to include many additional notes to describe their professional development and classroom experiences. In a recent article on the value of educational research, Burkhardt & Schoenfeld (2003) have suggested that educational research needs to be more directly linked to the practical needs of the education community. This requires that researchers make a serious attempt to understand the multiple contexts of the practice they are investigating. The professional development initiative described in this paper, as is the case with all human activity is context sensitive and the scope of the research design needs to be able to investigate those contexts in order to understand the implementation. The preliminary findings of this study suggest that if the research is designed with the community of participants in mind, as well as their contexts, it has the potential to substantially alter attitudes and practices. That being said, we are well aware of the deep challenges to change but remain optimistic that professional development initiatives such as this one can, if the partnerships are nurtured, lead to the successful implementation of educational reform.

Endnotes

1. The following researchers form the team: Marie Josée Berger, Renée Forgette-Giroux, Barbara Graves, Martha Koch, Claire Maltais, Christine Suurtamm, and Nancy Vézina.

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THE PROCESS AND INFLUENCES OF DISTRICT LEADERS BECOMING MEMBERS OF A PROFESSIONAL TEACHING COMMUNITY

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Objective

This paper describes how district leaders became members of a professional teaching community. Their influences on the joint enterprise of the professional teaching community and the district leadership community are highlighted. I do this by identifying shifts in district leaders participation in the professional teaching community, and also the joint enterprise of the professional teaching community. In addition, I also identify shifts in the practices of the district leadership community.

Theoretical framework

A professional teaching community is typically conceptualized as a group of teachers working together with a group of researchers in order to develop generative teaching practices (Cobb, McClain, Lamberg & Dean 2003; Wenger 1998; Franke & Kazemi (in Press); Grossman, Wineburg and Woolworth 2000; Leherer & Schauble 1998; Warren and Rosebery, 1995). Generative teaching practices involve teachers making pedagogical decisions based on student reasoning (Franke & Kazemi, in Press). Teaching for conceptual understanding is a complex activity because it involves figuring out how students are reasoning and making pedagogical decisions to support student learning by building on what students know and can do. A professional teaching community can become an arena for teachers to figure out how to support student learning by using each other as a resource (Cobb, McClain, Lamberg & Dean, 2003). A professional teaching community is a community of practice (Wenger, 1998) within the institutional setting of the school district. Cobb, McClain, Lamberg & Dean, 2003 identify math leadership community, school leadership community and the professional teaching community as primarily influencing math instruction within the institutional setting of the school district. The math leadership community is made up of distinct leaders such as the math coordinator and math specialists that work with the schools to support math instruction. The school leadership community is at the school level includes the principal, vice principals and department chairs.

A community of practice can be identified as having a joint enterprise, mutual engagement and norms of participation (Wenger, 1998). When a member from one community of practice interacts with another community, they act as brokers (Wenger, 1998). In other words, a broker helps facilitate meaning by coordinating different perspectives among the communities through boundary encounters and boundary objects (Cobb, McClain, Lamberg & Dean 2003. Math instruction within an institutional setting is a distributed activity. Therefore, the influence of brokers in professional teaching communities needs to be understood. According to Gameron (2003), a professional teaching community needs resources in order to be able to sustain itself. In this paper, I specifically explore how district leaders became part of the professional teaching community and their resulting influences on their participation in the district leadership community. This paper is significant because it sheds light on how district leaders and teachers can be brought together to develop mutually beneficial relationships in order to maximize the strengths and resources of the professional teaching community and district leadership community to influence and improve math instruction.

Methodology/Evidence

The data presented in this analysis is from ongoing collaborations with a group of middle school teachers in a southern state for the past three years. A research team including myself from Vanderbilt University have been meeting with the teachers once a month for six times during the school year and also during the summer for a workshop. The research team facilitates professional development sessions to support teachers develop generative teaching practices for mathematics instruction.

The school district is an urban district that serves 60% minority student population. It is located in a state with a high stakes accountability program. The school district had received external funding to support reform of mathematics instruction. The district leadership community is made up of a mathematics coordinator and 4 math specialists who serve 8 middle schools. The district leaders as this school district coordinate math reform in the district. The math specialists coordinate the professional development in the district and they also provide classroom assistance.

The data collected include videos of the professional development sessions during the school year and also the summer sessions from 2001-2003. The data also includes interviews of the district leaders conducted by the research team. The district leaders interviewed include the math coordinator and the math specialist who works with the schools represented in the professional teaching community. The methodology used to collect and analyze data includes the snowballing methodology (Spillane, 2000) and a Bottom-up strategy (Talbert and McLaughlin, 1999). Shifts in the district leaders participation in the professional teaching community and the district leaders community in relation to the joint enterprise of the communities is documented.

Results

The following shifts in participation describe the process that district leaders became members of the professional teaching community (see figure1).

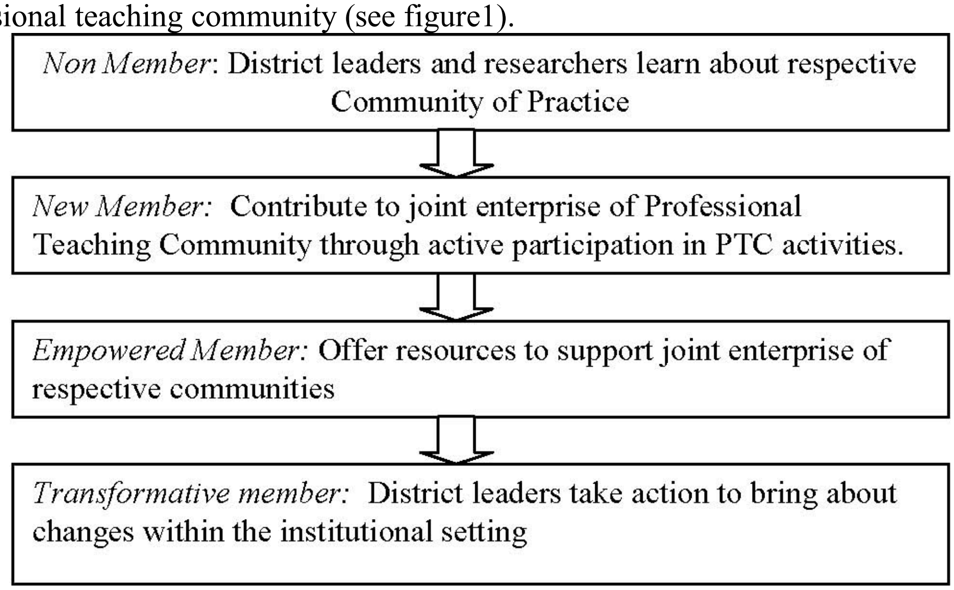

Figure 1. The process district leaders became members of PTC

Non-member: District leaders and researchers learn about respective CoP

District leaders did not initially participate in the professional development sessions. They helped researchers conduct professional development sessions by helping with the logistics. At this point, the researchers wanted to create a non-threatening atmosphere for the teachers so that they would feel comfortable expressing their thoughts and ideas. Therefore, the district leaders were asked not to attend the sessions. The researchers interviewed the district leaders in order to understand what it was like to teach within this institutional setting of the school district. The researchers were interested in understanding the role of the institutional context as it relates to math instruction.

The district leaders described their roles, duties, the district wide agenda for math reform and information about boundary objects related to math instruction. The district leaders roles and duties involved supporting teachers teach reformed math within the school district. The Math coordinator supported math reform by focusing on the implementation of reformed curriculum district wide. The math specialist focused on providing assistance to teachers in specific schools at the request of the principals and also conducting professional development sessions within the school district.

During this time, the joint enterprise of the professional teaching community shifted to supporting principals become instructional leaders so that teachers would have opportunity to plan together, collaborate and problem solve to support student understanding. This joint enterprise evolved as teachers during the professional development session assessed student thinking and understanding on basic mathematical concepts. They concluded that even though students were able to solve the problem procedurally, they had difficulty understanding the mathematical concept. Therefore, the teachers felt that they needed support on how to focus on student understanding.

The district leaders communicated the challenges they faced implementing math reform district wide. The math specialist had difficulty assisting several schools at once due to time constraints and physical location of the schools. The math coordinator shared her frustration of implementing the reformed curriculum district wide. Some teacher did not use the reformed curriculum and the professional support provided. The researchers appreciated their concerns and offered alternate ways for district leaders to think about the challenges they encountered. The researchers challenged district leaders current assumptions of math reform as implementation. Furthermore, we shared our ongoing analysis of the institutional setting particularly with regard to the lack of professional networks. We communicated the potential of the PTC to further the district leaders agenda by sharing ongoing analysis of the institutional setting, and highlighting the PTC as a resource. For example, we raised the issue of the teachers from the PTC becoming teacher leaders. The district leaders began to conceptualize the PTC as a potential resource for their agenda of math reform. At the same time, the researchers perceptions of how the district leaders can provide resources to enhance the agenda of the professional teaching community also changed.

New member-Contribute to joint enterprise of PTC through active participation in PTC activities

The district leaders participation in the activities of the PTC characterizes new member participation. During this phase, the district leaders started to attend and participate in the activities of the professional development sessions of the professional teaching community.

Their participation included defining the domain and identifying issues within the Professional teaching community. This means that they were figuring out what the PTC was about and what issues that they were grappling with. For example, the district leaders initial assumptions about the PTC as providing professional development from the perspective of implementation was in conflict with the emerging joint enterprise of the PTC that was exploring what it means to teach for student understanding. Therefore, the teachers and researchers shared stories of activities that they had engaged in and what they had learned about student understanding with the district leaders. Story telling became a means of communication. By doing so, the teachers and researchers communicated to district leaders why they were working on an activity to communicate with the principals the need for instructional leadership.

The district leaders participation in the PTC professional development sessions influenced the emerging joint enterprise of the PTC as well. For example, the math coordinator provided suggestions on how to communicate with the principals. She provided insights from a district leaders perspective. The Math coordinator was able participate in the PTC sessions because she was beginning to develop a deeper understanding of the joint enterprise of the PTC. By contributing her insights, she was also shaping the joint enterprise of the PTC as well.

In addition, the district leaders began to view the PTC as a potential resource to further their own agenda of mathematics reform. This took place as they were able to listen to different perspectives about math instruction within the district and they were able to communicate issues of math reform from a district leaders perspective. For example, a discussion took place within the PTC on how to assess students for conceptual understanding. The math specialist pointed out that aligning assessment with teaching was a district wide issue.

As district leaders began to view the members of the PTC as potential advocates to further their agenda of reforming math within the district, understanding the concerns and issues of the teachers became necessary.

Empowered Member-Offer resources to support joint enterprise of respective communities

District leaders participation shifted to that of an empowered member. An empowered member feels like he has a voice and the potential to take action to bring about change. The district leaders started to develop an understanding of the joint enterprise of the PTC. They began to *offer* material and leadership resources and opportunities to further the joint enterprise of the PTC and agenda of the District leaders as well. For example, the math teacher recruited teachers to take leadership roles within the district to conduct district wide professional development sessions. Several teachers expressed an interest in taking a leadership role within the district. The Math coordinator communicated that she had material resources such as video cameras that the teachers could use to video tape each other teach. She also informed the teachers that they could take an active role within the district to make decisions with regard to revising the district prescribed instructional plan for math instruction.

Transformative member-take action

Transformative membership involves actually taking action to bring about change. In other words, it is not just saying that you are going to do something but involves actually doing it. District leaders *provided* material and human resources to further the joint enterprise of the PTC and also the district leadership community. For example, the district coordinator provided funding for a school principal, two teachers from the PTC and the math specialist to visit another school district in a Western State to learn about instructional leadership with regard to teacher leaders and principals as instructional leaders. The math specialist worked with several teachers from the PTC to facilitate and provide district wide professional development.

Shifts in district leaders participation in the CoP

District leaders initially approached district wide professional development as implementation of curriculum. Professional development consisted of doing activities with reformed materials, and presentations on why the teachers should use reformed materials. There was a shift to conceptualizing professional development to focus on student understanding and how to plan for it. There was a focus on examining student work and thinking as grade level meetings.

Discussion

The shifts in the nature of interaction between the district leaders becoming part of the Professional teaching community and the joint enterprise of the Professional teaching community and district leadership community were jointly constituted. According to literature on knowledge management (Axelrod, 2000;Ashkenas, Ulrich, Jick &Kerr, 2002; Wenger, McDermott, Snyder 2002), as peripheral members from different communities of practice become full members of a community of practice, it becomes a knowledge generative community with a shared meaning. The joint enterprise of the PTC evolves so that it reflects multiple viewpoints and agendas of different Communities of Practice. Furthermore, it affords material and human resources that help sustain the professional teaching community.

Therefore, researchers must take into account the institutional setting of the school district and the individuals who influence math instruction such as district leaders in order to maximize resources and support to bring about changes in math instruction when attempting to establish professional teaching communities through professional development. These relationships do not naturally occur. Rather, what brings people together is a purpose for participation that is mutually meaningful.

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MATHEMATICS TEACHERS' PERCEPTIONS OF THEIR CHANGE: AN INQUIRY THROUGH PSYCHOLOGICAL TYPE PREFERENCE LENSES

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The purpose of this study was to investigate teachers' perceptions of change in their conception and practice of mathematics teaching and learning throughout their careers and to explore the role of their psychological type preferences in these perceptions of change. The findings from two teachers showed considerable influence of their psychological type preferences in the way they changed their teaching and learning mathematics. We argue that efforts to bring about development in teaching practices must consider knowledge of the teachers' psychological types along with the teachers' prior knowledge and conceptions about teaching and learning.

Introduction and Background

According to the normative-reeducative perspective and naturalistic view of change (see Richardson & Placier, 2001), every teacher experiences minor or major changes throughout their career. This can be through: (a) natural circumstances such as, discussions with teachers or experiences in or out of classroom, or (b) intervention programs. These circumstances and programs may be perplexing and lead teachers to examine and reflect on their beliefs about teaching and learning mathematics. Nevertheless, while the literature further suggests that change in teachers' conceptions and practices is gradual and idiosyncratic, we still do not know much about issues of why some teachers change more than others, and why some professional development programs or other circumstances bring about a significant change for some teachers but not others (Grant, Hiebert, & Wearne, 1998).

Further, research on change in teachers' beliefs and individual differences in change raised the need for exploring the change in individual teachers' conceptions and practices together with considering teachers' individual functioning and personality characteristics (Cooney, Shealy, & Arvold, 1998). Very little attention, however, has been given to personality characteristics and individual differences in psychological functioning in investigating teacher change (Evans & Hopkins, 1988). In this study, we explored teachers' perceptions of changes in their conception and practice of mathematics teaching and learning throughout their careers, and attempted to understand the role of their personality characteristics in these perceptions of change.

Methods and Data Sources

This study reports data on two experienced high school mathematics teachers' change histories. The Myers-Briggs Type Indicator (MBTI) (Myers, McCaulley, Quenk, & Hammer, 1998) was used to identify personality characteristics of the teachers (Mr. Miller and Mrs. Jones) while their perceptions of change in their conceptions and practices about teaching and learning mathematics were explored using semi-structured and informal interviews, an reflective survey, and classroom observations. The main focus of the interviews was the critical incidents that had a bearing on the teachers' change. Data were analyzed during and after collecting data. A list of codes was developed using all data. Based on the focus of the research, the codes were revised, refined, and sorted into four coding categories: (a) needs and expectations, (b) support, (c) constraints, and (d) change orientation. Each teacher's profile was developed using these categories, and significant facets of the teacher's mathematics life histories were illustrated.

Finally, the teachers' MBTI types or psychological type preferences were identified, and accordingly the teachers' change histories were evaluated in the light of their MBTI types.

Results and Conclusions

The analysis of the results suggests that Mr. Miller and Mrs. Jones changed their conceptions of mathematics teaching and learning at several points in their career. However, the way of their change was considerably different. After the change in the state's curriculum, accumulated personal experiences that involved Mr. Miller's successful teaching attempts with investigative materials, and dissatisfaction with students' previous learning and test results, he experienced a significant change in his conceptions about teaching and learning. Furthermore, the close and comprehensive support of the school principal, the university faculties, and colleagues strengthened his new conceptions. The natural circumstances that affected Mrs. Jones's conception of teaching and learning were the ones in which she was involved in the process of designing and experiencing the activities. Further, the incidents where ideas were exchanged evenly in a risk-free context (e.g., working with student teachers), where she could discuss the applicability of ideas based on her imaginative projections of their use in her classroom, seemed to prompt and support her change.

These findings indicated that these teachers' psychological type preferences show considerable promise in understanding issues regarding their change processes. Mr. Miller's change was considerably prompted and supported by two factors − cooperation and accumulated experiences − which were key indicators of his psychological type preference, Extraversion-Sensing-Feeling-Judging (ESFJ) (Myers et al., 1998). Changes in Mrs. Jones's conceptions came from three factors − being part of the process and influencing the change, independence, and reasonableness − which were in accordance with her psychological type preference, Extraversion-Intuition-Thinking-Perceiving (ENTP).

The findings of this study showed that to achieve development and, in some cases, to promote a movement toward being a reflective connectionist (Cooney et al., 1998), when teachers face new or perplexing situations and attempt to make sense of these situations, it is necessary to support their needs and expectations associated with or arising from their psychological type preferences. Findings also point to the need for further research into the associations between teachers' change processes and their psychological type preferences. Efforts to bring about development in teaching practices must consider knowledge of the teachers' psychological functioning along with the teachers' prior knowledge and conceptions about teaching and learning. The results suggest the need for systematizing the existing useful, however disordered and obscure bulk of information about effective professional development programs, to develop an integrated system which takes individual differences into consideration.

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THE OTHER SIDE OF "TEACHING AS ONE WAS TAUGHT:" A CASE OF A STUDENT FROM REFORM-ORIENTED SCHOOLING IMPLEMENTING TRADITIONAL INSTRUCTION

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Researchers have documented challenges teachers face when enacting reform practices (Cohen, 1990; Fennema & Scott Nelson, 1997). Some claim that teachers teach in ways similar to how they experienced teaching during their own schooling and hold beliefs consonant with such practices (Borko & Putnam, 1996; Thompson, 1992). For most current teachers, that instruction was more traditional than reformed. Hence, the phenomenon of "teaching as one was taught," is a significant obstacle to reform. Some evidence suggests that teachers who experienced reform instruction are better-positioned to enact reform-oriented practices (Schifter, 1993; Stigler & Heibert, 1999). What happens, however, if someone who experienced reform-oriented instruction attempts to teach in traditional ways? As students who experienced reform enter the teaching profession, what will happen if they are in school settings that promote traditional instruction? This study examined a teacher's practices and beliefs (whose own mathematics education had been reform-oriented) as he tried to implement traditional instruction.

Theoretical perspective and methods

Similar to prior research (Cohen, 1990; Fennema & Scott Nelson, 1997), this study examined factors shaping a teacher's practices as he implemented practices novel to him. This participant-observation research was approached from the perspective that beliefs shape teachers' practices in important ways (Thompson, 1992). The study was conducted at a large U.S. university. The participant, "Alex" (a pseudonym), was a mathematics doctoral student and had five semesters of teaching experience but was teaching a content course for pre-service elementary school teachers for the first time. Instructors for this course teach small (25-student) sections and are supervised by a faculty member. Course supervisors hold periodic meetings and conduct observations. I was a faculty member assistant to the course supervisor and I conducted observations of some instructors.

I first observed Alex during the fifth week of the semester. During the 90-minute class, I took fieldnotes on his practices and interactions with students. Afterwards, we discussed the class. I visited Alex's class again, took fieldnotes, and met with him afterwards. In between visits, we discussed lesson plans via email. After the semester, I interviewed Alex to obtain data about his beliefs and instructional practices. The interview was audio-recorded and transcribed. Data analysis of fieldnotes generated descriptions of Alex's typical instructional practices by first constructing "parsings" (Schoenfeld, 1999) of each class. This representation captured instructional routines and how time was apportioned during class. Notes from post-observation discussions and interview data were sources of warrants for claims about Alex's prior learning and teaching experiences as well as his beliefs about learning and teaching.

Evidence

First observation: Initial observations of Alex's class suggested that he was a novice teacher having a difficult time engaging with students. He struggled to maintain students' attention, with many talking amongst themselves and doing work for other classes. Alex's dominant instructional routine entailed presentation of rules, followed by presentation of examples illustrating the rules. From our discussions, I confirmed this routine to be Alex's typical practice in this and previous courses.

The day's topic was geometric transformations. Alex began by presenting rules for rotation around the origin. He then showed examples of rotations, requesting computational answers, point by point, from students. When students struggled to find an answer, Alex provided it himself. He repeated this routine for 60 minutes. The last 30 minutes involved discussion of an exam.

Post-observation discussion: I suggested that Alex give students more opportunities to solve and discuss problems. He believed that doing so was "not the way teaching should happen" in this country.

In addition, he believed (from conversations with colleagues) that posing challenging problems generated frustration and negative student reactions. He also thought it was inappropriate to "put students on the spot" by asking for explanations since that could be interpreted as confrontational. He believed his role was to present rules and solve simple problems and that other activities were likely to be criticized by faculty and students. Alex was generally disappointed with his teaching, but believed he just needed to "make clearer presentations" to improve. Since what Alex reported were *not* the course supervisor's intentions, I encouraged him to have students work on problems collaboratively and present their solutions. We discussed lesson plans and strategies for engaging students. Since researchers have documented significant difficulties that even strong and experienced teachers have implementing reform-oriented practices, it seemed unlikely that a relatively inexperienced teacher who was struggling as much as Alex would succeed in transforming his practices in a short period of time.

Second observation: It was, therefore, quite surprising that during my observation several weeks later, I watched Alex orchestrate dramatically different learning opportunities for his students. The classroom atmosphere was different, as were the ways Alex engaged students. The topic was volume and surface areas of cylinders and cones. Instead of presenting rules and examples, Alex began by discussing shape characteristics and asking for definitions of various figures. Next, he showed paper models and then derived formulas with ideas volunteered by students. For the remaining hour, students worked collaboratively on problems, presented solutions, and engaged in whole-class discussions. Problems involved finding and comparing areas and volumes. All problems entailed use of formulas discussed earlier, but each also required reasoning about shapes both before and after using the formula.

Alex's learning experiences: From our discussions, I discovered that Alex's teaching practices during my first observation were not ones he had experienced as a learner. He had active learning environments throughout his mathematics schooling and believed he and others had learned well from working on challenging problems together, discussing and presenting their thinking. He believed such environments were "more like how learning math really happens" and that the traditional practices he had tried seemed straightforward, but said he did not actually understand their role in students' learning.

Conclusions

Findings suggest that ability to "teach as one was taught" is not limited to those who experienced traditional instruction. Reform-oriented experiences and beliefs appear to be extremely resilient– resilient enough to withstand the influences of several years of teaching in a context that promoted traditional instruction. This resilience is also evidenced by how easily Alex transformed his unsuccessful traditional practices into the more reform-oriented ones with which he was familiar. These findings are encouraging: if the mathematics education community succeeds in helping current teachers develop reform-oriented practices, their students may be powerful levers to successful implementation of reform.

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DEVELOPING MATHEMATICS TEACHING ASSISTANTS' CONCEPTIONS OF ASSESSMENT

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This research study looked at the ways in which mathematics teaching assistants' (TAs) conceptions of assessment developed in the context of an ongoing seminar focusing on assessment. A weekly seminar provided the forum for teaching assistants to design, pilot, revise, and reflect on assessment tasks. Interviews, audiotapes of the seminar, and collected assessment artifacts, (e.g., quizzes designed by the TAs) provided evidence that although the TAs' responses to broad questions about assessment did not change, they expanded their views of assessment to include informal, formative, and non-traditional assessment measures and shifted their focus from surface features of item design to attending to student understanding. This study provides evidence that the analysis of assessment items can be a way for TAs to start examining their assessment practices and that such analysis can contribute to a change in conceptions.

Introduction and Rationale

Historically, graduate students have been expected to learn how to teach through one-day orientations and socialization into the community of higher education (Austin, 2002). Little is done to specifically prepare TAs for the challenges and responsibilities of teaching. In light of this, calls for research in the area of professional development for TAs have recently been expressed (Austin, 2002; Shepard, 2001) and many studies have been carried out (e.g., Austin, 2002) that have shown that TAs feel they have had inadequate preparation for faculty responsibilities such as teaching.

A particularly important aspect of teaching for which TAs are ill prepared is assessment, although assessment is fundamental to student performance in higher education and is an integral part of instruction (Shepard, 2001). Unfortunately, assessment in collegiate mathematics is dominated by summative written tests which capture a very limited view of a student's understanding.

The purpose of this study was to determine how encouraging TAs to focus on issues of assessment—such as the inherent complexities of item design and characteristics of high-quality problems—might change assessment conceptions. During the seminar, teaching assistants designed, piloted, revised, and reflected on assessment tasks as a means for developing their conceptions and practices surrounding assessment.

Methods

The overarching question guiding my study was: In what ways do mathematics teaching assistants' conceptions of assessment develop in the context of an ongoing assessment seminar? To study the different aspects of this question I looked at the seminar's impact on the TAs' conceptions regarding:

1. The general purposes of assessment, i.e., why they assess their students,
2. The forms assessment can take, i.e., what counts as assessment,
3. The specific intentions for an assessment, i.e., what they anticipate they will learn about their students and why they designed the assessment the way they did,
4. The design and adaptation of assessment tasks.

The study took place during a mandatory weekly seminar for calculus TAs. Five of the six attending TAs agreed to participate in the study. It is important to note that these TAs were not teaching the traditional lecture/discussion format; rather, each TA was responsible for his or her own course. Each participating TA was required to contribute an assessment task to be piloted, critiqued, and edited by all seminar participants twice during course of the seminar. As a group, we critiqued the assessment, offering suggestions and ideas. The TAs then had the opportunity to revise their original assessment based on the input from the seminar members and to share the revised assessment with the group the following week.

For the sake of brevity, the conceptual framework will not be discussed here; suffice it to say that Niss' (1993, pp. 1-30) "assessment modes" provided the framework for analysis of TAs' interview responses and their developing conceptions of assessment.

Results

When asked very general questions about their assessment conceptions, the TAs' responses changed little if at all over the course of the seminar. For example, when asked why he assessed his students, one TA stated, "I don't know. I guess I never really thought too much about why…I just sort of went along" and when asked the same question 15 weeks later, said, "I don't know. I never really thought about it. Probably because I have to, mostly." These seem dire results, to be sure. However, when the same TA was asked specific questions about what types of assessments he used, i.e., what counts as assessment, his responses broadened from, "I've never really done anything but just give short quizzes, give the standard two-hour exams or collect homework and check off that they did it and correct a few mistakes and give them some feedback, sort of traditional things" to "I had them turn in written-up group assignments that counted towards their grade…in terms of homework, they can do as much or as little as they want…their grade comes from whether they really understand, which is hopefully measured by quizzes, exams, and group projects."

Finally, when the same TA was asked about the characteristics of a good assessment item, he initially said, "I try to find a problem that can be done quickly and I don't pick ones that require a lot of thinking" and later, "There's the issue of not too hard, not too easy, the issue of can everyone do something on the test, even if they can't do the whole question or they can't do the hard part of the question, can they do something to show me that they learned something."

Although these results describe only one of the five TAs, the changes in others were similar: little, if any change occurred on broad questions of assessment, and specific, individual changes emerged on more explicit questions of the TAs' assessment conceptions.

The results do not indicate a complete change in TAs' conceptions of assessment; hoever, this study provides evidence that some development in this area is possible in the context of a focused professional development experience that provides support research shows TAs want and need.

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PROJECT MENTOR:
ASSESSING THE GROWTH OF MATHEMATICAL CONTENT KNOWLEDGE

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Background

In the Pacific region served by Pacific Resources for Education and Learning (PREL), lack of access to four-year degree granting programs, coupled with a rapidly growing student population, results in the increased hiring of inexperienced and under-qualified teachers. The MENTOR Project addresses this problem by establishing a mentoring program for novice teachers aimed at developing in them the knowledge, skills and dispositions necessary to be effective teachers of mathematics, thereby decreasing the length and trauma of their induction into teaching and increasing their commitment to the profession. The Project commenced in January 2003 and runs through August 2007.

Description of the Project

Based on training led by PREL personnel, Project mentors are providing summer institutes as well as in-class mentoring, inquiry group sessions, and lesson study for novice teachers. The Project is creating two cadres, one a group of well-trained and effective mentors of novice teachers, and the other a group effective novice teachers who have an understanding of how to implement standards-based mathematics instruction and who exhibit a strong commitment to the profession.

The ability of the MENTOR Project to achieve its stated purpose of nurturing effective mathematics instruction in novice teachers is dependent upon the achievement of a number of goals, including but not limited to:

- developing experienced mathematics educators' understandings of the roles and responsibilities of mentors, as well as of effective mentoring processes;
- developing experienced mathematics educators' skills as mentors, as well as their abilities to design and implement professional development models that foster professional growth in novice teachers;
- increasing mentors' and novice teachers' mathematical content knowledge, as well as their understanding of associated pedagogy;
- increasing novice teachers' ability to plan, implement and assess instructional sequences that reflect an understanding of the principles of standards-based mathematics learning and teaching;
- developing novice teachers' and mentors' abilities to reflect critically on their practices and on their growth as educators.

This presentation focuses on the third goal: measuring the growth of mathematical content knowledge.

Research Questions

1. Is there any impact on the novice teachers in their content knowledge, teaching practices and assessment skills?
2. Is there any impact on the mentors in their content knowledge, teaching practices and assessment skills?

This presentation will focus on an analysis of the scores obtained in 2003 and 2004, and will compare with scores of the University of Hawaii preservice education students.

Design

Project staff developed an instrument to measure the mathematics content knowledge of the participants. This test instrument consisted of 32 items with 24 multiple-choice items and 8 open-ended items that tried to find out the mathematics content knowledge of the mentors and novices. During the summer of 2003, the instrument was administered to both mentors and novices. Data collected will be used as base-line test data of the Project. The novice teachers are to be retested during the summer of 2004, and the mentors in the summer of 2005. The instrument was also used to collect data from selected groups of preservice mathematics education students at the University of Hawaii in the fall 2003.

Analysis and Findings

All 35 mentors and 76 novices of the project first year participated in the administration of the test instrument. Forty-three preservice college students who were taking the education courses at the University of Hawaii were also asked to take the assessment during one of their class periods.

Cronbach's alpha was computed to check the internal consistency of the test instrument. It was found to be 0.9016.

Mean scores of novices, mentors, and college students were 17.7, 25.9, and 28.4 respectively. One-way ANOVA showed that these means were significantly different ($F_{2, 151} = 58.2$, $p = 000$). To find out which pairs of means were different from each other, t-tests were performed. For mentors and novices, means were significantly different from each other ($t = 6.36$, $df = 109$, $p = .000$) with mentors scoring higher than novices. For mentors and college students, means were also significantly different from each other ($t = 2.3$, $df = 76$, $p = .024$) with mentors scoring lower than the preservice college students. Thus, the mean score of college students were significantly higher than that of the novices ($t = 10.5$, $df = 117$, $p = .000$).

Discussion

The result showing the novice teachers scoring significantly lower than the mentors and the UH college students are not surprising. The novice teachers across the Pacific, on average, have two years or less of college training. Many begin teaching directly from high school. One goal of the MENTOR Project is to raise the novice teachers' level of understanding of mathematics.

The fact that the UH college students scored significantly better than the mentors draws attention to the fact that mentors' mathematical education is not as strong as MENTOR Project staff had hoped for and believed. The result was somewhat surprising though not totally unexpected. Analysis of the test results identified specific areas of weakness (e.g., operations on fractions) in the mentors' mathematical background and these will be focused on during future MENTOR Project institutes.

Endnote

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CHARACTERISTICS OF TWO KEY TEACHERS IN A K-3 TEACHER PROFESSIONAL DEVELOPMENT CONTEXT

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In a group professional development project, teachers' participation is an important structure of the professional development process because it influences each other's learning experiences in the group. Wenger (1998) defines learning as follows: "For individuals,…learning is an issue of engaging in and contributing to the practices of their communities" (p. 7). Therefore, the way the teachers participate and interact with others can be considered as an important factor of their own learning as well as the development of their group's understanding. In fact, some teachers play a significant role in helping the group move forward and make a pivotal contribution to the group. These teachers serve as key members by creating learning opportunities not only for themselves but also for the whole group.

This paper describes two key teachers' cases while they are participating in a one-year professional development project: Kathy, a beginning teacher, and Jenny, an experienced teacher. While the two teachers had different backgrounds in terms of teaching and professional development experiences, they revealed common characteristics in ways they participated in the project. The purpose of the paper is to identify these key teachers' characteristics and role in the group teacher learning context.

Theoretical Framework

The current predominant theory of learning comes from a social perspective that posits that learning takes place in a social context and that participation in the social context is an important aspect of learning. Participation, however, is not only a social process but also a personal experience. When looking at social context and social relationships among group members, we also come to look at how individuals work together and react to other individuals or the whole group. Therefore, the process of teacher professional development should be inferred by looking at both social and individual aspects (Simon, 2000). The two cases discussed in this paper are situated in the group professional development context. As such, I needed to look at how the group of the teachers developed their expertise of teaching mathematics while inferring the two particular teachers' characteristics and role in the group. By doing this, I came to see how the way the teachers participated in the project activities influenced the development and refinement of the whole group's understanding of the mathematics the teachers taught and the pedagogical issues and strategies that they explored.

Data Collection and Analysis

This study is part of a large ethnographic study that investigated the shared culture of forty-two K-3 teachers as they worked together to resolve their teaching concerns and develop their expertise in teaching mathematics through a one-year professional development project. For the inquiry of this paper (identifying the characteristics of the two key teachers), I used the following data sources: observation data (field notes and course transcripts), teacher reflections and working group papers, Pedagogical Content Knowledge Tests (PCT) and Mathematical Understanding Tests (MUT), post-questionnaires, interview transcripts, and e-mail conversations.

When analyzing the data, I first summarized each activity or response and then identified key aspects of each data source. I also made connections between the key aspects within each data source and then looked for relationships of the key aspects among the data sources. In particular, I identified each of the two key teachers' characteristics and then compared them and looked for common themes.

Results

There were similarities as well as differences between the two key teachers. By actively participating in the project, both teachers utilized the project opportunities to develop their expertise of teaching mathematics even though they had different backgrounds, experiences, interests, and goals for participating in the project. Both of them, whether experienced or not, were aware of their struggles, set a solid goal to improve their teaching, and wanted to resolve their teaching problems through the project.

Individual Characteristics

As a beginning teacher, Kathy shared her concerns and teaching problems. Yet, she did not know how to make changes at the outset of the project. While discussing her interests and concerns with other teachers, she noticed useful strategies that she could implement in her classroom. In fact, she applied in her own classroom what she learned from other teachers and what she gleaned from readings throughout the project. She found what did and did not work for her. Finally, she developed her own ways to teach mathematics effectively, such as facilitating classroom discussions. Kathy, although not an expert teacher, had the skills and understandings that might eventually make it possible for her to become a lead teacher. In a sense, Kathy's case provided a glimpse of a lead teacher in the making.

Jenny was an experienced teacher. She provided ideas and strategies that other teachers could use in their classrooms. She also continued to explore issues and concerns through the project activities. She further refined her notions of teaching using a reformed curriculum. Especially, through participating in the project, she had opportunities to develop her confidence in doing mathematics. In fact, her participation in mathematical activities was implicit at the beginning of the project. Once she became confident in sharing her ideas, she actively participated in mathematical activities and contributed to the group's mathematical learning.

Common Characteristics

Both teachers demonstrated some similarities as they participated in project activities. They were key in contributing to the shared meanings with respect to mathematics and mathematics pedagogy in the community of learners. They were active math problem solvers and flexible mathematics thinkers. In addition, they were reflective teachers who thought about ways of teaching mathematics in order to continue to improve their teaching practices. With these characteristics, the two teachers actively participated in the project and influenced other teachers to do so as well.

With regard to mathematical activities, Kathy provided important ideas for other teachers to consider as they solved problems. She was not satisfied with just getting an answer. She questioned the reasons why certain ideas "worked" per se. She always wanted to generalize the patterns that she found. She also provided different perspectives to look at problems. This way of exploration about mathematical ideas encouraged other teachers to further investigate the ideas and created more opportunities for them to learn mathematics.

Kathy also created possible learning opportunities for herself as well as other teachers to explore pedagogical issues. With her clear goals for the project, she made comments about her teaching problems and shared her struggles. While discussing with her, her fellow teachers began to "see" and possibly make her problems of practice their own. Kathy made it possible for her colleagues to address issues, such as how to facilitate students' participation in discussions. She actively led her group while working on her group project. Not only did she suggest pedagogical issues that they needed to explore, but she also made suggestions about how they could investigate the issues.

Once Jenny became confident in sharing her mathematical ideas, she actively contributed to other teachers' mathematical learning. She thought flexibly and provided ideas about numbers and their relations. She wanted to clarify mathematical statements that other teachers made. This effort, in turn, may have encouraged other teachers to refine their claims. By providing her interpretations about formal expressions of mathematical ideas, such as the p/q definition of rational numbers, she also helped other teachers see the connection between formal and informal ways of viewing and thinking about mathematical ideas.

As an experienced teacher, Jenny provided many ideas and strategies that other teachers could use in their classrooms. She suggested possible questions to ask to facilitate students' verbalization during discussion. By providing possible ways of using certain materials and the purposes of certain activities, she also helped beginning teachers realize how to use certain materials and activities. While sharing her experiences from her classroom, she facilitated discussions among the teachers and encouraged other teachers to think about certain pedagogical issues, such as using representations.

To summarize, the main characteristics of the two key teachers were: active participants with specific goals, active math problem solvers, flexible mathematics thinkers, and reflective practitioners. On the one hand, by being participants with such characteristics Kathy and Jenny not only made their own learning opportunities maximize, but also provided opportunities for the group of the teachers to establish shared meanings about mathematics and mathematics pedagogy. On the other hand, the project activities provided opportunities for the key teachers to draw these characteristics to develop their expertise of teaching mathematics.

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THE TALK OF SCAFFOLDING: COMMUNICATION THAT BRINGS ADULT LEARNERS TO DEEPER LEVELS OF MATHEMATICAL UNDERSTANDING

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Scaffolds are the supports provided by knowledgeable others to help a student move from a current level of performance to a more advanced place. Essential to scaffolding within instruction is the use of language for mediation (Albert, 2000; Wertch, 1980). Language provides the medium for interchanges between scaffolders and learners allowing for individual construction and co-construction of knowledge. This study explores language for mediation by considering the talk of scaffolding between adult learners as they develop deeper levels of mathematical understanding during a professional development program designed to increase mathematical content knowledge. The goal is to investigate the type of talk that best scaffolds adult learners and moves them forward in their strengthening of mathematical content knowledge. This research looks at the question: What does scaffolding look like in the professional development of mathematics teachers and does content knowledge improve for the teachers?

The mathematical topics for the professional development seminars include the following: basic concepts of number and number operations; geometry and spatial sense; data analysis; and patterns, functions and algebra. Problem solving and mathematical reasoning skills are emphasized within each topic development. The setting for this study is a mathematics learning community of 15 middle grade mathematics teachers serving urban, ethnically diverse, and low-income populations within Catholic and private schools located in the Northeast. The guiding framework for this study applies a pre-post-test design to include: use of pre- and post-tests to assess increased content knowledge; and use of pre and post open-ended reflective survey questions to assess experiences and beliefs about collaborative inquiry. Observation protocol techniques and audio-taping of dialogue are used to gather information about participants as they engage in the seminar activities. The unit of analysis for the intervention seminar activities focuses on the talk that takes place as learners actively engage in questioning, clarifying, reformulating, suggesting, and summarizing their thinking and understanding of the content and an interpretative approach is used to analyze the data generated from the intervention seminar activities. Descriptive statistics is used to indicate the mean score and the variability of scores for the sample of subjects.

Due to space limitation, only preliminary results of the qualitative data analysis are presented. When learners (teachers) encounter a mathematical problem that is beyond what they are apt to understand on their own, the facilitator or more knowledgeable colleague assists the learner, providing support directly or indirectly through hints, suggestions, models, questions or a combination of these scaffolding techniques. For example, one of the seminar activities asked teachers to use base-ten-blocks to construct a rectangular array of the problem 13 x 22, to identify the partial products, and to discuss what the model suggests about mathematical teaching and learning of algorithms. Andrew could model 22 x 13 with base-ten-blocks as well as solve the paper-and-pencil algorithm, but when asked by the facilitator to identify the partial products and to relate the concrete model to the paper-and-pencil algorithm, he was not able to make the connection. In this instance, the concrete model posed greater difficulty for him than the abstract algorithm. Ellen scaffolded Andrew by carefully explaining and modeling how she solved the problem with the base-ten-blocks. She provided a step-by-step explanation in which she would first drew an illustration of the model, point to the concrete representation of the partial product, and then perform the algorithm to show the partial product (See Figures 1 and 2). At each step of the problem, she would ask Andrew "Are you following me? If so, show me." He would reply, "Yes!" as well as illustrate the procedure that Ellen had explained to him.

```
  13
x 22
   6 = 2 x 3
  20 = 2 x 10
  60 = 20 x 3
 200 = 20 x 10
 286
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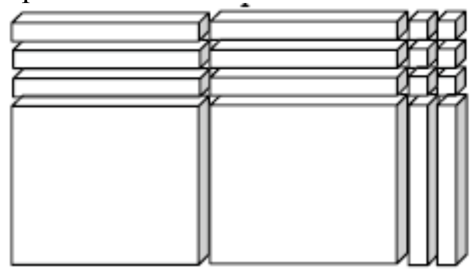

Figure 1. Algorithm with Partial Products Figure 2. Illustration of 13 x 22 with base-ten-blocks The scaffolding that occurred within learning conversations between the participants facilitate their thinking through framing, encouraging, refocusing and prompting. The dialogue strategies used by Ellen provided a means of ongoing assessment of Andrew's understanding of the content. For Andrew, his prior experiences in learning and teaching mathematics may have been based on a symbolic rather than concrete representation of numbers founded only on the rote memorization of rules or formulas. Rote learning may have assisted the teacher/learner of mathematics in performing the operations correctly; but in learning to answer the problem correctly, he may have never understood the problem solving process of mathematics. In this example, interaction and language between Ellen and Andrew serve as mediators to alter or generate new knowledge, creating the zone of proximal development. The zone of proximal development brings the scaffolder, Ellen, and the learner, Andrew, together with the mathematics content, where Andrew is assisted in acquiring the necessary tools of understanding abstract concepts through the use of concrete models.

According to Vygotsky (1978), assessing a learner in the zone of proximal development provides a better prediction of that learner's performance, than results obtained from a conventional test. With dynamic cognitive assessment, learners' knowledge and understanding grow and change as they make meaning of contextually mediated mathematics practices and activities (Albert, 2002). It follows then that an emphasis on social context and collaboration are necessary elements for meaningful mathematics professional development activities. Thus, a promising contribution of this study is the representation of teachers' learning process vis-à-vis concrete models and dialogue thereby enabling a process of self-understanding and self-reflection that may transform teachers' conceptions of the way their students are learning and understanding mathematics.

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ELEMENTARY TEACHERS' PROFESSIONAL DEVELOPMENT AS THE BASIS FOR BUILDING CONNECTIONS BETWEEN COMMUNITIES

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This paper provides a brief analysis of a three-year elementary teacher development program designed primarily to enhance teachers' content knowledge of mathematics. Participation in a program of this type offers opportunity for the development of community on many levels and further demonstrates a program capable of impacting teacher practice and individual classrooms.

In this research we share one teacher's experience incorporating what she discovered in the teacher development program as a mathematics learner, into her classroom as a mathematics educator. The participants attend a three-hour class each week, spanning three fall and winter semesters, which integrates five mathematical content courses and a methods course. Teachers work in small groups of five to seven members. The classes are taught by a university mathematics education professor who uses an inquiry based teaching style as mathematics content is explored by the teachers in a setting placing the teachers in the role of mathematics learners. Further papers presenting this research will demonstrate how this program offers teachers the opportunity to build connections within a community of learners, a community of mathematics educators and a community of education researchers, where as this report only touches on the initial connections made between the first two communities.

Theoretical Framework and Methodology

Data for this study is collected through the use of videotaping within the classroom setting. A focus group of five to seven individuals is videotaped every class period during each semester. Videotape is transcribed and verified for accuracy. The video data is then coded by key aspects of the teacher's conversations concerning topic or mathematical concepts related to the building of understanding. Undergraduate and graduate researchers document through field notes the general activities and conversations occurring in each group. Also available for analysis is the class work and notes taken by each teacher, lesson plans created by each group as well as any student work produced when the teacher presented the lessons to their elementary class. The analysis of data collected in this paper is viewed under the following lens: How does a teacher development program aimed at increasing teachers' mathematical content knowledge through the building of mathematical communities affect participating teachers' practices?

This analysis takes into consideration the philosophy that learning is a social process. Considering the development of communities in the examination of a teacher development program, offers a format for closer analysis of the community phenomenon that may develop within this type of setting. As had been suggested, opportunities for teacher learning and change in a community of learners depend a great deal on the participants' willingness to share their ideas and critically examine their own and their peers' ideas (Crespo, 2002). An examination of the data offers an indication of the "substance and nature of the learning that occurs within" this type of program (McGraw, Arbaugh, Lynch, & Brown, 2003).

One Example

RoAnn is a fifth grade teacher with fourteen years experience in the elementary classroom setting prior to entering this program. The following experience RoAnn related in the second semester after she took the opportunity to use an open ended task from the teacher development program as a resource for a lesson in her own elementary classroom.

Instructor: "I want whatever you do in this classroom to have applications both as a university student and as a teacher in the classroom. There has to be a link between what we do in this classroom and how this experience may enrich your schools. Start looking at what it is that we are doing in this class that … pertains to the mathematics that your students are learning."

RoAnn: "I'll just give a little testimonial. I handed out the desert task and said ' We're going to do a story problem'. Everyone went 'UGH!' We have been working on it for four days and everyone says, 'This is the funnest story problem!"

Instructor: "Have you come around to graphing yet?"

RoAnn: "We did the CBR [calculator based ranger] today with the actual graphing and they were so excited."

RoAnn provided work from her students on this task. Later in class the teachers were given graphical representations and asked to create a story that would describe them, RoAnn responded, "I think my students can do these." RoAnn later shared the sentiments of one of her students in a discussion with her small group as she showed the other teachers work the students had produced. She recalled to the group one student had begun the day by asking when were they going to do math that day. She asked him what time of the day they always did math and reminded him that this day would be no different. She also shared a request the students made that solicited laughter from her group; the students requested the task for homework in order to have more time to work on it. Following RoAnn's account, another teacher in her group expressed similar student enthusiasm when he presented the same task in his elementary class.

Results

The opportunity to incorporate university mathematical learning and in class tasks into the elementary classroom is evidenced by RoAnn's account in her own elementary classroom. The group adaptation of classroom tasks into the development and trials of lesson plans develop connections between a community of learners and a community of educators. This community effort models the type of activity these teachers will employ as numeracy coordinators within their respective schools. At the end of the project, the outcome of this program will be more apparent. The larger study will offer a longitudinal look at the development of communities of practice within as well as the connections between a community of mathematical learners, a community of mathematical educators and a community of mathematical education researchers.

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EXPLORING MIDDLE SCHOOL TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE OF FRACTIONDS AND DECIMALS

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This study explores the pedagogical content knowledge of fractions and decimals of three six-grade mathematics teachers. The research findings point out that sequencing appropriately and connecting mathematical ideas appeared to be a serious challenge for the teachers. We propose that professional development programs should aim in making teachers knowledge of mathematical ideas more explicit by fostering proofs, argumentation, and reasoning with symbols. This paper reports on the research component of a project that aims at developing and sustaining a professional development program that will support middle school mathematics teachers throughout their career. In order to develop effective means (workshops, study groups, summer institutes) for improving teachers' instructional strategies and student achievement in mathematics, we engaged in a study of the current status of teachers' pedagogical content knowledge as reflected in their everyday classroom practices. Particularly, we focused on the experience of three six-grade teachers in teaching the notions of fractions and decimals.

Rationale

It is widely recognized that high quality teacher preparation, both in content and pedagogy, is critical for the improvement of student outcomes in mathematics (NRC, 2001). Several studies (Brown & Borko, 1992; Ball & Bass, 2000) have concluded that content knowledge and understanding of the methods of inquiry in mathematics is at the core of effective teaching and learning. Current research in teacher development (Shulman, 1986, Ball & Bass, 2000) suggests that teachers should possess knowledge that integrates content and pedagogy, called pedagogical content knowledge. In mathematics, this kind of knowledge may include useful representations, unifying concepts, clarifying examples and counter examples, helpful analogies, and important relationships and connections among concepts (Grouws & Shultz, 1996). Thus, pedagogical content knowledge is essential for planning and executing lessons that facilitate effective students learning.

Methodology and Results

The research took place in a rural middle school in the northeast. The school had three six-grade teachers who all agreed to participate in our study. In the course of a school year, we made regular classroom observations and we had weekly meetings with the teachers discussing teaching strategies and students' learning. During the classroom observations and discussion meetings we took detailed notes and wrote reflective memos which were regularly analyzed and the findings were further discussed in due course with the teachers on the weekly meetings.

One of the most noticeable observations of our study was that the teachers experienced difficulties in organizing and sequencing the mathematical ideas in a coherent and connected way; they had difficulties in identifying the conceptual prerequisite necessary for the introduction of new concepts and connecting it to other previously studied mathematical concepts. For instance, the teachers intuitively felt that the suggested by the textbook sequencing of topics, first decimals then fractions, is inappropriate and they decided to reverse the order of introducing the concepts. However, poor understanding of the mathematical content (particularly, reasoning with symbols) as well as lack of experience in developing their own lessons and curriculum materials prevented the teachers from making the relations between fractions and decimals explicit despite the appropriate choice of sequencing. Thus, we noticed that often mathematical ideas were taught in isolation from each other. For instance, all of the teachers failed to relate simplifying fractions and finding common denominator to the previously studied prime factorization of natural numbers. Instead, they introduced rather complicated and unrelated to prime factors algorithms for finding greatest common divisor and least common multiple.

Further, we found out that all of the three teachers were trying to employ innovative instructional approaches incorporating problem solving, classroom discourse and hands-on activities. However, often their instructional activities were designed and carried out as an end in themselves and did not lead to the desired conceptual understanding of mathematical ideas and relations among them. They had difficulties in engaging the students in exploring the concepts on their own. When introducing operations with fractions, manipulatives and problem situations were used mainly for illustration of readily given rules rather then as means for exploration and discovery.

Conclusions

The research findings of this study point out that sequencing appropriately and connecting mathematical ideas appeared to be a serious challenge for the teachers. These findings suggest that the activities of a professional development program addressing teachers' pedagogical content knowledge should focus on enhancing teachers' ability to connect mathematical ideas through constructing and deriving new ideas from prior notions employing reasoning with symbols, proofs and argumentation. The analysis of the data also suggests that teachers need further instruction and opportunities for exploration on how to use effectively visual and hands-on activities in a classroom instruction. More specifically, teachers need help in making the transition from using manipulatives and hands-on activities only as means for illustration of directly introduced mathematical concepts to utilizing these tools as means for exploration and discovery leading to students' deep conceptual understanding of mathematics.

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STUDENT ISSUES AND PERSPECTIVES INVOLVING MATHEMATICS EDUCATION IN ELEMENTARY AND MIDDLE SCHOOL SETTINGS

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The attitudes we possess towards mathematics affect how we approach, persist, and succeed at the subject (Thorndike-Christ, 1991). This paper will report on a study that surveyed 1,691 students in grades three through eight about their enjoyment of and confidence with mathematical problem solving. Findings reveal gender and grade level differences that warrant the attention of mathematics educators so that pedagogical practices can be tailored to nurture more positive attitudes towards problem solving.

Background and Purpose

Students who come to enjoy and value mathematics increase their achievement, persistence, and confidence with the subject (Gottfried, 1985; Lehmann, 1986; Meece et al., 1990; and Pokay and Blumenfeld, 1990). Students also engage in and enjoy mathematics more if they expect to be successful (Dickenson and Butt, 1989), and generally avoid the subject if possible when they perceive their ability to do mathematics as poor (Hilton, 1981; Otten and Kuyper, 1988). The study discussed in this paper sought to further existing research through an investigation of student attitudes specific to their enjoyment of and confidence with mathematical problem solving. Research questions included: 1) To what extent do students enjoy mathematical problem solving and does this vary across grade level and/or gender? and 2) To what extent are students confident in their mathematical problem solving ability and does this vary across grade level and/or gender?

Awareness of such attitudes towards problem solving is of both practical and theoretical importance. Solving problems about the surrounding world has always been central to mathematical thinking, and recent PreK-12 curricula place emphasis on such tasks (NCTM, 2000). Attitudes were found to be shaped in great part by the learning environments one experiences (Graham and Fennel, 2001), and teachers who understand their students' attitudes are better able to create learning environments conducive to positive attitudes and better achievement (Middleton, 1995). The study described in this paper deepens the knowledge base and serves the mathematics learning community by providing educators with insights into students' attitudes towards problem solving so that classrooms conducive to enjoyment of and confidence with problem solving can be nurtured.

Methods and Results

A sample population of 1,691 students in grades three through eight from ten coeducational elementary schools in Queens, New York was randomly selected to participate in this study. The students were classified with average to low mathematics achievement levels based on standardized testing scores and were from families with middle to low socioeconomic status. Their ethnic backgrounds consisted of 7% Asian, 30% Afro-American, 13% Caucasian, 46% Hispanic, and 4% Pacific Islander.

The students were categorized into twelve groups according to grade and gender, and were administered a survey consisting of 12 statements that gauged responses on a 5-point Likert scale. The statements reflected pedagogical practices advocated by The National Council of

Teachers of Mathematics (NCTM, 2000), and were clustered into two categories, namely Cluster 1(Enjoyment of Problem Solving), and Cluster 2 (Confidence with Problem Solving). The gathered data was recorded and overall mean responses to each cluster of statements, as well as mean responses to each survey statement, were compared to gather information about developmental patterns in responses throughout the grades and between genders. Independent samples t-tests were used to determine the existence of any significant differences in overall mean responses to each cluster of statements throughout the grades and between genders.

Concerning Question 1, it was concluded that students' enjoyment of mathematical problem solving was in need of improvement. Decreases in levels of enjoyment were noted as grade levels increased for both females and males. Significant grade level differences were found for both females and males between grades four and five, as well as between grades six and seven. However, when searching for patterns in mean responses, it was noted that the majority of student groups (8 out of 12) agreed or tended to agree that they enjoyed problem solving situations that connected to other discipline areas such as social studies and science.

Concerning Question 2, it was concluded that students' confidence with mathematical problem solving also needed improvement. Although significant differences did not emerge, it was noted that as grade levels increased, females became less confident than males in their problem solving ability. However, when searching for patterns in mean responses, all of the student groups agreed or tended to agree that they were confident in their ability to find alternate methods of solution when problem solving.

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PRINCIPLED USE OF CLASSROOM ARTIFACTS IN PROFESSIONAL DEVELOPMENT

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The Turning to the Evidence (TTE) project investigates teachers' use of classroom artifacts in two professional development contexts: Fostering Algebraic Thinking seminars (Driscoll et al., 2001) and Videocases for Mathematics Professional Development seminars (Seago et al., in press). The project follows teachers participating in one of four groups over the course of 12, 3-hour professional development sessions during the 2003-4 school year (two of the groups use Fostering Algebraic Thinking materials and two use Videocases for Mathematics Professional Development materials). A total of 56 teachers and two mathematics supervisors participate in project seminars. The project investigates the participants' use of classroom artifacts during the seminars and compares performance on pre- and post-program measures of artifact use and mathematics knowledge.

This study grew out of the observation that, while there is currently considerable interest in using classroom records and artifacts as a tool for mathematics teachers' professional development, as a field we know surprisingly little about what teachers actually learn by working with artifacts or how they integrate their learning into daily classroom practice (Ball, 1996; Wilson & Berne, 1999). Furthermore, recent work has reinforced the caution that organizing professional development around the examination of classroom artifacts does not, in itself, guarantee significant teacher learning any more than the use of manipulatives in the classroom ensures that students will develop deep mathematical understanding (Ball, 1992, Bulgar & Schorr, 2004). Like manipulatives, classroom records and artifacts are only tools; professional development must help teachers learn to use these tools to infer students' thinking, emphasizing the identification and interpretation of evidence when using artifacts to explore learning and teaching. For example, a recent report by Warren-Little et al. (2003) indicates that "looking at student work," a frequent component of professional development, is used in many different ways and with differing levels of success. This work begins to provide a purchase on the question of how to use classroom artifacts effectively.

Part of the TTE work, which can complement Warren-Little et al.'s findings, is to articulate principles of effective use of classroom artifacts in professional development. The PME poster session would present the principles we have identified provisionally, along with seminar data regarding changes in the ways teachers use artifacts to investigate student thinking and their own practice.

Some of these principles are general, while others relate to specific goals regarding the use of artifacts to examine *mathematical thinking* and to examine *teachers' practice*.

General principles include:

- Clearly defined purpose for using the artifacts; alignment of analysis and discussion with purpose
- Distinction between description and interpretation of artifacts
- Grounding of discussions in evidence from the artifacts
- Attention to alternative interpretations of the evidence Examples of principles specific to mathematics or practice include:
- Using artifacts to investigate both mental processes and the underlying mathematical ideas; being clear about the distinction

- Using artifacts to consider factors contributing to teachers' in-the-moment decisions

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EVOLVING PRACTICES OF SIXTH GRADE MATHEMATICS TEACHERS

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In this poster we illustrate the evolution of three learning activities developed in collaboration with two sixth grade mathematics teachers during a year-long professional development project. This project aimed to (1) develop a cohesive year-long sixth-grade mathematics curriculum that addresses the California state mathematics standards, and (2) facilitate and support two sixth-grade teachers in the implementation of, reflection on, and revisions of the developed curriculum. The project began with intensive professional development collaboration in summer 2003 with our two mathematics teachers. During this 20-hour, one-week workshop, we collaboratively developed and adapted mathematics learning activities for their sixth-grade courses. These activities were designed to address more than one concept and sought to make connections among concepts. Throughout the following academic year we met with these teachers as they pilot tested, reflected on, and refined the activities and curriculum.

The elements of this poster will include a description of the professional development project and the stages involved in adapting and refining existing activities for use in the sixth grade classroom. We will display each of three original activities, the changes made during the summer workshop with the teachers, and the changes that resulted after piloting the activities with each of their sixth grade classes.

The three learning activities presented in this poster revolve around various concepts related to rectangles. The first two activities involve a birthday party business, called Perfect Party Place, which provides and sets up card tables for birthday parties. Each table is square and seats one child on each side. These square tables are arranged into a single rectangle at each party. Depending on the location, the tables must be set up differently each time. The first activity involves exploring the possible rectangular table arrangements for a party with 18 children and identifying patterns in table arrangements, dimensions, and areas. This scenario stipulates a fixed perimeter and requires an investigation of different areas that correspond to the perimeter of 18. The second activity involves exploring possible table arrangements for a party that can accommodate 24 tables and identifying patterns in table arrangements, dimensions, and perimeters. A connection can be made between the dimensions of the different tables and the factors of 24. The third activity, Rectangle Ratios, involves the exploration of similar rectangles. Students are given a collection of 14 rectangles and asked to sort them into "families". This is followed by exploration of ratios of length to width and how this relates to the idea of similarity of rectangles.

Over the year-long course of our professional development activities, we found that it is critical for teachers to engage in the entire process of design, implementation, reflection, and revision of instructional materials in their own classrooms. It is not sufficient for teachers to see this process modeled in a professional development workshop/institute. They must be involved in all phases in this process to effect significant, lasting improvements in their knowledge, perspectives, and practices.

A PROTOTYPIC ADVANCED MATHEMATICS EDUCATION PROGRAM THAT CONNECTS COMMUNITIES OF PRACTITIONERS AND RESEARCHERS

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While Georgia State University's Masters in Education (MEd) and Educational Specialist (EdS) degrees both focus on pupil learning and are grounded in the Five Core Propositions of the National Board for Professional Teaching Standards (NBPTS), the outcomes differ. While MEd candidates become more masterful at teaching, EdS candidates advance as master teachers toward disciplinary leaders and supervisors. This presentation describes how effective professional development, communities of learners, and mathematical proficiency influenced strategic improvement in an existing EdS program.

Included will be a description of how mathematics educators at GSU responded to an unusual approach for admission into the EdS program by a self-selected cohort of secondary mathematics teachers across four schools in one school district. Outcomes include a prototypic field-based EdS program in secondary mathematics education, 89% success rate of submissions for NBC, action research as a supervisory approach designed to continue beyond the life of the program, and the fostering of a community of learners.

The elements of this poster presentation are a proposed alternative application process for self-determined cohorts into the EdS program, the program description, program of study, residency, scholarly inquiry, evaluation matrix, and evaluation data. Essential components of the program are The Reflective Teaching Model for mathematics professional development (Hart, Najee-ullah, & Schultz, 2004), the Mathematical Task Analysis framework (Stein, Smith, Henningsen, Silver, 2000), NBPTS protocol for submission for National Board Certification, and action research as a supervisory approach.

The format for the poster presentation will include a paper-based poster display complemented by computer-based audio-visuals. In addition, a paper will be provided including greater detail.

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BEYOND IMPLEMENTATION: IMPROVING TEACHERS' USE OF AN INNOVATIVE MIDDLE SCHOOL MATHEMATICS CURRICULUM

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When school districts adopt innovative curriculum materials, some professional development support is typically offered to familiarize teachers with the new materials, but this support is often too brief and incomplete to support effective and masterful use of the curriculum materials to promote student learning. Frequently missing is explicit help for teachers to support student engagement with complex intellectual activity in the classroom, a feature of mathematics instruction that is often absent in U.S. classrooms (Stigler & Hiebert, 1999).

BIFOCAL (*Beyond Implementation: Focusing On Challenge And Learning*) addresses the continuing professional development needs of middle school mathematics teachers who are experienced users of one innovative mathematics curriculum for the middle grades, *Connected Mathematics*. The project is aimed at many aspects of teacher's work, but this poster focuses on only one -- lesson planning. We choose this focus because it appears to be a fruitful site for inquiry into how teachers prepare to use intellectually challenging tasks with students and how they anticipate the enactment that will occur. Prior research on challenging mathematics tasks in classrooms has demonstrated that (a) student work with complex tasks sometimes conforms to a teacher's intentions, but often it does not (*e.g.*, Stein, Grover & Henningsen, 1996); (b) teachers' actions and reactions are often crucial in determining if student intellectual engagement occurs at a high level or is allowed to decline (*e.g.*, Henningsen & Stein, 1997); and (c) the degree to which teachers maintain intellectual demands has important consequences for student learning (Silver & Stein, 1996).

Teachers from 5 middle schools in the Detroit metropolitan area met monthly between May 2003 and May 2004. They completed several cycles of a modified lesson study process -- selecting a target lesson, using a structured set of questions to assist in planning, teaching the lesson, reflecting on their instructional moves in relation to evidence of students' thinking and understanding, and discussing their lessons with colleagues.

To examine changes in teachers' practices related to lesson planning, we analyzed textual and video artifacts collected across the first year, including teachers' lesson plans, post-instructional reflections, reports of lessons to colleagues, and reflections on those reports. Using data from 12 teachers, we have analyzed the components of lessons that draw their attention in planning, the resources on which they draw when planning, the nature of their engagement with resources for planning, and their self-assessment of the impact of planning on students' opportunities to learn.

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CURRICULUM NEGOTIATION AND PROFESSIONAL DEVELOPMENT IN ONTARIO MATHEMATICS EDUCATION

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In response to low achievement results among Grade 9 Applied-level mathematics students in standardized provincial assessments, the Ontario Ministry of Education has funded the development of a unique support document designed to provide middle school mathematics teachers with extra planning and teaching resources. Having been involved as part of the writing team for this Targeted Implementation and Planning Supports: Grades 7, 8, 9 Applied Mathematics *(TIPS) (OME, 2003) document (math education research), I also have chosen to track the development and implementation of TIPS as the focus of my doctoral research thesis.*

Related Research Literature

Much has been written regarding educational reform and change theory (Fullan, 2003; Apple, 2001; Levin, 2001). Although highly complex as a socio-political phenomenon, educational reform is consistently portrayed in the literature as requiring *quality professional development*. This common factor is generally depicted as featuring a well-researched and presented rationale, the provision of adequate time and resources, and long-term support mechanisms. All of these aspects, when combined effectively, attempt to alter the fundamental pedagogical beliefs of teachers, and thereby effect, in the words of Fullan, "second-order change." Although substantial research has been conducted surrounding the area of professional development for teachers (Guskey, 2002), it seems that very little has focused specifically on professional development for mathematics educators (Loucks-Horsley & Matsumoto, 1999), and even less in terms of the Ontario mathematics education scene. Drawing upon the curriculum negotiation model developed by MacDonald and Walker (1976) and later extended by Pitman (1981), I have analyzed and further developed this model while tracking multi-level participant perceptions of the TIPS document itself and of the related professional development taking place in boards.

Research Methodology and Methods

The methodological frameworks which I am using are those of Grounded Theory (Glaser & Struass, 1967) and Auto-ethnography (Hayano, 1979). At this stage in the research, all 65 participants have been interviewed and data is now being analyzed for emergent themes using qualitative software. By examining the perceptions of multi-level participants in the TIPS process, I hope to gain significant insights into the nature of curricular dissemination as well as into issues surrounding the effectiveness of professional development models.

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SUPPORTING THE DEVELOPMENT OF MIDDLE SCHOOL MATHEMATICS TEACHERS' EVOLVING MODELS FOR THE TEACHING OF ALGEBRA

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Many middle school mathematics teachers equate the teaching of algebra with demonstrating procedures for symbol manipulation, simplifying algebraic expressions and solving and graphing linear, quadratic and more complex equations. Most students' first experiences with algebra in the US are in a traditional algebra course offered at the seventh, eighth or ninth grade level. (*Principles and Standards for School Mathematics*, 2000) Rather than traditional symbol manipulation instruction, students at all levels should have opportunities to model a wide variety of phenomena mathematically, to represent, explore, and understand quantifiable relationships in multiple ways.(Kaput, 1999) In order for this learning change to take place in classrooms, teachers' instructional models of teaching algebra must change. (Doerr & Lesh, 2003) This study investigates those models as they evolve in the daily practice of middle school teachers as they design, use, and revise reflection tools to guide their teaching of algebraic thinking and modeling at the middle school level.

The ideas offered in this poster presentation are preliminary results from a research project in progress. The aim of this study is to document and articulate the change and growth of teachers as they use their classroom practice as a learning environment for their teaching. It adopts a design experiment method (Brown, 1992) in which the participating teachers are designing, implementing and revising reflection tools for analyzing their practice as they design learning environments for their students to learn a "new" algebra. These tools describe teacher understanding in several areas of fostering students' understanding of algebraic thinking including mathematics content, psychological connections, instructional connections and historical connections. Psychological connections involve the development and understanding of student thinking. Instructional connections involve the understanding of the ideas as they are developed in curriculum materials and the classroom environment, and the historical dimension describes the context or situation and time frame for the development of students' and teachers' ideas.

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LESSON STUDY AS A VEHICLE FOR TEACHER CHANGE

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At first appearance Lesson Study is a simple idea. Teachers come together to set overarching goals for their students. These goals are then used to collaboratively plan research lessons (lessons to be studied by the teachers). After a research lesson is developed, a teacher from a planning group teaches the lesson while the other members of the group observe. The team debriefs on the lesson. Finally, the group rewrites and adapts the lesson making improvements and changes noted during the debriefing. The process of Lesson Study is, however, more complex than first appears. Lesson Study allows teachers to observe student learning and make databased improvement from lesson observations, but it is about more than producing and studying lessons. It is the experience of collaborative goal-setting, planning, observation and lesson discussion that contributes to professional growth and teacher change.

Lesson Study originated in Japan and has recently become of interest in the United States. However, the culture of the educational systems in Japan and the U.S. are radically different as illuminated by the video studies of the Third International Math and Science Study conducted in 1999. If Lesson Study is to work as a professional development format for teachers in the United States, U.S. teachers must begin to assume responsibility for their own learning.

The City Schools of Decatur (CSD) began a Lesson Study project during the 2003-2004 school year with volunteers from grade-three teachers. There were several issues to be considered if Lesson Study was to work in Decatur: (a) the CSD text series did not lend itself to Lesson Study planning; (b) no common planning time was available for the teachers; (c) the third grade teachers had limited mathematical content knowledge; and, (d) the teachers had little experience being in charge of their own learning.

The work began in the summer of 2003 with the teachers coming together to develop group norms for how the group would work together, to develop a common vision of good teaching, to develop their research goal, and to plan the initial *research lesson*. During this time, the project facilitators attempted to relinquish their perceived leadership role and establish themselves only as a resource. The teachers were not use to being in a professional development activity where they were expected to take the lead. This was the first opportunity many had had to share pedagogical knowledge, negotiate teaching strategies and discuss student learning and outcomes with their colleagues. As the year progressed teacher leaders emerged from the group and they were able to function with little outside support.

This poster session will share the process of teacher development and change over the academic year of the project in CSD. Artifacts from the project as well as video segments of lessons and teacher processing will be provided.

REFLECTION AND COLLABORATION THROUGH LESSON STUDY: THE AFFECTS ON STUDENT ACHIEVEMENT IN MATHEMATICS

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A partnership was created between California State Polytechnic University and a local high poverty high school, with funding provided by the CSU System, to improve math placement scores for at risk students on the Entry Level Math Exam. Math department faculty worked with high school math teachers to implement a lesson study group on campus to encourage collaboration and reflection. Teachers in the project focused on the geometry curriculum and, as a team, developed methods to improve instruction. With considerable emphasis on the content and delivery teachers were successful in engaging students and improving achievement on standardized tests. Test results indicate that students in the targeted classes performed significantly better than any other group. The establishment of learning communities and collegial collaboration has empowered the department.

Nogales High School is in a district east of Los Angels with approximately 3000 students in grades 9 through 12. It has a very diverse student population with high poverty and low student achievement. It has been identified by the state as an underperforming school. With strong administrative support we formed a lesson study group of four math teachers with the focus on Geometry. The teachers were given one period of release time for collaboration and classroom visitation. In the true spirit of lesson study, the university was a support system and a resource but the teachers were the creators of all lessons. The teachers used a plan, teach, reflect, and apply cycle with teacher observations. This process produced different modes of instruction including lecture, projects, activities, group work and technology with the goal of improving student engagement and achievement.

Data was collected from benchmark exams and a one-way analysis of variance was performed on the scores of the 451 geometry students. The students who had teachers in the lessen study groups scored significantly better on the exams at a level less than .001, and their mean score increased from 51% to 72%. Additionally, the only subject in the mathematics department that showed any improvement from the first benchmark to the second was Geometry – all of the others decreased. In doing the analysis it was noticed that there were some big jumps in the low-end students. On the first benchmark 51% of the students scored at or below 50% while on the second benchmark only 13% scored at or below 50%. If we look at or below 40% the percentage of students in these areas drops from 26 to 5. Thus, the lowest performing students benefited the most from the improved lessons and differentiated instruction.

Empowered by the success of their students and the rewarding collaborations the culture of the high school math department has undergone a substantial and impressive change that continues today.

Teacher Education-preservice

DEVELOPING MATHEMATICAL JUSTIFICATION: THE CASE OF PROSPECTIVE ELEMENTARY SCHOOL TEACHERS

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The present study examined data from a mathematics course for prospective elementary school teachers in which the instructor routinely required that students justify their solution methods as a way of helping them deepen their understanding of number and operations. The purpose of the study was to examine the notions of justification that prospective elementary school teachers possessed at the beginning of the class, and the process by which their individual and collective notions of justification developed throughout the course. In this paper, we will illustrate the results using examples from a unit on multidigit multiplication.

Purpose

The role of reasoning and justification in mathematics learning has gained great attention since the release of the *Standards* by the National Council of Teachers of Mathematics in 1989 and 2000. Research studies provide compelling evidence that elementary school children, in a supportive environment, are capable of both reasoning mathematically and learning to justify their solutions (Lampert, 1990; Ball, 1991). Perhaps establishing a classroom environment for prospective teachers in which justifying mathematical claims is expected, increases the likelihood that as teachers they will be able to guide their future students to do likewise. To accomplish this end, more needs to be known about how to develop prospective teachers' knowledge of justification and what constitutes a valid justification for them. This study focuses on justification in the area of computation for several reasons: 1) the fundamental role of computation in elementary school mathematics; 2) the tendency of prospective teachers to have memorized rules and procedures with little or no understanding of why these work (Conference Board of the Mathematical Sciences, 2001); and 3) the potential of computation as an area that can highlight students' proficiency in *adaptive reasoning* – described as the ability to think logically about the relationships among concepts and situations and to justify and prove the correctness of a mathematical procedure or claim (National Research Council, 2001).

Theoretical Framework

Past research involving mathematical justification focused primarily on proof. Researchers found that rather than being used as a tool for learning mathematics, proof in school has traditionally been taught as a formal, meaningless exercise (Alibert, 1988). As a result, research studies found that students' conceptions of proof were quite limited. For example, many students asserted the truth of a statement based on checking a few cases, or did not understand the role of counterexample in refuting a conjecture (Bell, 1976; Galbraith, 1981). Research studies also found similar views and conceptions about proof among K-12 teachers (Knuth, 2002; Martin & Harel, 1989). To enhance teachers' conceptions of proof, Knuth (2002) concluded that teachers needed, as students, to experience proof as a meaningful tool to study mathematics.

Several researchers have attempted to characterize levels of proof and justification (Balacheff, 1991, Van Dormolen, 1977, Simon & Blume, 1996). Simon and Blume (1996) described four levels first proposed by Balacheff (1987) as follows:

a) Naïve empiricism: an assertion based on a small number of cases,
b) Crucial experiment: an assertion based on examining a case that is not very special,
c) Generic example: an assertion based on a particular case, but the case was used as an example of a class of objects; and
d) Thought experiment: an assertion detached from particular examples and begins to move toward conceptual proofs.

When analyzing the data from their classroom teaching experiment, Simon and Blume (1996) expanded the above levels to include more primitive levels when student responses did not address the issue of justification or when they appealed to external authority for validation. Furthermore, researchers have identified three major roles for proof: to prove, to explain, and to convince (Bell, 1976; Hanna, 1990; Hersh, 1993). In the context of K-12 mathematics, there is a growing consensus that proof should not be taught as a meaningless exercise that is used only to formally establish the truth of a statement, but should also be taught as a tool that can be used to explain why a statement is true. The last aspect, proofs that convince, brings in the social aspect of working in a classroom community.

In the present study, we focused on mathematical justification for the purpose of explaining thinking and convincing others that thinking is valid, and thus we viewed the development of mathematical justification in a classroom setting as both a social and cognitive process. During this process, a community of learners was engaged in answering "why" questions for the purpose of developing mathematical understanding (Yackel & Cobb, 1996; Simon & Blume, 1996). Simon and Blume (1996) found that both the taken-as-shared knowledge and the conceptual understanding of the community members shaped what they accepted as valid justification. Thus the current study sought to investigate these two phenomena further by focusing on the notions prospective elementary teachers had and developed through a mathematics course.

Participants and Context

This study was conducted in the first of three mathematics courses designed specifically for prospective elementary and middle school teachers. The materials for this course were developed for "A Problem and Reasoning Based Curriculum Project for Elementary Educators," a project funded by the National Science Foundation and co-directed by Angela Krebs, Rheta Rubenstein and the third author. This course focuses on number and operations, and seeks to encourage the development of: multiple reasoning strategies for whole number and fraction computation; general conjectures about our number system and computation; patterns and relationships among strategies and conjectures; and justifications for all strategies and conjectures that arise. It is important to note that the decision to focus on reasoning and justification in this particular course, and the instructional approaches used to achieve this goal, have been greatly impacted by the previous work of the second author and her colleague, Kate Kline, who have had extensive experience working with both prospective and practicing elementary teachers to generate convincing justifications for computation procedures.

The second author taught the class that was used for this study. All forty students enrolled in this section participated in the study. [Two of these students dropped the course after the first exam.] Ten students, who frequently participated in class discussions and represented a wide range of grades on their first writing assignment, were asked to consider being interviewed for the study. Four of these students agreed to participate and were included in this study. Final course grades for these four students ranged from C+ to B+.

Data Collection and Analyses

Four types of qualitative data were collected for this study: instructor's notes for each lesson, videotapes of all lessons, copies of writing assignments and exams from all students, and a series of three taped interviews conducted with the four case-study students. The first author conducted all interviews with the case-study students.

This study sought to investigate the following three research questions:

1. What are the important features or aspects of the students' mathematical justifications?
2. What was the nature of the process prospective elementary school teachers went through as they developed notions of mathematical justification?
3. What were factors that influenced these developments, and what was the nature of their influences?

To begin answering these research questions, we first focused on identifying classroom episodes that focused on justification. Analysis of the data was an iterative process where the researchers identified emerging themes and hypotheses, revised existing and/or developed new codes, and then looked for confirming and disconfirming evidence of these hypotheses (Strauss, 1987). Analysis of instructor notes and student written work shed additional light on individual differences that were not always apparent during the class discussion. Finally, the interview data provided us with the opportunity to examine individual students' notion of justification at different points throughout the semester. Given the wealth of data, we chose to focus this paper on the first research question in one domain: multidigit multiplication.

Results

Justifying why a specific computation method works

The multidigit multiplication unit lasted four days (lessons six through nine), and followed a similar unit on addition and subtraction. This unit was begun by having prospective elementary teachers find different ways to solve "18x26" using reasoning, with two constraints: 1) no standard algorithms; and 2) each computational step of their strategy done mentally. After working in small groups on this task, students posted four different solution methods for the class to examine, analyze and ultimately explain. After the students had an opportunity to contemplate these different methods, the instructor initiated a whole class discussion focusing on explaining why these computational methods worked. Students initial attempts to explain typically focused on describing the steps of the procedure, e.g., "to find 18 x 26, I separated the 18 x 26 into 18 x 20 and 18 x 6." When pushed further for justification, a student suggested that 18 x 26 could be thought of as 18 sets of 26. This student then elaborated, stating that each 26 contained a 20 and a 6, therefore18 sets of 26 was the same as 18 sets of 20 and 18 sets of 6. Essentially this method was an application of the distributive property (a + b) x c = (a x c) + (b x c). Since this property does not work for all operations (e.g. $(a+b)+c \neq (a+c)+(b+c)$), the justification of why this property works requires prospective teachers to specifically utilize the meaning of multiplication. In this case, the student imagined 18 x 26 as 18 equal groups of 26. [At other times, students utilized the area/rectangular array meaning of multiplication to justify their thinking.]

While using the meaning of an operation helped justify a simple method, such as the one described above, to the majority of the class, this kind of justification was not necessarily sufficient for more complex methods. Consider the somewhat more complicated method of solving "18x26" by adding two 10x26's and then subtracting two 26's. While the justification of this method was “self-evident” to some, to others it was not. One student explained this method through a story starting with 20 bags of 26 marbles:

> *Ten bags of 26, ten bags of 26, so she had twenty bags of 26. Then she realized that two bags she counted was from her friends, and so she took those away.*

Another student asked why 8 was nowhere in the computation. This student might have a fixed image of 18 groups of 26 and anticipated seeing 8 groups of 26 in the computation. A third student was also very confused; she asked why they took away 2 sets of 26 if they originally rounded 18 up to make 20. This student seemed unable to separate the different meanings of 18 and 26 in the multiplication sentence: one indicated the number of groups; the other indicated how many in each group. The instructor asked the class what might be done to make the justification more clear. Drawing pictures was suggested to help facilitate the justification. A picture of 20 circles with the numeral "26" written in each was posted on the board. Then, the last two circles of "26" were crossed out to illustrate the subtraction of two 26’s. Note that the introduction of the picture leads to a potential alteration in the explanation for getting rid of two bags of 26. In the student’s story, the reason for getting rid of the two bags is that they are “from her friend,” while the picture could be used to suggest that you have 20 bags, but were only supposed to have 18 bags, thus you get rid of two of those bags.

In summary, when justifying why a specific method works for solving a given computation problem, a viable and convincing justification for this class seemed to include one or more of the following aspects: interpreting the given problem based on one of the meanings of the operation, embedding the computation in a story context, and creating representations to illustrate the steps. Furthermore, as indicated by the above examples, what was convincing to some students was unclear to others. Even though referring to the meaning of the operation, utilizing representations, or making a story seemed to be what the class relied on to address the confusion, not all students were able to apply these aspects of proof in forming their own justification.

Individual differences in justifying a particular computation method using equal groups meaning of multiplication

One goal of the course was that students would become adept at justifying their thinking with both the equal groups and the area/array interpretations of multiplication. Although some of the tasks assigned to students allowed them to choose an interpretation, we also provided assignments in which students were required to generate two distinct justifications for the same solution (one utilizing each interpretation), and other assignments that required a justification utilizing one particular interpretation of the operation. So, for example, one writing assignment required that students: analyze the erroneous thinking of Thomas, a 5th grader; find a way to correctly solve the problem (17 x 36) by starting the same way Thomas had, using 20 x 40 (Schifter, Bastable & Russell, 1999); and then provide two written justifications, one utilizing the area/array interpretation, and the other utilizing the equal groups interpretation. We found it most productive to do in-depth analyses of the justifications based on each interpretation separately, as different issues arise. For the purposes of this paper, we will focus exclusively on the analysis of those justifications that utilized the equal groups interpretation of multiplication.

On the first exam, students were asked to solve the following problem:

> *Solve 24x38 by using a reasoning procedure that begins with the following starter: 20x40. Clearly show every step of your reasoning procedure and justify why it works by using the equal groups interpretation of multiplication. Note: Each of your steps needs to be solved mentally.*

The analysis of student solutions for the exam problem indicated that there were five different levels of performance based on the mathematical correctness and clarity of the justification. The following summarizes the characteristics of each level.

	Levels of Justifications	**Number of students (%)**
Level 4	Justification is clear and logically correct.	12 (30%)
Level 3	Justification is mostly clear and logically correct. Student may have glossed over, or omitted, some aspects of the justification	13 (32.5%)
Level 2	Parts of the justification are mathematically incorrect.	7 (17.5%)
Level 1	The majority of the justification is mathematically incorrect.	2 (5%)
Level 0	No justification is offered.	6 (15%)

A common omission for the Level 3 justification is not being consistent about keeping track of the results of adding or subtracting the number of equal groups. For example, one student wrote the following as part of her justification,

I started out with 40 groups of 20 (40x20=800). Then I added 4 to each group because the original problem is 24 x38. (4x40=160). Then I added 800 and 160 and had a total of 960. But I had 40 groups and I was only supposed to have 38. So I took off 2 groups of 24, which left me with 38 groups of 24.

The justification would have been more clear if the student had explained what 24x38 meant (38 groups of 24), and what 960 would be equivalent to (40 groups of 24).

When analyzing the data classified as Level 1 or 2, we identified three common errors. The following Level 2 examples illustrate these errors.

*We started with 20x40 to get easier numbers to work with. We now see that instead of 38 groups of 24, we took 40 groups, so we have 2 extra groups of 24. So we need to subtract those 2 extra groups from the 800 so 800-24=776, 776-24=752. So we now have 752. We also see that instead of taking 24 groups of 40, that we only took 20, so we need to add those groups of 40** back into the answer so, 752+40=792, 792+40=832, 832+40=872, 872+40=912. [** The student initially wrote 38 here, then erased and put 40.]*

The first common error was to add or subtract groups of different sizes, for example, taking away 2 groups of 24 from 40 groups of 20. The second type error was that of switching the roles of the numbers in the middle of the justification, for example, 40 was referred to first as number of groups, and later as the size of the group. [It is important to note that we have examples of mathematically valid justifications (coded as either Level 3 or 4) that involve reversing the roles of the two numbers at some point during the justification.]

From our data, the third most common mistake was: treating the increase or decrease of each of the two numbers in the multiplication problem as independent changes. For example, many students thought that changing 20x40 to 24x38 required decreasing 2 groups of 24 and increasing 4 groups of 38. Some of them, like the above student, changed the increases to 4 groups of 40 in their attempts to make the numerical answer correct. In these cases students did not appear to recognize the need to modify their justification to make the reasoning behind the computation steps correct.

Discussion

As indicated earlier, our analyses of whole class discussion as well as student written work led us to features of justification that differed from those described by previous research studies, which seemed to focus primarily on the source of conviction, such as single or multiple pieces of empirical evidence or a thought experiment. In our study, we found that many prospective teachers had developed a sense of what constitutes a valid justification based on the meaning of the operation and an understanding of the related properties. Their arguments were explanatory in nature and often supported with diagrams, story lines, or a combination of both.

What accounts for the differences between previous research on justification and our work? We believe that previous studies on proof and justification, even when conducted in specific content areas such as geometry and algebra, focused on identifying the characteristics and schemes of students' (or teachers') generated proof that were content-independent. Furthermore, they placed heavy emphasis on "generalizability." While it is important to use these general characteristics and schemes to think about the overall instructional goal of fostering students' ability to justify and reason across K-12, we believe that they are insufficient to guide instructional decisions on specific content for two reasons.

First, there is a growing consensus that it is important to provide elementary students the opportunity to develop their own reasoning strategies for computations before (or in place of) teaching them conventional algorithms. Thus it is important for the elementary teachers to know how to judge reasoning strategies based on the mathematical concepts embedded in the strategies. For example, our analysis showed that a mathematically sound justification for why a specific strategy for multidigit multiplication worked required specific attention to, and robust understanding of, the meaning of multiplication. When justification entails the use of the equal groups interpretation of multiplication, one has to understand that the two numbers being multiplied play different roles, thus altering one or both numbers may have different effects. Though multiplication is commutative, one cannot add 3 groups of 5 and 5 groups of 4, without reconceptualizing 5 groups of 4 to 4 groups of 5, thus obtaining the 7 groups of 5. In this instance, it is clear that the interpretation of multiplication as area/array is more productive in explaining why multiplication is commutative. Thus the importance of being fluent with generating justifications that utilize various meanings of the operations. [Though beyond the scope of this paper, we noticed that different issues arose when analyzing prospective teachers' justifications utilizing the area/array interpretation of multiplication.] All these ideas are fundamental to the concept of multiplication and are needed to support the development of mathematical reasoning with multidigit multiplication.

Second, we are convinced that elementary school students are capable of developing justifications that possess the power of "generalizibility" when placed in a learning environment that supports mathematical reasoning, and we believe that these students should be provided with opportunities to develop such notions (e.g. Ball's students justifying why odd numbers + odd numbers = even numbers). However, we do not see these as short-term goals for elementary students. Rather, elementary teachers must pay special attention to the role of meanings of operations in forming reasoning strategies and justification in arithmetic contexts, as well as to the role of definitions in forming conjectures and making justifications in geometrical or statistical contexts. In addition, they need to help their students develop a sense of what is entailed in a good justification.

Implications

Based on our current analysis, we suggest the following implications for both teaching and research. First, one major result of this study is a framework for the characteristics of content-specific justifications. More studies are needed to further examine characteristics of the justifications in this area, as well as to extend this work to other mathematics topics. Second, studies are needed to explore ways to help prospective elementary teachers develop the capacity to make connections among language, symbols/notations, and diagrams for justifications based on one particular meaning or definition, as well as justifications based on other meanings and definitions. For example, future teachers should be able to show relationships between justifications of the same method based on equal groups and area/array interpretations of multiplication.

Finally, although we were encouraged by the finding that about two-thirds of the students were successful at developing relatively clear and valid justifications, we were not convinced that all were solid multiplicative reasoners. Consider the following example of a student's thinking about how to solve 24x38 by starting with 20 groups of 40. The student began by taking 2 from each of the groups (of 40), and with these "extras" she formed 2 new groups of 20. Thus she had 20 groups of 38 and 2 groups of 20. In comparing this result with a picture of 24 groups of 38, she recognized that by adding 2 more groups of 18 to the 2 groups of 20, she would get 2 groups of 38. Thus giving her 22 groups of 38. And then two more groups of 38 would give her 24 groups of 38 in total. Note that instead of operating on the groups, she operated on the singletons in the groups. Thus the nature of the thinking process appeared to be more additive than multiplicative. Future studies are needed to investigate ways to support the development of multiplicative reasoning.

Endnotes

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PRE-SERVICE TEACHERS EXPERIENCING MATHEMATICS THROUGH MOON PROJECTS AND SPINNING TOPS

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This article describes inquiry-based environments we created in our mathematics methods courses where we designed projects that allowed pre-service teachers to explore, investigate, analyze, and communicate "investigable" realms of physical and mathematical phenomena. Our goal was for students to experience the value of learning in an inquiry-enhanced environment, so they would utilize this instruction in their future classrooms. Pre-service teachers were expected to pursue their own conjectures, collect data, think critically, and communicate findings. Our resulting data indicates how an inquiry-based environment enhanced the learning of mathematical content and educational strategies for pre-service teachers.

Framing Literatures

Current research has shown that inquiry and higher-order thinking are directly connected (Kelly, 1999). "Inquiry-based education sets the stage for bringing balance to how we instruct children and adolescents. Not only do such frameworks open the way for diverse modes of thought, but they also encapsulate the often unsung reality that much of our knowing emerges from unknowing" (McMillan and Wilhelm, in progress).

For a number of years, inquiry has been a major focus of instruction in science and science education. The National Science Education Standards or NSES (NRC, 1996) view inquiry as a teaching approach beyond that of a process skill where "science as inquiry" is considered one of NSES' eight categories of content standards. The NSES state that students "should have the opportunity to use scientific inquiry and develop the ability to think and act in ways associated with inquiry, including asking questions, planning and conducting investigations, using appropriate tools and techniques to gather data, thinking critically and logically about relationships between evidence and explanations, constructing and analyzing alternative explanations, and communicating scientific arguments" (p. 105).

So is inquiry the same in mathematics education? The current and revised National Principles and Standards for School Mathematics or PSSM (National Council of Teachers of Mathematics, 2000) clearly emphasize inquiry-oriented teaching as one of the best ways to facilitate mathematics learning, but do not specifically list "inquiry" as a content or process standard. However, the idea of inquiry is woven throughout the PSSM text where it describes how one might go about enacting the PSSM's five content standards and five process standards. For example, the PSSM process standard of *Reasoning and Proof* states,

> Doing mathematics involves discovery. Conjecture – that is, informed guessing-is a major pathway to discovery. Teachers and researchers agree that students can learn to make, refine, and test conjectures in elementary school. Beginning in the earliest years, teachers can help students learn to make conjectures by asking questions: What do you think will happen next? What is the pattern? Is this true always? Sometimes? Simple shifts in how tasks are posed can help students learn to conjecture…. To make conjectures, students need multiple opportunities and rich, engaging contexts for learning (p. 57).

Both Pierce and Dewey regarded inquiry as a "public process carried out by a community of investigators," and they defined it as the process of settling doubt and fixing belief within a community (cited in Skagestad, 1981, p.24; Pierce, 1877/1982; Dewey, 1933). Bishop (1988) indicated that this perspective on inquiry is especially significant for mathematics education, in that it is commonly viewed as only a collection of facts and procedures within a techniques oriented classroom.

Richards (1991) offered a notion of "inquiry by design" and argued that "students will not become active learners by accident, but by design, through the use of the plans that we structure to guide exploration and inquiry" (p. 38). Richards also described how little mathematics education literature exists that could potentially assist mathematics teachers with enacting this challenging task.

Borasi and Siegal (1992) defined an inquiry cycle as an instructional experience organized to engage students in inquiry as a four-part cycle of problem sensing, problem formation, search, and resolution. Specifically, this inquiry cycle consists of setting the stage; developing and focusing one's question; identifying appropriate approaches, resources, and tools for exploring the question; carrying out the research; collaborating with others; communicating with outside audiences; identifying problems; and offering invitations for new beginnings (p.380).

Schifter (1998) indicated that when students' thinking becomes the central focus, the classroom process is more difficult to manage and less predictable. She elaborated with, "If they are to build their instruction around children's thinking, the skills teachers need to acquire include interpreting students' mathematical ideas, analyzing how those ideas are situated in relation to the mathematics of curriculum, and challenging students to extend or revise those ideas so as to become more powerful mathematical thinkers" (p. 56).

Pre-service teachers need to experience the "practice" of inquiry-based learning by becoming a *full participant* during investigations facilitated in methods courses. Most pre-service teachers have not had personal experiences with inquiry-based learning during their K-12+ education, so they lack the background to design and to implement this kind of instruction.

Research Study and Methods

Our research study took place within a series of mathematics methods classes at a West Texas University over a period of four semesters. The participants consisted of 82 pre-service middle level and secondary mathematics teachers, eight pre-service middle level and secondary science teachers, and two pre-service middle level math/science teachers. All participants described in this study were instructed by first author, J. Wilhelm. The methods students were required to conduct several inquiry activities for their methods course. Two of the activities were an initial Moon investigations assignment with a follow-up Moon project and a spinning top investigation. Both activities were chosen for their motivating capabilities as well as their inherent interdisciplinary (science and mathematics) makeup. All activities required students to keep journals and encouraged students to document their thinking processes in a narrative format.

For our study's focus, we pursued the following questions:

- How will providing pre-service teachers with open-ended inquiry observations and writing tasks assist them in making sense of physical phenomena and assist them in their generation of new research questions?
- What will pre-service teachers learn from the inquiry investigations, and will they plan to use such investigations within their own classrooms?

- What mathematical content emerges through these investigations and what are the conjectures pre-service teachers communicate?

This study is of a qualitative research design. We used the process of triangulation, which involved ethnographic methods. Data collected included inquiry surveys, student journals, summative narratives, videotaped lessons, and final student project products. Student records were used to document their evolution of thought and development of understanding. The actual Moon phase enterprise was based on the writings and research of Eleanor Duckworth (1996). In addition, a classroom benchmark lesson (the Moon finale) was conducted with a guest astronomer. This Moon finale was videotaped and transcribed in order to record students' questions and responses concerning their Moon phase understandings. Summative narratives (midterm and final essay responses) were implemented to determine students' self-perceptions of which particular project investigations were the most conducive in aiding their formulation of thought and understanding.

The summative narratives were also designed to allow students an opportunity to explain which course components they deemed as most significant and to describe what they specifically valued in using inquiry projects within their own future classrooms. Narrative responses included discussions concerning what students learned from their inquiry investigations and if they planned to use such investigations in their own classrooms.

Initial Moon Assignment

Methods classes began with an initially vague project where students were asked to conduct daily Moon observations. Students would observe the Moon's phases as well as the sky for approximately five weeks. They were required to make sense of their Moon observations and were given an invitation to write "whatever they wish and think is relevant." After the five-week period students participated in a Moon finale with a guest astronomer. The purpose of this Moon finale was to disrupt incorrect Moon phase understandings and/or to confirm correct understandings.

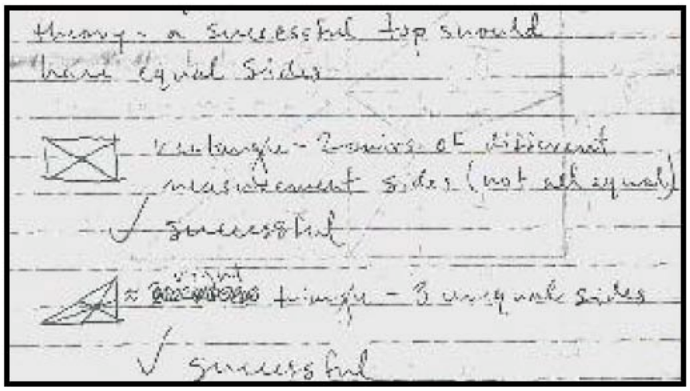
Much mathematics can and were considered when trying to understand the cause of the Moon's phases. Mathematics included geometrical configurations of the Moon, Sun, Earth system; calculating the Moon's orbital speed around the Earth; calculating the Earth's and Moon's orbital speed around the Sun; examining the elliptical orbital paths of the Moon about the Earth and the Moon and Earth about the sun; measuring in degrees the location of the Moon in the sky relative to horizon or relative to other objects in the sky (stars or planets); discovering the differences between sidereal time (the amount of time it takes the Moon to orbit the Earth and appear at the same place on the Celestial Sphere) and the lunar month (the amount of time between successive new Moons, or full Moons, or any two successive similar phases); possible relationships that occur between the angular displacement of the Moon from the Sun and the percent of the Moon lit. Many questions emerged from the methods students due to their keen observations, and these questions were used as springboards for their follow-up Moon projects.

Spinning Tops

Prior to the students' follow-up Moon projects, but after the initial Moon assignment, students conducted a more focused inquiry investigation with spinning tops. The inquiry with tops was used as a scaffolding assignment to aid in their success of the up-coming Moon projects. The tops activity was designed to follow Llewellyn's (2002) constructivist inquiry cycle where students conduct initial exploration, develop a driving research question/conjecture, design a plan for investigation, collect data, analyze data, communicate results, and develop a new question to research that emerged from their first investigative cycle. The assignment began with methods students brainstorming about specific features of a “successful” top. Most groups listed criteria such as: 1) spins at least five seconds, 2) spins perpendicular to the floor, 3) spins on a point, and 4) has a funnel or cone shape.

Working in groups of three and four, students were then asked to create their top. With a variety of materials and tools available, students created, tested, revised, and collected data with their tops. After initial explorations each group formulated a conjecture or question to test or answer.

Figure 1 – Pre-service teachers test their conjecture that a successful top must have sides of equal length.



Some mathematical questions explored by the pre-service teachers involved the geometry of the top (see Figure 1), the ratios of axis length below the top’s body to the axis length above, varying dowel rod lengths while holding the top’s diameter constant (see Table 1), and the location and distribution of weight (such as weight along rim or weight centrally located, see Figure 2).

Table 1 – Pre-service teachers explore the effect of axis length on spin time of top and investigate how the spin time is affected by the ratio of dowel length below top to dowel length above top.

Diameter of plate – 18.4 cm; Total length of axis – 8.7 cm.

Amount below plate	Amount above plate	Ratio of amounts	Spin time
1.2 cm	7.1 cm	0.17	25.00 sec
1.7 cm	6.4 cm	0.27	19.12 sec
2.3 cm	5.9 cm	0.39	14.20 sec

Diameter of plate – 18.4 cm; Total length of axis – 18.6 cm.

Amount below plate	Amount above plate	Ratio of amounts	Spin time
2.7 cm	15.9 cm	0.17	8.80 sec
3.9 cm	14.6 cm	0.27	2.80 sec
5.2 cm	13.4 cm	0.39	2.50 sec

After pre-service teachers proposed their question or conjecture, they planned how they would collect and analyze their data. All groups were required to generate a representation of their data collection. As shown in Figures 1 and 2 and in Table 1, students chose to represent their data through sketches and narration, tabular format, and with a bar graph.

Figure 1 displays a group's conjecture that in order for a top to be "successful," it must have equal-length sides. This group found that this conjecture was untrue since right triangular and rectangular (geometric shapes with unequal sides) shaped tops were just as good at spinning as their counterparts with equal lengths.

Figure 2 – Pre-service teachers discover that their most successful top's body was located one inch from the floor and its weight was equally distributed along the top's rim.

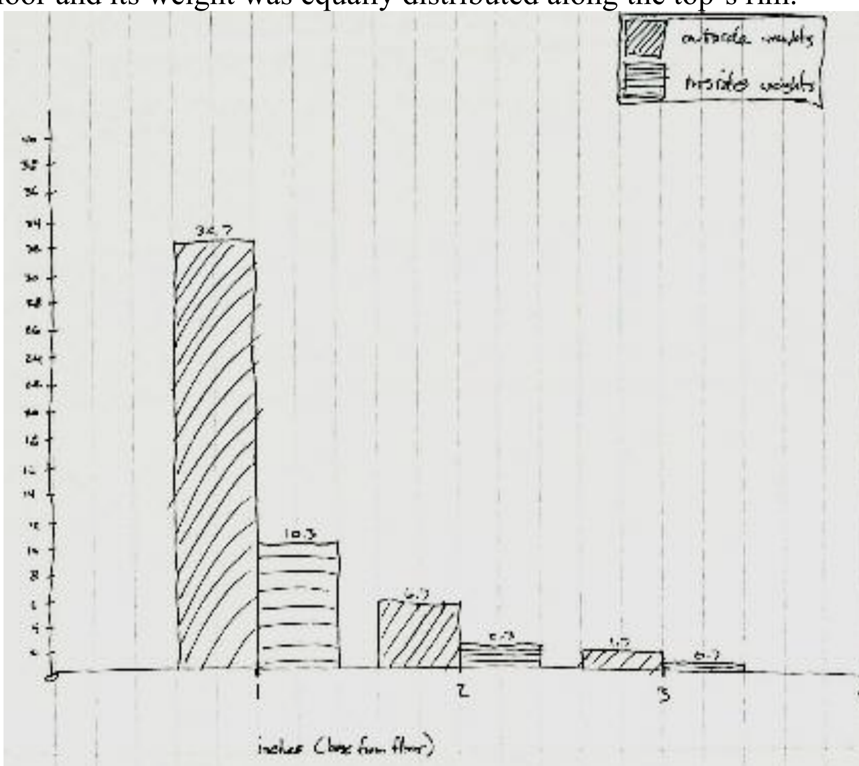


Table 1 illustrates two interesting features that emerged from another group's data collection. First, it appeared that the best spin time was obtained for the top with an axis length nearly equal to the radius of the top (a spinning plate). The second interesting relationship involved the ratio of the axis length below the spinning plate to the axis length above the spinning plate. The ratio that produced the best spin time was 0.17, which occurred when using both an axis length of 8.7 cm and an axis length of 18.6 cm. This group concluded that further data collection would be needed to test both of these new emerging mathematical relationships. Figure 2 displays a bar graph where this group discovered that their most successful top's body was located one inch from the floor with weights (coins) evenly distributed along the top's (a paper plate) rim.

Most students found the tops investigation accessible and interesting due to the fact that it contextualized many mathematical concepts, such as ratios, limits, statistics, and geometrical relationships. Students also indicated that they would conduct mathematical inquiry with tops within their own classrooms. In fact, one pre-service teacher used the tops investigation during her student teaching assignment at an area middle school. She conducted the tops activity over a period of two days. The cooperating teacher within the classroom remarked that one seventh-grader reported working on the assignment at home, which was something that he had never done throughout the entire year in his mathematics class.

Follow-up Moon Projects

After students experienced the constructivist inquiry cycle with the tops investigations, they were ready to continue the cycle with their Moon explorations. Many questions emerged from the methods students' initial Moon observations assignment and were used as a springboard for their follow-up Moon projects. The following are examples of some of the emerging questions that were incorporated into follow-up Moon projects:

- How does the night sky compare between city and rural viewing?
- Does human and/or animal behavior change with the phases of the Moon? If so, in what ways?
- What are the phases and positions of other planets in our solar system relative to Earth?
- Do the moons of other planets have the same phases as Earth's Moon?
- What would happen if the Earth had no Moon?

Students investigated and reported on topics of light pollution, relationships between the rise and set times of the Moon and Sun, phases of the Earth (as seen from the Moon), crime rate during a full Moon, birth rate during a full Moon, tidal forces caused by the Moon, cow milk production at various Moon's phases, planetary phases, and Jupiter's moons' phases. These follow-up inquiry investigations combined statistical mathematics, scientific data collection, and narrative perspectives in uniquely blended unison. Table 2 displays the mathematics utilized in each of the investigations in order to answer their research questions.

Conclusions

Students displayed throughout their inquiry investigations an increased ability of communicating what they do know, and questioning what more they need to know by coupling their experiential learning with opportunities to construct new thoughts and questions. They communicated what they knew through sketches, narration, and tabular and graphical representations. They looked for patterns and relationships and were beginning to discover a "reality connection" between theoretical ideas and the real world.

Students were asked what they considered to be the purpose of the inquiry activities enacted withing their methods course. Two methods students reported the following:

Student one – *"I think its purpose is to make the students go deeper and dig deeper on a subject than they would normally in a regular classroom. For inquiry-based activities, you can go in all different kinds of directions and you could have all different students in different groups researching different aspects of a certain concept."*

Student two – *"For inquiry, you can go and figure out what you want to know. It's like what we did in the Moon investigations. There are so many questions that I wanted to know just by doing that little project. By doing the observations, I developed questions on my own. I think by now I will remember more what I have found out by having questions I wanted to know rather than just something you wanted us to know."*

Most students stated that they wished to incorporate inquiry activities within their own classrooms.

The results from our study provide specific evidence of how an inquiry-based learning environment enhanced the learning of mathematical content for pre-service teachers. In addition, the experiences and the results from this study provide additional information for teacher educators regarding how to enrich the design of the instructional format used in methods courses for pre-service teachers.

Table 2 – Embedded mathematics within students' follow-up Moon projects.

Student follow-up Moon project	Mathematics utilized
How does light pollution affect what we can see?	**Star counts from city parking lot, from in-town home, and from rural location.** **Percentage differences in star counts.**
What relationships can be found between the rise and set times of the Moon and Sun?	**Collected data from the local paper and a naval website on the Moon and Sun rise and set time.** **Graphical representation of data to compare the differences in the changes in time.**
How does the Moon affect crime?	**Research statistics from Internet.** **Collected data on number of crimes per day committed in Lubbock, Texas in 2002.** **Graphical representation of data comparing the average number of crimes per day with the actual number of crimes per full Moon day (in Lubbock).**
Is there a correlation between the full Moon and cattle behavior?	**Research statistics from Internet and from personal interviews.** **Graphical representation of data (scatter plots and pie charts).**
What are the phases and positions of other planets relative to Earth?	**Examined planetary elliptical orbits, ecliptics and angles of inclination, planetary rotations, and relative orbital sizes.**

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HELPING PRESERVICE ELEMENTARY TEACHERS DEVELOP FLEXIBILITY IN USING WORD PROBLEMS IN THEIR TEACHING

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This paper focuses on an intervention approach involving investigation of the structure and mathematical meaning of arithmetic word problems [WP] as a basis for helping preservice elementary teachers to develop flexibility in their thinking and use of WP in their teaching. Pre-intervention, intervention and post-intervention activities and the study of the approach are discussed. The findings suggest that the approach was effective in enhancing the participants' views of WP and providing them with a basis for being flexible in their teaching of WP. The study also provides examples of the way preservice teachers' could think about WP without any intervention and a structure for an intervention approach, both of which can make a contribution to inform and enhance preservice teacher education.

This paper reports on a study that investigated a way for preservice elementary teachers to develop flexibility in their thinking and use of word problems in their teaching of mathematics.

Related Literature and Theoretical Perspective

Word problems [WP] are an integral aspect of school mathematics. They can be used as a basis for application and integrating the real world in mathematics education. They can motivate students to understand the importance of mathematics concepts and help students to develop their creative, critical and problem solving abilities. Traditional WP and traditional instruction, however, are critiqued for not accomplishing these goals in a meaningful or effective way. For example, there is concern expressed about students' lack of sense making (Silver, Shapiro, & Deutsch, 1993) or suspension of sense making (Verchaffel, Greer, & DeCorte, 2000) when working with contextual problems. Whether or how this situation changes will likely depend on the teacher. Chapman (2002, 2003) showed that teachers' conceptions and beliefs about WP play an important role in their teaching approaches. For example, traditional teachers had a limited view of WP and lacked the flexibility to teach them meaningfully. This suggests that helping teachers to extend their views of WP could make a difference to their teaching.

Preservice elementary teachers are likely to have limited views of WP given that they may have experienced them in a predominantly traditional way as students of mathematics. Studies have suggested that preservice teachers' mathematics-related views or beliefs, in general, are usually problematic in a variety of ways. These studies have focused on the preservice teachers' mathematics-related beliefs in a variety of contexts, for example, their learning of mathematics (Gorman, 1991), their understanding of specific mathematics concepts (Verschaffel, 1996), the nature of their beliefs structure in teaching mathematics (Cooney, Shealy & Arvold, 1998), changes in their thinking about mathematics and its teaching (Mayers, 1994) and their learning and implementing of teaching methods (Cooney, 1985, Raymond, 1997). Despite the growing number of such studies on preservice teachers, there is little focus explicitly on WP. However, other deficiency-oriented studies have directly or indirectly highlighted difficulties they have with WP. For example, Contreras et al. (2003), in their study of 139 preservice elementary teachers' solution process to additive WP found that the majority of them contained incorrect solutions due to incorrect interpretation of the solution in relation to the problem situation. Given the central role of WP in learning mathematics, there is need for further work on preservice teachers' thinking and sense making of WP and ways of effectively extending or enhancing their thinking. This study is intended to make a contribution in this regard.

Theoretically, the study is about preservice teachers' learning and is framed in a social or interactive perspective of learning. An interactive perspective on teaching and learning has been discussed by several people including Bauersfeld, 1979; Dewey, 1916; Lave & Wenger, 1991; and Vygotsky, 1978. This perspective promotes the position that learning takes place in a social setting and thus emphasizes human interactions as a key factor to facilitate learning. It is a perspective that is reflected in current reform perspective of mathematics education that advocates the importance of teacher-student and student-student interactions (NCTM, 1991; Kilpatrick et al. 2001). Thus engaging preservice teachers in this way also allows them to experience learning in a way they are encouraged to provide for their own students.

Research Process

The study was framed in a qualitative, naturalistic research perspective (Creswell, 1998) that focused on capturing and interpreting the participants' thinking about a phenomenon (WP in this situation) as a basis of evaluating an intervention experience.

The participants were twenty preservice teachers who had completed their practicum, were in their final semester, expressed fear of WP, and had no instruction or theory on WP prior to this experience. They were all interested in improving their understanding of WP and participated fully in all of the activities. They were taking no other math, math education or math related courses. The work was completed in about 9 hours in class plus out-of-class work over 3 weeks.

The pedagogical activities the participants were engaged in were organized as: pre-intervention, intervention, and post-intervention activities, briefly summarized here.

Pre-intervention Activities:

This involved four individual tasks and one group task. The individual tasks consisted of the following:

1. Creating an original WP for an elementary grade and reflecting on and describing what he/she thought about to create the problem.
2. Creating a WP that is similar to the following problem, then reflecting on and describing what she/he thought about to create the problem. Golf balls come in packs of 4. A carton holds 25 packs. Marie, the owner of a sporting goods store, ordered 1600 golf balls. How many cartons did Marie order?
3. Comparing and contrasting this golf ball problem to the following: Jerry, a new elementary school teacher, wants his students to sit in groups on the carpet section of the classroom floor for certain activities. After doing some investigating, he found that the carpet area was 144 square feet and decided to allocate 6 square feet for one student. How many groups will Jerry be able to form if he wants 3 students in a group?
4. Comparing the problem created in item 1 to the golf ball problem to explain whether or not they involved the same mathematics concepts.

For the group activity, they first shared and compared their individual work with each other in small groups and individually recorded any meaningful differences they noticed. They then worked collaboratively to create different WP to reflect the meanings of addition, subtraction, division and multiplication for whole numbers and identifying what meanings were conveyed for the operations.

Intervention Activities:

Based on the theoretical perspective of the study, the intervention consisted of purposeful activities in social settings (Dewey, 1916) with a focus on peer collaboration. The activities were designed based on the work of Riley, Greeno & Heller (1983), Greer (1992) and Verschaffel & DeCorte (1996). This work provides an exhaustive list of examples of different problem situations modeled by addition, subtraction, multiplication and division. There are 4 situations for addition and subtraction: change, combine, compare, equalize. However, these can be framed so that the unknown represents a result, change, start, super, sub, difference, compared, or reference set in a direction that is an increase, decrease, more or less. For example, the problem "Rod has three marbles. Todd gives him five marbles. How many does Rod have?" is a change situation, the unknown is a change set and the direction is increase. For multiplication and division there are two situations: asymmetric and symmetric. The former deals with equal groups and multiplicative comparisons, the latter with rectangular pattern or Cartesian product.

Problems were developed for each situation and grouped as addition situations (8 WP), subtraction situations (5 WP), algebraic situations (5 WP), multiplication situations (4 WP) and division situations (5 WP). The addition WP consisted of four situations, with 2 WP per situation – one that could be modeled in terms of groups and the other linear measures. For the other operations, the number of WP equaled the number of situations for each based on a group model. Students had to make up examples of WP for the corresponding linear model. The algebra situations involved WP with missing addend or missing minuend. Examples of problems used:

> I have some marbles, I give a friend three and have five left. How many marbles did I start with?
>
> I have 35 marbles, which are 15 marbles less than my friend has. How many marbles does my friend have?
>
> Simple structured problems were used to allow the participants to focus more on how to unpack, by interpreting and modeling, the problem situations and not on getting the answer.

The participants worked in groups. They modeled each situation concretely, drew pictures of the mathematical processes, determined the meaning of the operations in the context of the situations involved, and represented the processes symbolically and with verbal mathematics language (e.g., the sum of). The groups shared and compared their findings, questioning each other and justifying their positions. The researcher/instructor facilitated the process by posing questions to guide their attention to what they were not noticing.

Post-intervention Activities:

This included working individually to compare two multi-structured WP; unpack multi-structured WP; create original WP for all the different situations and solve and represent them pictorially, symbolically and verbally; review the most commonly used textbook in the local school board for the presence or absence of WP of the different structure of WP situations (each student was assigned only one of grades 2-4); and write journals on the meaning of the experience for them.

Data Collection

Data collection was built into the pedagogical activities described above. A pre-post intervention design was used. The pre-intervention activities were used with a different group of preservice teachers a year earlier and were found to be reliable in capturing relevant information on unpacking WP situations. This finding was what motivated this study. The activities provided data of the participants' thinking of WP, for e.g., what they attended to in terms of the structure of the WP, how they interpreted the WP, and how they were able to relate the WP situation to meaning of the arithmetic operations. The intervention activities produced data on their developing understanding of the WP situations in a variety of ways. The post-intervention activities provided data on their post-intervention thinking of WP (similar to the examples noted above for the pre- intervention), how they performed with more complex structured WP than used in the intervention and how they viewed the overall experience. The data thus consisted of copies of all of their written work for the pre and post intervention activities and the intervention activities. There were also field notes of their group discussions and whole- class discussions.

The analysis focused on obtaining evidence of the participants' thinking and performance before, during and after the intervention. Pre-intervention data were coded by the researcher and a research assistant working independently. The post-intervention data were coded by the researcher and a different research assistant who did not have access to the pre-intervention data, working independently. This allowed for comparison and validation of the findings. Coding involved, for example, identifying significant statements about their thinking of WP and the experience with the intervention and what they attended to in working with WP [e.g., interpreting, solving, comparing, creating, identifying and modeling them]. The researcher also summarized their way of describing and representing the meaning of the operations for their intervention work. This was compared to the pre and post intervention findings for triangulation. The coded information was categorized based on common themes and frequency of occurrence and used as a basis to draw conclusions.

Findings

Pre-intervention:

The pre-intervention activities revealed how the participants made sense of arithmetic WP. For example, they viewed them as consisting of cover stories and operations. The cover story should relate to students' real world experiences and the operations should be what the students already know. Thus WP are used only to practice the operations after they are taught. The difficulty of the WP was judged by its format and size of the numbers. The cover story was reduced to isolated words to identify operations to find a solution. Specific examples of the participants' thinking for the pre-intervention tasks follow.

In creating a WP, the participants' thinking included: skills to teach, operation to use, topic [students] just finished, two concepts students learned, how relates to students, real world, a scenario, visual types of problem [they had] seen, type of problems [they had] seen before, kids prior knowledge, not obvious, simple, how difficult, and more than one operation. Relevance to students was the most common in terms of their social activities and mathematical knowledge.

In creating a similar WP to the golf ball problem [described earlier in the pre-intervention activities], they focused on changing the objects in the problem by thinking about, for example, what come in cases, objects they like, good memories, and what kids can relate to. There was very little duplication in their choices of objects that included: coke, paper plates, oranges, tennis balls, cookies, disks, eggs, and candies. There were six categories of problems in terms of the story, numbers, and operations reflected in the WP they created to represent a similar problem.

5. Similar story, same numbers and operations: For example, Dave ordered disks for his computer. Disks come in packs of four. A box holds 25 packs. Dave wanted 1600 disks. How many boxes should he order?

6. Similar story, different numbers, same operations: For example, Anne must supply the candy store with 1200 chocolate bars. The bars are packaged in cases of 12 bars per box and are shipped in cases of 4 boxes per case. How many cases must she order? Different story, same numbers and operations: For example, a ship was carrying 1600 golf balls and dropped them in the ocean. Golf balls come in packs of four and each carton holds 25 packs. How many cartons fell into the ocean?
7. Different story, numbers and operations: For example, oranges come in cases of 50. Ladaws needs 1050. How many cases do they need to order?
8. Different required, numbers and operations: For example, eggs come in cartons of 12. A case of eggs holds 10 cartons. Amy the storeowner of Safeway, ordered 25 cases of eggs. How many eggs did Amy order?
9. Similar story and operation, numbers give remainder: For example, badminton birdies come in packs of 6. Each box contains 36 packs. A school needs 360 birdies. How many boxes does the school need?

The changes in the cover stories of the WP, as seen in the above examples, also corresponded to changes in the mathematical situation and structure from the original problem. These examples provide evidence of how the participants made sense of "a similar problem".

In comparing the golf ball problem [#1] and the floor area problem [#2] given earlier in the pre-intervention activities, examples of what the participants attended to are:

10. Format of the problem: For example, #2 was wordier than #1.
11. Level of difficulty: For example, #1 seems easier; easier division; #2 seems more complex, math seems more complicated because of square feet; seems like more factors to consider; seems more overwhelming.
12. Arithmetic operations: For example, both require multiplication and division; #2 is multiplication and division and #1is division.
13. Relevance to students' life: For example, #1 may be more meaningful to kid's real life; students might like context [for #2].
14. Objects: For example, #1 contains golf balls, packs, cartons; #2 contains students.
15. Mode of learning: can better visualize it [#2].
16. Solution strategy: For example, same operations, different order; #1 equals multiplication then division [25 X 4, 1600 /100], #2 equals division then multiplication [144/6, 24X3]; both involve division and multiplication reasoning; 2-step process, determine how many people/golf balls fit in something, then compute, but different skills needed to approach problem solving; same steps to solution – you multiply the individual by the "grouping" then divide by the total.
17. Numbers: For example, different magnitude of number; for #2 slightly more difficult numbers, might require a calculator.

These ways of unpacking the two problems are representative of the participants as a group. Each participant articulated a subset of these aspects of the problem, but the group sharing and whole-class sharing allowed them to become aware of the complete list and whether there were errors in their thinking about the mathematics of the problem and how to resolve it.

For the pre-intervention group activity, described earlier, the groups were able to create examples of WP for each of the four arithmetic operations. The following examples for one group are representative of the type of WP situations they considered for each operation.

If you have three apples and two oranges, how many do you have altogether?

If you have 25 smarties and you eat three how many will you have left?

A group of 7 boys had 3 hockey cards each. How many did they have in total?

There are 25 smarties in a box. Five kids need to share them. How many does each child receive?

There was no issue in solving such WP, but there were difficulties associating mathematical meanings of the operations with them or interpreting the problem situations for them in terms of the mathematical structure.

Post-Intervention:

The post-intervention activities revealed greater depth in how the participants viewed WP. For example, they now viewed WP as consisting of a cover story of what the problem is about, a mathematical structure associated with the meaning of the given and unknown quantities involved, and a semantic structure associated with the way in which an interpretation suggests particular mathematical relationships. They viewed the problem context not only as a social story but also a mathematical story. They seemed to be more knowledgeable about the mathematical story based on their journal responses; their successful performance interpreting multi-structured WP and comparing multi- structured WP of the same and different structures with depth; and their ability to represent different versions of WP concretely, pictorially, verbally, and symbolically to show the meaning of the operations. They indicated they could make more sense of the use of the textbook as a resource for WP. They also now viewed the role of WP in their teaching as a basis to explain/explore the meaning of the operations and to introduce the math concepts to students, and to interpret real-world situations mathematically. The algebra situations were new to them in the context of elementary school mathematics. Categorizing them this way made it more meaningful for them in interpreting these situations. They explained they would have avoided such problems because on the surface they seemed confusing and they were not sure how to help students with them until now. The following excerpts from two participants' responses is representative of their thinking:

18. This makes so much sense now. I always looked only at the numbers and try to figure out the operations from certain words in the problem. But I didn't understand about the problem situation and the structure of the problems and how you use that to make sense of the problem. Now it makes more sense to teach these problems to help students understand them and relate them to the real world.
19. I felt inadequate after the first set of activities, but after studying how to interpret these kinds of problems, I feel a lot better about it.

Thus, they also expressed increase in confidence working with WP in their teaching compared to their earlier experience of trying to avoid them during their practicum teaching. They talked about being more comfortable in allowing students to create their own problems. They wanted to teach in a way that reflected the experience with this intervention approach, in particular, helping students to interpret WP and not simply look for the operations as they did in their practicum teaching on the few occasions some of them attempted to work with WP.

Conclusion

The findings suggest that an intervention approach based on investigating the structure of arithmetic WP could enhance preservice elementary teachers' view of WP and teaching WP. The approach seems capable of providing preservice teachers with a basis of being flexible in their teaching. However, how they are able to implement this into their practice needs further study, for example, in understanding how to select and use appropriate WP and representations in a meaningful way and to evaluate students' work to make sense of their thinking. Finally, the study provides examples of the way preservice teachers could think about WP without any intervention and a structure for an intervention approach, both of which can make a contribution to inform and enhance preservice teacher education.

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LORTIE'S APPRENTICESHIP OF OBSERVATION REVISITED

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Our primary purpose in this paper is to examine Lortie's notion of apprenticeship of observation and the ways in which it has been used in contemporary research. We revisit Lortie's Schoolteacher study and review the data from which he drew his conclusions about the power of observation in shaping teachers' practices. Although it is intuitively appealing and has been widely cited, we contend that apprenticeship of observation lacks the explanatory power to account for the perpetuation of certain instructional practices. We provide data from a recent study to offer a contrast with Lortie's findings.

When Daniel Lortie's sociological study of teaching titled *Schoolteacher* was published in 1975, it was heralded as "some of the most trenchant, unique, and helpful research ever done on the profession of teaching" (Lortie, 1975, back cover). In the nearly three decades hence, Lortie's work has been cited time and again in more modern studies of teaching and teacher education–both general studies and studies specific to mathematics education. The portion of Lortie's work that has garnered more attention than any other is his notion of the "apprenticeship of observation" (Lortie, p. 61) and his oft-quoted observation that "the average student has spent 13,000 hours in direct contact with classroom teachers by the time he graduates from high school" (p. 61). The catchphrase "apprenticeship of observation" has since become synonymous with the claim that teachers teach as they were taught and has been widely used to explain the apparent lack of influence of teacher education programs on teachers' beliefs and practices. However, we are apt to side with Wideen, Mayer-Smith, and Moon (1998), who referred to this frequently cited passage as an example of Byrne's "snark syndrome"; an idea that takes on the air of authority through repetition, instead of empirical evidence. In this manuscript it is not our aim to discredit the work done by Lortie; rather we aim to examine Lortie's original words, question the way in which they have been invoked, and suggest ways that Lortie's work might be used more productively in future scholarly endeavors.

Rereading Lortie in the Original

Lortie introduced the term *apprenticeship of observation* in the book's third chapter, the topic of which is the limits of socialization in preparing teachers for the classroom. He noted, "There are ways in which being a student is like serving an apprenticeship in teaching; students have protracted face-to-face and consequential interactions with established teachers" (p. 61). However, he went on to note that students have a particular perspective on teachers and the task of teaching, namely that of an audience member. Although there is a relationship between students and teachers as a result of their daily interactions, students are not privy to their teachers' reasons for and reflections upon their actions. Students are on the receiving end of what teachers do and are therefore only in a position to notice teachers' actions and their influence on them as students. They are not in a position to be reflective and analytical about what they see, nor do they necessarily have cause to do so. Students are likely to express pleasure, appreciation, dislike or other affective responses toward a teacher or particular practices but not to assess thoughtfully the quality of the teaching they experience.

This type of apprenticeship stands in stark contrast to the more traditional notion of an apprenticeship in a trade in which the apprentice is privy to the thinking and reasoning of the master while observing the master at work. Further, in a traditional apprenticeship the master coaches the apprentice as he learns to ply the trade. Lortie noted that what students learn about teaching via observation "is intuitive and imitative rather than explicit and analytical; it is based on individual personalities rather than pedagogical principles" (p. 62).

Thus, Lortie concluded, the metaphor has significant limits. Although Lortie used the term apprenticeship of observation repeatedly, he clearly did not mean to suggest that spending 13,000 hours observing teachers results in students who have analyzed and critiqued the work of teaching from the perspective of a teacher. Rather, students acquire both detailed and generalized notions of what "good" teaching is based on how particular kinds of teaching have affected them.

Despite Lortie's careful analysis of the use of the term apprenticeship, he then went on to make a rather lofty claim about the results of apprenticeship of observation. He speculated that teachers' practices are based on imitation of their teachers, "which, being generalized across individuals, becomes tradition" and "transcends generations" (p. 63). Lortie seemed to be aware that this claim was intuitive but unfounded because he asserted that "It would take complex research to confirm this analysis" (p. 63). Despite the fact that such research has not been conducted, many an author has invoked Lortie's generalization, thus supporting the perpetuation of the snark syndrome.

A Closer Look at Lortie's Data

Lortie offered seven quotes from teachers in his study to confirm his claim that former teachers influence the practice of current teachers. Five of the seven quotes provided evidence that a specific practice by a specific teacher had an effect on a student that was carried into their teaching careers. For example, a fifth-grade teacher recalled an elementary school teacher who was especially understanding of her fright at moving to the school from out of state and noted that in her own classroom she always assigns a buddy to new students to help them feel comfortable. The other two quotes were from teachers who believed that their teaching practice was shaped in some non-specific way by their collective experiences as a student. These quotes do not provide convincing data to support Lortie's claim of the "continuing influence of former teachers" (p. 64) in the overpowering and negative sense that is often portrayed in the literature. Lortie's study is often invoked to decry the dismal and overwhelmingly negative state of teaching in this country. Lortie's work is also used to support the assumption that there is a cycle of intellectual poverty in teaching because future teachers cannot be expected to break away from the traditional teaching they have experienced as students. Although intuitively appealing, Lortie's claim that teaching practices transcend generations is not substantiated by the evidence he provided. In fact, Lortie himself went on to say that "there is little reason to expect that any group of teachers-to-be will share common images or proclivities" (p. 66).

Many authors who have invoked Lortie's generalization to explain preservice teachers' resistance to change as a result of teacher education coursework may have assumed that preservice teachers possess uniformly traditional views of teaching, in general, and mathematics teaching, in particular. Lortie's work has been uncritically appropriated by teacher educators from a variety of disciplines despite the fact that neither the subject of his book nor the purpose of his data collection was specific to a particular discipline. It is reasonable to question whether a prospective literature teacher and a prospective mathematics teacher might have different views of teaching based on their choice of subject matter. Lortie's work spanned grade levels, but a careful analysis of his generalization by schooling level is appropriate. It is quite plausible that a prospective elementary teacher (who is going to be a generalist teaching multiple subjects) would have different views of mathematics teaching than a prospective secondary mathematics teacher. At a minimum, the latter is likely to have been highly successful in mathematics in school while the same cannot be guaranteed of the former.

Additionally, claims about the overpowering influence of apprenticeship of observation are based on an assumption that mathematics is monolithic. It is certainly reasonable that a prospective teacher could have differing views about teaching geometry versus teaching fractions. Although some students' experiences with mathematics are uniformly positive or negative, the majority of students likely have differing levels of success with different topics within mathematics. We do not deny that preservice teachers have ideas and beliefs about what it means to teach mathematics when they enter a preparation program, but we are doing a disservice to preservice teachers when we invoke Lortie's apprenticeship of observation as a "one size fits all" explanation for the views that they bring to teacher education programs.

A Contemporary Application of Lortie

Data collected from a four-year study of novice elementary school teachers suggest that preservice teachers are cognizant of the myriad experiences they have had as mathematics learners and the specific roles that teachers have played in those experiences. In contrast to the picture that people paint by invoking Lortie and his ideas, we believe that future teachers are capable of being analytical about their goals for their teaching practices in light of their prior experiences. In fact, Zeichner and Gore suggested that some preservice teachers "focus more directly on their own learning as students and deliberately seek to create in their own teaching those conditions that were missing from their own education" (1990, p. 333). Thus, even when preservice teachers have had negative experiences as learners, they are able to use these experiences to shape their ideas about teaching practices in positive ways.

In support of our claim that preservice teachers are capable of being analytical about their experiences during their apprenticeship of observation, we offer data from mathematics autobiographies written by preservice elementary teachers during their first week of a mathematics methods class. The assignment was open-ended, but it was suggested that they comment on their vision of an ideal mathematics teacher. This question is very similar to a question Lortie posed in which he asked teachers to describe an "outstanding" teacher they had.

Of the 14 teachers on whom we collected extensive data, only two made no mention in their autobiographies of specific teachers or general teaching practices that had either inhibited or facilitated their mathematics learning. Both were very successful in mathematics and mentioned an overwhelming influence of their parents. In Figure 1 we have classified the remaining 12 teachers' autobiographies based on whether they mentioned specific teachers or general teaching behaviors that either inhibited or facilitated their learning. Two preservice teachers mentioned instances that fit in all four cells of the table below.

We submit that there is more to the "apprenticeship of observation" notion advanced by Lortie than simply being exposed to certain kinds of teaching/learning and being destined to repeat those practices. We found that preservice teachers were quite adept at articulating what they did and did not like about their prior schooling experiences and why. They were aware that their prior experiences were not monolithic and were able to describe elements of mathematics courses in which they learned well and those in which they did not learn well. The preservice teachers reported on mathematics learning experiences from throughout their elementary, middle school, high school, and college years that impacted their thinking about teaching mathematics. Some traced their entire careers as students, making note of specific events and teachers across time, while others pinpointed one incident or teacher that made a lasting impression. In some cases below, we have also provided evidence from the teachers' practices that suggest that their entering ideas about teachers were manifested in their later practices.

	Inhibit	Facilitate
General Teachers Andrew	Stephanie	Eileen
Specific Teacher Megan	Becky Katherine Sherrie	Elizabeth Molly Mary Rodrick Shelly
No mention of teachers	Jennifer Heather	

Figure 1: Preservice teachers' accounts of teachers' influence on their mathematical learning.

Both Sherrie and Becky related specific actions by teachers that negatively impacted their learning of mathematics. Sherrie had a history of being unsuccessful with mathematics, and she related an experience from sixth-grade that she believed led to her placement in remedial mathematics classes where she "continually remained bored and unchallenged." Sherrie related her fear and humiliation in mathematics class in this excerpt from her autobiography.

> One day as I worked on a problem at the chalk board she yelled "No, No, No! Sherrie, that is wrong. We have been over this for weeks!" Embarrassed, I sat down in my chair and never volunteered again to go to the board.

We observed Sherrie during her first year of teaching and noted that she was very supportive and understanding when students gave incorrect answers. She asked students to explain both correct and incorrect answers and supported students both verbally and nonverbally as they gave explanations. She encouraged peers to assist one another and sometimes allowed children to "pass" when they struggled with an answer. Sherrie's actions suggest that she was deliberately attempting not to replicate teaching practices she had experienced as a student. Ross (1987) found similar examples of teachers deliberately enacting practices that were counter to those they had experienced as students. Ross argued that preservice teachers are "highly selective" in picking and choosing from among the models they have seen in order to meld several practices into the type of teacher that they become.

Becky, who had been successful in mathematics, as evidence by her participation in Advanced Placement Calculus in high school, shared her experiences with a teacher who only accepted answers that were in the format she was expecting. Becky's frustrations with this teacher were captured by the following incident:

> I was checking over my test and I saw one question that had the right answer but had received no credit. I asked her why she counted my answer wrong when even the key showed that I had arrived at the correct answer. She responded to me saying, "That's not how they want it solved on the AP exam." Her attitude was what made me cry more than the fact that I had lost points. I thought the whole thing was ridiculous. I felt that I deserved those points because I had discovered an alternate way to solve the problem. I understand her point but I still do not believe that I should have been penalized for creativity and being able to make connections that had not been taught in class.

This incident and others led to Becky dropping the calculus course at the end of one semester, a decision that she later regretted. In observations of Becky's teaching during her preservice years, we saw numerous examples of her valuing students' thinking and asking for multiple solution methods. She consistently asked students to explain their reasoning in addition to giving the final answer, and she frequently asked, "Did anyone do it a different way?" In one observation, she took a substantial amount of time to have students compare the answer $16\,\frac{2}{3}$ and $16.\overline{6}$ to realize that they are the same and either format was acceptable. In the same lesson, there were three other instances in which she engaged students in discussions about multiple solution strategies or ways to represent an answer.

Several preservice teachers reminisced about mathematics teachers who made learning fun, who connected mathematics to its real world uses, who were patient, and who had a passion for what they were doing. Shelly's comments about her seventh-grade mathematics teacher are representative of several students' writing.

> I have always loved Literature, but it wasn't until seventh grade that I learned to love Math. My teacher was Mrs. McDonald and I thought the world of her. She presented ideas and concepts in such a way that everyone understood. If there ever came a time that we didn't understand a problem, she would go over it again and again until we did. She never talked down to us or made us feel stupid. Math class was always fun and entertaining, as well as educational. We worked in groups, as well as individually; we listened to Mrs. McDonald lecture, as well as teach other students ourselves. We were exposed to so many new and exciting things.

From multiple observations of Shelly's practices during her first two years of teaching, we have evidence that she treated her students with respect and provided multiple ways for her students to learn a particular piece of content. For example, in a lesson on sequencing numbers with first-grade students, Shelly used whole group instruction, a song, partner work with physical manipulatives, and individual written work.

Eileen offered a more general comment about her experiences as a mathematics learner and teacher actions that helped her be successful. She also connected her experiences to her views about what a good teacher should do.

> I have a need to be challenged; that is how I learn best…. A teacher should encourage you. He or she should encourage you to try new things, to bring up your grades, or to always give your best. He or she should challenge you. I have found that if I am put into a classroom where I am not challenged, I become lazy. Therefore, I learn a great deal less than I would have if I had been giving all my effort.

This excerpt shows that Eileen reflected on her own learning and made a connection to her teachers' actions. During observations of Eileen's teaching, we noted that she was adept at using individual extensions to challenge students who worked at a faster pace than others. She also pitched her questions to students at a level appropriate for their development, asking more demanding questions of higher achieving students.

Five preservice teachers made direct and specific connections between their prior learning experience and their future teaching. In two cases, the preservice teachers noted that a particular teacher had influenced them to become a teacher. Elizabeth wrote

> My favorite teacher from my last 16 years of school was my sixth-grade math teacher, Mrs. Weber. She was amazing! She made math fun, interesting, and beneficial for everyone in the class. Mrs. Weber was actually one of the greatest influences on my decision to become a teacher; she most definitely deserves the label as an "ideal teacher." Mrs. Weber was

> compassionate, intelligent, honest, fair, fun-loving, energetic, and caring. Everyone in her class looked forward to math every day because she possessed an amazing ability to make learning so much fun. Even the students who had always claimed to "hate" math found a way to enjoy Mrs. Weber's math class. I can only hope that one day I will be able to get my students so excited about learning mathematics.

In the lessons we observed in Elizabeth's classroom during her first year of teaching, we noted that she almost always had a fun activity planned for the whole class portion of the lesson, followed by small group work. These activities often involved games or manipulatives, and Elizabeth made an explicit point of telling students that what they were about to do would be fun.

> Molly was influenced in her career choice by her high school calculus teacher. Ms. Anderson was the best teacher I have had in the area of getting us to visualize problems and apply them to real life. She also was one of the best teachers I had in general, because she knew each of us personally, and based her teaching off that. In fact, I count Ms. Anderson as one of the reasons that I am going into the teaching profession. She made learning fun, and she loved her students, two characteristics I feel are necessary to be a good teacher. …The ideal teacher is one who makes learning fun, shows the students that learning applies to real world situations, and cares about the students.

In observations of Molly during her preservice program, we did not see evidence of her enacting these statements. Molly's teaching tended to be very directive and worksheet-driven. Students were allowed to use manipulatives, but only briefly and in the ways she prescribed. We did not observe her making any real world connections with her students. This discrepancy may reflect differences between high school and elementary school mathematics. We have not yet had an opportunity to see Molly in her own classroom, so it is possible that her actions were not a reflection of her intentions but were constrained by the culture of the classroom in which she was placed for student teaching. On the surface, Molly appears to fit Lortie's stereotype because, despite her preservice preparation program, she seems to have adapted "traditional" teaching methods. However, it is not clear whether Molly's methods are the result of the context in which she has taught, her beliefs, or other factors.

Two preservice teachers wrote explicitly about how their experiences as learners had shaped the teaching practices they hoped to use. Stephanie had negative experiences as a learner and reported having great anxiety about mathematics.

> Now that I sit and realize that most of my math anxieties were caused by a combination of not understanding the concepts and a few bad teachers, I am reminded that this is one reason I want to be a teacher. I want to be a good teacher that knows how to explain things so that each new chapter doesn't cause students to be afraid, but to help them be excited about the new things they're going to master.

Andrew was generally successful as a mathematics student, and he realized the importance of mathematics to students' future educational opportunities.

> When looking back upon my school years, most of what I recall is math and what happened there. Thus, the subject of math can make or break a student in grades, which in turn affects the attitude toward it. So much emphasis is placed on math that it often becomes the deciding factor on educational advancement, as is witnessed in standardized tests for entrance into college. Hence, it is essential that the teacher has a love for math and can instill that mathematical affection in the student body, which unfortunately seems not to have been the case in the past. Therefore, it is my goal to help end this trend.

The passion with which these preservice teachers viewed their experiences is evident in their writing. It is worth remembering that these autobiographies were written during the first week of a mathematics methods class; thus, the preservice teachers were writing to a total stranger. Yet they did not hesitate to share their long-held and deeply rooted experiences with mathematics learning. In the next section of the paper we suggest ways that Lortie's work might be productively used in a more modern context.

Lortie Today

The data presented above suggest that we have been too quick to dismiss preservice teachers' prior experiences as simply leading them toward more traditional views and practices of teaching. We have blithely cited Lortie over the years and used his apprenticeship of observation as a convenient buzz phrase to prop up our arguments about the ineffectiveness of teacher education and professional development. Invoking Lortie's apprenticeship of observation as an explanation for the failure of teacher education programs and practices leads to a downward spiral in which teacher educators are either absolved of all responsibility for making change or are rendered powerless by the trump card of prior experience. We do a disservice to preservice teachers when we so readily take a shallow view of their prior experiences.

We have had over a decade of significant reform in education in the form of content standards, new pedagogies, and new curricula. These reforms, while not uniformly implemented, surely have had some impact on the students who now come to us as future teachers. Teacher education programs and those who enter them have changed as well in the nearly 30 years since Lortie's initial study. While we still have much to do to optimize teacher education experiences, we cannot claim that they are the same as they were in Lortie's day.

The challenge before the teacher education research community is to revisit Lortie's work with a critical eye. If we accept Lortie's exhortation that "it would take complex research to confirm" (p. 63) that apprenticeship of observation exerts an immutable influence on the development of future teachers, then we need to design studies to unpack the notion of apprenticeship of observation and its influences on future teachers. Alternatively, we might decide that this is not fertile ground for future research because it does not move us forward in our thinking about teacher education. In that case, we need to collectively raise questions about teacher education that warrant further investigation, and we need to cease citing Lortie as an "excuse" for some of our research findings. Whichever direction we decide to move, we owe it to the teacher education community to read the original words of Lortie and to more carefully portray preservice teachers in all of their richness and complexity.

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HIDDEN MATHEMATICS CURRICULUM AND TECHNOLOGY IN K-12 TEACHER EDUCATION

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An emerging issue in mathematics education reflected in the PME-NA goals is that of improving mathematics teaching and learning by developing more complex and deep teacher understanding of mathematical concepts that have traditionally only been addressed superficially. In an attempt to address this issue, the concept of hidden mathematics curriculum is introduced. Theoretically, it resides at the confluence of Freudenthal's theory of learning mathematics as advancement of the culture of mankind and Vygotskian theory of learning in a social context. This paper shows how computing technologies can mediate pedagogy based on the hidden mathematics curriculum approach in the context of the preparation of K-12 teachers.

Introduction

The intent of this paper is to introduce to the PME-NA community the notion of hidden curriculum in mathematics teacher education proposed recently by the authors (Abramovich & Brouwer, 2003a, 2003b, 2004). The general notion of hidden curriculum can be traced back more than three decades (Jackson, 1968) and has received much attention in foundational educational research (Ginsburg & Clift, 1990). While a hidden curriculum framework in a traditional sense explores tacit features that structure life in schools, hidden mathematics curriculum includes tacit concepts and structures that underlie a variety of school mathematical activities. The notion of hidden mathematics curriculum attempts to bridge practice and research as it is based on the observation that many mathematical activities across the K-12 curriculum, seemingly disconnected from a naïve perspective, are, in fact, permeated by a common mathematical concept or structure, traditionally hidden from learners because of its complexity. Such complexity may be either procedural or conceptual in nature. The authors' approach to investigating the idea of hidden curriculum in mathematics teacher education is to find and work with a series of topics found across the curriculum that from a deeper perspective may be described by a common mathematical concept. Thus a hidden curriculum approach connects research and practice within mathematics teacher education. Technology has great potential to enhance this approach through appropriate pedagogical mediation.

It is often observed that teachers of mathematics do not have sufficient mathematical background to see the general concepts behind particular phenomena. This lack of understanding contributes to the communicating of mathematics to learners in a disconnected fashion, so that elementary students believe that the problem solving work that they are doing is limited to their grade level. By the same token, secondary students cannot see the connection between problem solving they are currently engaged in and their earlier mathematical experience. Thus, teachers can use the knowledge of hidden concepts and structures in the mathematics curriculum to extend the curriculum in both directions. This suggests the importance of exploring a hidden curriculum framework across all levels of mathematics teacher education.

Theoretical framework

The notion of hidden curriculum may have a profound impact on mathematics teacher education provided that prospective K-12 teachers (pre-teachers) are given the opportunity to learn advanced ideas in a social context of competent guidance enhanced by appropriate technology tools. A pedagogic mediation of technology-integrated hidden mathematics curriculum framework supports the advancement of Freudenthal's (1983) theoretical construct of the didactical phenomenology of mathematics as "a way to show the teacher the places where the learner might step into the learning process of mankind" (p. ix). In other words, technology-enabled learning in a social milieu of expert-novice relationships opens windows into the hidden meanings of, otherwise perceived as elementary, mathematical concepts.

Such a focus on expertly assisted learning of mathematics by pre-teachers brings to mind one of the basic tenets of Vygotskian pedagogy which considers social interaction as the primary educative mechanism and conceptualizes learning as a transactional process of developing informed entries into a culture with the support of more capable members, or agents of this culture (Bruner, 1985). As far as a mathematical culture is concerned, the above notion of hidden curriculum may serve as a powerful intellectual link between the two concepts of Freudenthal's didactical phenomenology of mathematics and Vygotsky's zone of proximal development (ZPD) that learning by transaction creates. The latter component was based on the assumption that human learning is essentially a social process in the sense that what one can do with the assistance of a more knowledgeable other fully characterizes one's cognitive development. Vygotsky (1978) argued that learning by transaction creates the ZPD and proposed "a new formula, namely that the only 'good learning' is that which is in advance of development" (p. 89).

At this confluence of pedagogical and psychological theories, the combination of Freudenthal's pedagogy of learning mathematics as advancement of the culture of mankind and Vygotskian theory of learning in a social context provides theoretical underpinning for the didactical framework of hidden mathematics curriculum. More specifically, the pedagogy of revealing hidden curriculum messages to pre-teachers in a social milieu of computer-enabled learning creates the ZPD that, in turn, provides the basis for one's profound understanding of mathematics. It is through educated assistance at the points of the zone where such assistance is needed (Tharp & Gallimore, 1988) that one develops the skills and confidence needed for its competent teaching. Therefore, it appears that a hidden curriculum framework enhanced by technology has potential to become a vehicle for what Vygotsky has called "good learning." In what follows, through a series of illustrations, the authors will show how this potential can be realized.

Background

Motivated by work done with elementary and secondary preservice and in-service teachers in various mathematics education courses as well as by field observation of school mathematical practice, this paper shows how technology tools, such as spreadsheets, computer algebra systems, and dynamic geometry software, enable informal journeys into hidden aspects of the formal content of the school mathematics curriculum. It provides evidence how a hidden curriculum framework supports pedagogical mediation of formal mathematical reasoning by elementary pre-teachers, including the development of proof schemata. Also, it demonstrates how the framework can be put to work in the context of preparation of secondary mathematics teachers that includes the development of research-like experiences in mathematics through technology-enhanced problem posing. The paper shares specific examples of student work that illustrate the above points. The conclusions of the paper are based on the analysis of classroom observations and portfolio assessments from a variety of mathematics education courses incorporating technology-enhanced hidden mathematics curriculum framework.

Illustration 1: Partition of integers as hidden curriculum

One profound concept that unites many of the topics found across the school mathematics curriculum, including arithmetic, algebra and geometry, is the partition of integers. As mentioned earlier, it may be due to the complexity of mathematics behind this concept that it has not been explicitly highlighted in the curriculum as such. Yet, partitioning problems often emerge from simple situations like the one that pre-teachers face on the first day of a graduate education class when they are asked to arrange themselves into working groups. An obvious constraint is that the groups should be as close to the same size as possible. Interestingly enough, whereas some may view this task as a division-with-adjustment problem (divide 6 into 20 to see which interval between two consecutive integers the quotient belongs – these integers determine the sizes of the groups), it can be solved using mathematical reasoning on a kindergarten level by using manipulatives (say, square tiles) as tools to think with. For example, in the case of 20 students and six groups a diagram pictured in Figure 1 is a result of such a hands-on solution.

Figure 1. Solution to the student group problem using square tiles.

While kindergarten mathematics was apparently hidden in this upper elementary level problem situation, advanced mathematics can be developed from this situation as well. First, this includes its algebraic representation in the form of the system of two simultaneous equations 3x+4y=20, x+y=6 (coefficients in the first equation result from the inequality 3<20/6<4). Second, a graphic representation may be developed so that the point of intersection of two straight lines on the Cartesian plane can be interpreted as students being split into four groups of three and two groups of four. On even more advanced level, this situation can be understood in terms of partitions of integers; that is, as partitioning of 20 into the summands three and four. In general, the partition of a positive integer is its representation as the sum of counting numbers without regard to order. For this, a spreadsheet representation can be developed which enables one to accommodate up to three summands (Abramovich & Brouwer, 2003a).

Note that many problem situations include counting partitions of a positive integer *n* whose summands are taken from the sequence of counting numbers $a_1, a_2, \ldots, a_k$. A formal approach to counting such partitions deals with the method of generating functions that differentiates whether these numbers can enter a partition more than once or at most once. In the first case, this method enables one to find the number of partitions as the coefficient of r^2 in the expansion of the following product of geometric series

$$\left(1+r^{a_1}+r^{2a_1}+r^{3a_1}+\cdots\right)\left(1+r^{a_2}+r^{2a_2}+r^{3a_2}+\cdots\right)\cdots\left(1+r^{a_k}+r^{2a_k}+r^{3a_k}+\cdots\right).$$

Indeed, by presenting the product as a polynomial in powers of *r*, one can see that the coefficient of r^n coincides with the number of partitions of *n* into the summands taken for the *k*-element set $\{a_i\}$.

In particular, for the student group problem, such a product has the form $(1+r^3+r^6+\ldots)(1+r^4+r^8+\ldots)$. By expanding the product (in which the first and the second series include the terms up to r^{18} and r^{20} respectively) one can find that two terms, namely,

r^{20} and $r^{12}r^{8}$ have an exponent of 20. While the former term represents the case of five groups of four, the latter term points to four groups of three and two groups of four (six groups total), thus furnishing the solution sought.

Consider the case when each of the numbers a_i enters a partition of n at most once. In that case the method of generating functions consists of the construction of the product $(1+r^{a_1})(1+r^{a_2})\ldots(1+r^{a_k})$ and finding the coefficient of r in its expansion. Such an expansion can be carried out using a computer algebra system; for example, the Graphing Calculator 3.2 (GC) produced by Pacific Tech (Avitzur et al., 2002) is an appropriate tool. Not only it can perform graphing of functions and, most notably, relations but also it can do various symbolic manipulations, including expansion of the product of binomials. For example, consider the case where the students are to be seated at six tables designed for two, four, six, eight, ten, and sixteen people. Exploring in how many ways this task can be carried out through the method of generating functions (assuming that the order does not matter and once a table is taken, it should be used to full capacity) leads to the product $(1+r^2)$ $(1+r^4)$ $(1+r^6)$ $(1+r^8)$ $(1+r^{10})$ $(1+r^{16})$, the expansion of which with the GC is shown in Figure 2. One can see that the term r^{20} in this expansion has coefficient four, thus indicating that four possibilities for the students to be seated exist.

$$(1+x^2)\ (1+x^4)\ (1+x^6)\ (1+x^8)\ (1+x^{10})\ (1+x^{16})$$

$$x^{46}+x^{42}+x^{2}+x^{44}+3x^{36}+2x^{38}+4x^{28}+3x^{12}+\boxed{4x^{20}}+x^{4}+2x^{40}+4x^{30}+3x^{14}+3x^{32}+4x^{22}+2x^{6}+3x^{34}+4x^{18}+4x^{24}+2x^{8}+4x^{26}+3x^{10}+4x^{16}+1$$

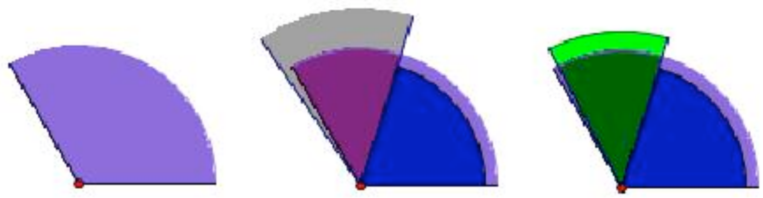
Figure 2. The Graphing Calculator as symbolic manipulator

In such a way, starting with a seemingly simple division-with-adjustment problem, the hidden curriculum approach enables one to see this problem from broader perspectives, both elementary and advanced. Indeed, the use of both manipulative and computing technology makes it possible to elevate pre-teachers learning of mathematics to "higher ground" (Bruner, 1985, p. 23) so that not only "the new higher concepts in turn transform the meaning of the lower" (Vygotsky, 1986, p. 202), but, in addition, the lower concepts can enhance understanding of the higher.

Illustration 2: Partition of fractions as hidden curriculum

In addition to square tiles, another form of concrete materials commonly used in contemporary elementary classroom is the fraction circle - a manipulative representing a unit fraction and shaped as a sector of a whole circle. Ironically, Van der Waerden's (1961) citation of Plutarch is worth mentioning in connection with these modern tools: "*The Pythagoreans also have a horror for the number 17. For 17 lies halfway between 16 … and 18 … these two being the only two numbers representing areas for which the perimeter (of the rectangle) equals the area*" (p. 96). A significance of this classic remark is in a hidden connection of its ancient geometric proposition and a hands-on activity recently observed in a elementary classroom in rural upstate New York – covering one-half of a circle with two fraction circles. Indeed, if x and y are integers which measure the sides of such a rectangle, then xy=2(x+y). Although Babylonians knew an algebraic solution to the last equation (Van der Waerden, 1961) it still may be difficult for an average elementary pre-teacher to comprehend. An alternative is to rewrite it as 1/2=1/x+1/y, to interpret this new equation as partition of the unit fraction one-half into the sum of two unit fractions, and carry out this partitioning as the above-mentioned hands-on activity.

In turn, this activity can be enhanced by the use of custom tools created within *The Geometer's Sketchpad*. By using these tools under competent tutelage one can not only solve the above problem but its natural extensions also. For example, one can be asked to find *all* ways of representing the fraction circle one-third as the sum of two like fractions as a strategy for finding all rectangles (with integral sides) for which the area is three times as much as its semi-perimeter. The above use of word "all" implies the need for students to provide formal justification of their results – otherwise known as mathematical proof. In this regard, note that Bell's (1979) claim that the essence of mathematical proof deals with the "public acceptability of the knowledge being discovered" (p. 368) is consistent with the Vygotskian notion of learning as a social activity. Asking pre-teachers to communicate their proof schemata through written speech using computer-generated images as a support system



1/3 does not equal 1/7 + 1/5 (too large) and does not equal 1/8 + 1/5(too small)--every other fraction smaller than 1/8 will also not help

Figure 3. Geometric representation of proof schemata

In fact, as mentioned elsewhere (Abramovich and Brouwer, 2004), the need for proof may be motivated by genuine student curiosity as in the case of finding all ways of covering the fraction circle representing one-third of a circle using two other fraction circles. As an elementary pre-teacher put it in a written form: "*Giving fraction circle 1/x a smaller denominator (making the fraction circle larger), will require that the fraction circle 1/y increase its denominator (making the fraction circle smaller) to maintain the equation 1/3=1/x+1/y. The combinations of fractions that are possible then are always finite given the nature of fraction circles. Once the initial fraction circle (1/3) is divided in half, the remaining possibilities for fraction circle 1/y are discovered by solving for 1/y given that 1/x can only be 1/6, 1/5, and 1/4. By making a bigger 1/x is 1/3 and … this will give no value to 1/y. It only remains to be seen whether all combinations [for 1/x] are possible*." It is remarkable to read in the teacher's proof schemata "It only remains to be seen …" when what remains of the proof is arithmetic of rational numbers, typically challenging for elementary pre-teachers. Indeed, the above cited proof was followed by a combination of geometric representation (Figure 3) and a numeric argument not included here for the sake of brevity. In such a way, experiencing success at high conceptual level – that is, by doing proof – expanded the pre-teacher's ZPD thus empowering him to tackle procedural details with confidence – a skill that should be expected from an elementary teacher, given current recommendations for elementary teacher preparation (Conference Board of the Mathematical Sciences, 2001).

Illustration 3: Hidden mathematics curriculum as a vehicle for developing research-like experience among secondary pre-teachers

The notion of hidden curriculum can be put to work in the context of technology-enhanced preparation of secondary mathematics teachers that includes the development of research-like experience in mathematics through problem posing (Abramovich & Brouwer, 2003b). This section illustrates how the GC and a spreadsheet can be used jointly to develop problem formulation skills within the framework of the secondary mathematics curriculum. It reflects on the authors' experience teaching an advanced secondary mathematics course that, in part, introduced elementary material from a deeper perspective. Consider the following variation of the student group problem: *In designing a new classroom with a capacity of 25 students, the dean found out that she could only order tables that seat either three, four, or five students. What combination of tables can be used to design this classroom?* While the indeterminate equation $3x+4y+5z=25$ can be solved through the method of generating functions, at an elementary level a spreadsheet can be introduced as a three-dimensional partitioning tool. In this regard, secondary pre-teachers were given the following problem posing assignment in the context of systems of three simultaneous equations: *Use the spreadsheet to modify the classroom design problem to fit the high school curriculum*. One such modification was suggested by a pre-teacher.

There are three ways to score points in football: a safety, a field goal, and a touchdown. Safeties are worth 2 points, field goals are 3, and a touchdown is 7 points if the extra point is made. The NY Giants scored 35 points on Sunday and made all extra points. The sum of the number of field goals and touchdowns is equal to seven times the number of safeties. Also, the number of field goals is equal to twice the difference between touchdowns and safeties. How many safeties, field goals, and touchdowns did the Giants score?

Figure 4. Spreadsheet solution to an indeterminate equation $2x+3y+7z=35$.

In developing this problem, the pre-teacher used the spreadsheet pictured in Figure 4 to consider possible valid relations between the variables. In particular, the contents of cells G8, G3, and C8 prompted the specific numerical structure of the problem for which an algebraic model is the following system of three simultaneous equations: $2x+3y+7z=35$, $y+z=7x$, $y=2(z-x)$. Note that consistent with current trends in using computer graphics for enhancing algebraic problem-solving skills in the classroom that make traditional skills "instrumentally rather than intrinsically valuable" (Confrey, 1998, p. 38), this system, using GC, can be solved graphically on the plane if one of the variables is taken as a slider-controlled parameter (Figure 5). Indeed, by gradually changing the slider variable one can arrive at the solution $(x, y, z)=(1, 4, 3)$, meaning that the Giants scored one safety, four field goals, and three touchdowns. This illustration demonstrates how combination of software tools used in conjunction with the hidden mathematics curriculum approach enables secondary pre-teachers' learning to pose and, using non-traditional skills, solve age appropriate problems. In this way, the approach is conducive to the teachers acting as a kind of curriculum developers in a technological paradigm.

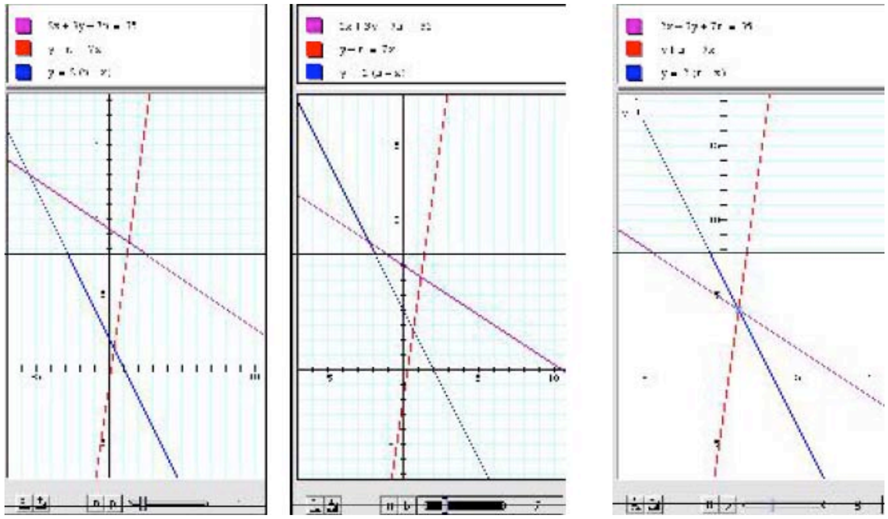


Figure 5. Graphical step-by-step convergence on a solution

Discussion

The notion of hidden mathematics curriculum advances several agendas within the realm of technology-enhanced mathematics teacher education. First, it promotes the interplay between theory and practice within the field by highlighting the deeper meaning in what is commonly perceived as routine school mathematical activity. As the student group problem shows, this deeper meaning may be extended in both directions: towards more and towards less challenging mathematics. The problem with fraction circles has a hidden connection to mathematics of the Pythagoreans and other appropriate examples from the history of mathematics can likely be found in this regard. Through technology and well-chosen examples pre-teachers develop an understanding of how these deeper meanings arise from common structures of the explicit curriculum and connect its different ideas and representations. In addition, in a technology-mediated intellectual milieu, achieving control over a concept occurs in a socially created ZPD where intuitive understanding of the concept meets the logic and formalism needed for its representation through a computational medium. The product of this human-computer interaction aided by competent tutelage is a solution, which, once internalized, becomes a part of one's consciousness.

Second, the notion of hidden mathematics curriculum enables one to introduce technology into mathematics teacher education programs in multiple ways. One option is to introduce various tools of technology into a program to support mathematical investigation and connection building without teaching special courses on technology. Another option is to develop a follow-up course on technology grounded in its educational application in which the pre-teachers concurrently learn technology and mathematics in the context of creating computational environments already familiar to them.

Third, this notion provides a structure through which one can systematically connect different mathematical ideas and representations and thus address and meaningfully incorporate the NCTM process standard "Connections" (National Council of Teachers of Mathematics, 2000). As one elementary pre-teacher mentioned in a course assessment: *"Certainly the math taught to me in the past helped in my understanding of concepts presented; however, the interconnection of mathematical concepts was something I do not recall being aware in the past. This may have been a fault … in the nature of the education system I grew up with but … it is hoped that I can implement skills learned in this course to … challenge students to see the relationships."*

The pre-teacher goes on to indirectly argue for deep mathematical knowledge by teachers in pointing out that *" ... as students learn [to ask questions] they will begin to challenge the teacher's understanding of the topic."*

In this respect, there are two specific aspects of the notion of hidden curriculum to consider. One is the author's belief that it enables pre-teachers' learning of mathematical concepts traditionally considered advanced which, in turn, allows them to more fully communicate mathematics to their students. The second reflects the above observation that the more teachers know, the more they need to know as their students ask more and better questions. In a similar context, this phenomenon was reported by Bruner (1985) who, referring to work done by Tizard and her colleagues, stated that "children … who ask the most searching questions are the ones whose parents are most likely to answer them fully and … the parents who are most likely to answer are the ones with children most likely to ask!"

(p. 31). Ironically, the hidden mathematics curriculum approach may challenge a feature of the traditional hidden curriculum; namely, "the underlying power relationship between teacher and child: the children seem to learn very quickly that their role at school is to answer, not to ask questions" (Tizard, Hughes, Carmichael, & Pinkerton, 1983).

Finally, a hidden curriculum approach that identifies deep concepts and structures of mathematics makes it possible to elevate pre-teachers' learning of mathematics to higher ground, both from elementary and advanced perspectives. Climbing to this new height creates in pre-teachers greater self-confidence in their abilities to teach mathematics. At the elementary level, traditionally poorly understood topics, like formal arithmetical operations with fractions, when highlighted from a different, sometimes advanced, perspective in which the pre-teachers experience success, leads to a greater understanding of and confidence in those topics. Using technology, pre-teachers can make significant progress in connecting their informal explorations with formal symbolic mathematics. As was mentioned above, experiencing success at a high conceptual level expands their ZPDs which builds confidence in their ability to handle procedural details. It is through the expansion of this zone that the notion of a hidden mathematics curriculum has the potential to significantly broaden pre-teachers' content knowledge at all levels, bring positive change in various teaching-related psychological phenomena, and eventually affect the way that mathematics is taught in the schools.

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EVIDENCE AND JUSTIFICATION: PROSPECTIVE PREK-8 TEACHERS' PROOF-MAKING AND PROOF-EVALUATING

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Approximately 280 prospective PreK-8 teachers responded to assessment items that were designed to investigate how students evaluate and create mathematical justifications. Analyses of responses for two items are discussed. Findings from the study suggest that students recognized the weakness in using examples to justify claims; however when asked to generate an argument for a claim, many used examples to make an argument or to further verify a general argument. Furthermore, it appeared that approaches valued by the students were different from what they perceived their instructors valued. The findings raise questions about students' understanding for the need to move beyond using examples to utilize the power of deductive reasoning.

"Does this always work? Sometimes? Never? Why?" (NCTM, p. 58, 2000). These are questions that teachers and students might ask in an effort to develop and evaluate mathematical arguments. Because extending to general cases requires more sophisticated reasoning, NCTM maintains that students "need to encounter and build proficiency" in creating and evaluating mathematical arguments across all grade levels, including an increased emphasis on making "deductive arguments based on the mathematical truths they are establishing in class" (NCTM, p. 59, 2000).

Although the teaching of formal deductive proofs is not recommended during the elementary and middle school years, teachers' views of what constitutes proof are vital to the process of developing communities of mathematical problem solvers. Using the definition of proving offered by Harel and Sowder (1998), "The process employed by an individual to remove or create doubts about the truth of an observation" (p. 241), we investigated how PreK-8 prospective teachers evaluated and created mathematical justifications. Ultimately, we explored,

What constitutes evidence and justification in mathematics from the PreK-8 prospective teacher's perspective?

Theoretical Perspective

Researchers hypothesize that students' abilities to justify and prove follow a developmental path that progresses from inductive to deductive approaches and greater generality (e.g., Simon & Blume, 1996). Waring (2000), building on the work of others (e.g., Balacheff, 1991; Fishbein, 1982), proposed six levels of understanding the concept of proof. Knuth, Chopin, Slaughter, and Sutherland (2002) extended these levels, levels that progress from a lack of awareness to an awareness of the need for a general argument. At lower levels, students seem satisfied that examples are sufficient to demonstrate the truth of a mathematical observation. At higher levels, students become aware of the need for but are not always able to produce a general argument. This overall progression is consistent with the notion that students progress from inductive to deductive approaches.

On the other hand, Martin and Harel (1989) found that when prospective elementary teachers evaluated mathematical arguments many of them simultaneously rated inductive and deductive arguments high. They concluded that the prospective teachers saw validity in using both types of reasoning to verify mathematical statements. They maintained "the inductive frame, which is constructed at an earlier stage than the deductive frame, is not deleted from memory when students acquire the deductive frame" (p. 49). Consequently, Martin and Harel suggested that students must activate both inductive and deductive frames to be convinced of a particular observation. However, because students were only asked to evaluate given arguments, the researchers questioned whether students would act differently when producing mathematical arguments. In the discussion that follows we explore the different types responses generated by students when they were asked to evaluate *and* produce mathematical arguments.

Methods and Data Sources

Approximately 280 students enrolled in four different mathematics courses for prospective PreK-8 teachers (Math 107 (n=126), Math 108 (n=38), Math 207 (n=86), and Math 305 (n=26)) during Fall 2003 semester provided the data for this paper. At the time of the study, the Math 107 curriculum included number and number operations, the MATH 108 curriculum included number theory and geometry, and the curriculum for Math 207 focused on algebra, functions, probability and statistics from a problem-solving approach. In addition to these three introductory math courses for prospective PreK-8 teachers, students concentrating in mathematics and science take 3 to 4 more 300-level courses. Some of the students in this study were enrolled in one such course, Math 305, for which, the curriculum included geometry from Euclidean and non-Euclidean perspectives and incorporated Geometer's SketchPad. These courses were only offered to students preparing to teach grades PreK-8 and thus, generally focused on content that was relevant to PreK-8 mathematics.

Data were collected through assessment items administered during one class period early in the semester. These assessment items, adapted from Knuth et al. (2002), were designed to identify students' competencies in justifying and proving. Here, we focus on two assessment items: one in which students were asked to evaluate given arguments (Item 1) and one in which they were asked to produce a convincing argument (Item 2). See Table 1 for abbreviated forms of these items.

In a broad sense, Waring's (2000) framework formed our lens for interpreting the data. Because the data were solely students' written responses, we could not identify confidently at which level they appeared to be functioning. However, we could observe if they attempted to justify an observation exclusively through the use of examples or if they highly ranked an argument that solely consisted of examples. This classification scheme provided some insight into how they convinced themselves and others of the truth of a mathematical observation. If students used or valued examples for justification, we identified them as functioning at level 2 or below and categorized them as inductive reasoners. If students at least attempted a general argument or highly ranked a general argument, we identified them as functioning at level 3 or above and categorized them as deductive reasoners (at least in regard to these items).

Results

Item 1. The first part of Item 1 asked students to indicate which of three arguments they liked best, understood best, would be most convincing to classmates, and which they thought would get the highest grade from the teacher. Our purpose in asking this question was to determine if students would identify a deductive argument as the best argument (i.e., Marian's argument). The data for this part of Item 1 (see Table 2) indicate an apparent disconnect between what the students seem to value and what they assume their instructors value. Most students indicated that not only did they understand Jamie's argument best (47%), but also that Jamie's argument, alone, would be most convincing to classmates (52%). Such students cited the visual representation as the primary reason for favoring Jamie's argument. However, when

Table 1. Abbreviated assessment items.

Item 1:

Three students are discussing whether the following statement is <u>*always*</u> *true.*

> The sum of any three consecutive whole numbers is equal to three times the middle number. For example, 4, 5 and 6 are consecutive numbers and 4 + 5 + 6 equals 15, which equals three time the middle number, 5.

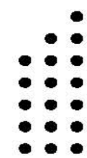
Their explanations are shown below.

Marian: I can take any 3 consecutive numbers. If n is the middle number, then n-1 is the number that comes before n, and n+1 is the number that comes after n. I write the following equation to show that when you add 3 consecutive numbers, you *always* get 3 times the middle number:

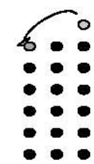
$$(n-1) + n + (n+1) = 3n$$

Since the 3 consecutive numbers add up to 3 times the middle number, n, I know Malcolm's rule is always true.

Jamie: I found a way using marbles. Imagine making three columns of marbles representing any three consecutive numbers. The first column represents the first number, the second column represent the middle number, and the third column represent the last number.



I can take the top marble from the last column and move it to the first column. This makes the number of marbles in each column the same as the number of marbles in the middle column.



Since the total number of marbles is always three times the number in the middle column, I know Malcolm's rule is always true.

Jeff: 5, 6, and 7 are three consecutive numbers and 5 + 6 + 7 = 18, and 3 x 6 = 18. 7, 8, and 9 are three consecutive numbers and 7 + 8 + 9 = 24, and 3 x 8 = 24. 569, 570, and 571 are three consecutive numbers and 569 + 570 + 571 = 1710, and 3 x 570 = 1710. Since it works in these three examples, I know Malcolm's rule is always true.

Item 2:

Mei discovers a number trick. She takes a number and multiplies it by 5 and then adds 12. She then subtracts the starting number and divides the result by 4. She notices the answer she gets is 3 more than the number she started with.

For example, suppose Mei starts with 7.	7
She would multiply by 5 to get 35.	7 x 5 = 35
Then she adds 12 to get 47.	35 + 12 = 47
She subtracts the original number, 7, to get 40.	47 – 7 = 40
Then she divides 40 by 4 to get 10.	40 ÷ 4 = 10
Ten is three more than 7, her starting number.	

(Shows another example generated by Malaika.)

Do you think they are right? How would you convince a classmate that you would always get a result that is three more than the starting number?

choosing the argument that would 'get the highest grade', 67% chose Marian's argument. Many of these students explained that Marian would get the highest grade "*because she used a mathematical equation*." A few cited the mathematical power of Marian's argument as indicated by one student's response, "*I think that Marian's answer would get the highest grade from the teacher because she does not base her answer on just one or several number sequences, instead she uses n to represent any number, thereby giving more credibility to her work.*"

Still other students believed more than one argument should receive the highest grade (16.8%). Students' reasons for multiple choices generally fell in one of two categories. Some students stated that the selected arguments are all equivalent/right, as one student commented, "They should all get the same grade because they explain it correctly." Other students stated that the selected arguments prove/justify the claim for all cases, as indicated by the following student's response: "*Either Jamie's or Marian's because they showed how this rule applies to all numbers not just by using examples such as Jeff did.*" This type of explanation provided evidence that some of the students interpreted Jamie's visual representation as a general argument and not as an illustration of one particular case.

Revisiting the results for the arguments students chose to convince a classmate, we found that about the same number of students favored Jeff's argument (19.2%) that favored Marian's argument (19.7%). Indeed, many students commented that they thought examples would be enough to convince their peers.

Table 2. Item 1 results (part one)

	Liked Best	**Understand Best**	**Convince Classmate**	**Highest Grade**
Marian (only)	128	76	54	185
Jamie (only)	111	129	144	38
Jeff (only)	30	63	53	5
Marian & Jamie	4	6	14	26
Jeff & Jamie OR Jeff & Marion	3	2	4	1
ALL 3	0	0	5	19
Total Responses	276	276	274*	274*

* Two students left these items blank.

The second part of Item 1 asked students to evaluate the convincing power of each of the three arguments from multiple perspectives. Due to limited space, we only report their responses from one point of view, that is, whether each of the three arguments showed that the mathematical statement was always true (see Table 3).

Table 3. Item 1 results (part two)

Student's argument shows Malcom's rule is 'always true'?		
Student	**Agree***	**Disagree***
Marian (only)	71	5
Jamie (only)	24	11
Jeff (only)	6	151
Marian & Jamie	122	0

Marian & Jeff	13	8
Jamie & Jeff	6	38
All 3	23	7

* The remaining students either responded "Don't Know" or left the item blank.

We found that the students could evaluate the mathematical power of arguments. About 83% ((71+122+13+23)/276) agreed with the statement that Marian's argument showed the mathematical statement was always true and 80% indicated they disagreed with the statement that Jeff's argument showed the mathematical statement was always true. Therefore, at least on the surface, these students understood the value in using a general argument to justify a mathematical claim. Likewise, most students understood that it was not valid to use specific cases to justify a claim. What is troubling is that 17% of the students believe that Jeff's argument somehow shows Malcom's claim to always be true.

Many students also valued the convincing power of Jamie's argument. In fact, 175 (63%) agreed that Jamie's argument showed the mathematical statement was always true. Based on the responses to the first part of Item 1, many of these students indicated that the visualization Jamie used would work for any three consecutive numbers and thus proves the statement. For example, in answering the question about whose argument would get the highest grade, one student commented, "*If I was the teacher, I'd give both Jamie and Marian the same grade. Both of them are well thought and correct. They both prove the statement. Jeff on the other hand didn't prove it, but simply gave examples.*"

Based on students' responses to both parts of Item I, their preference for Marian's but not Jeff's arguments, we concluded that 185 (67%), indicated a level of thinking about proof that was at least a level 3 on the Waring scale (i.e., were categorized as *deductive thinkers*).

As we categorized students as inductive versus deductive reasoners for Item 1, we began to notice apparent differences between the number of PreK-3 and PreK-6/6-8 students in each category. Consequently, we conducted chi-square tests to make comparisons between the number of early childhood students (PreK-3) categorized as deductive thinkers and the number of elementary (PreK-6)/middle school (6-8) students categorized as deductive thinkers within each course. We found a statistically significant difference ($p < 0.05$) between the numbers of students categorized as deductive thinkers in Math 107 ($p = 0.0001$) and Math 207 ($p = 0.0377$). Since Math 107 is the introductory mathematics course for the prospective early childhood and elementary/middle teachers, we might infer that these two groups begin their college experience at different entry points when evaluating mathematical arguments. Since the p value for Math 207 is much larger, we might be inclined to think that the PreK-3 students are 'catching up' to some extent by the time they reach the third course. However, since there was no significant difference found in the types of arguments that Math 108 and Math 305 students gave, and our findings are preliminary, such inferences are premature.

Item 2. This item asked students to decide whether or not a certain 'number trick' is always true and to explain how they might convince a classmate that the 'trick' always works. Overall, students' responses fell into two broad categories. (See Table 5.) Many students (40%) either tested a few cases and or, they offered extreme or random cases to show that the claim was true. Responses ranged from "*Do it with two different numbers…twice seems to be the number of times something works to prove someone right or wrong*" to "*showing multiple examples*." One student claimed, "*I don't really think there is a way to convince someone that it would work every single time. I would just try a bunch of numbers – even and odd, small and large – a wide variety of numbers to see if I kept getting the same results.*"

Table 5. Item 2: Approaches to convince a classmate.

Visual (picture, graph, blocks)[1]	"Don't know," "Unsure," "Opinion"	Examples only	Incorrect Counter-example	Examples & general argument	General argument only	Incorrect general argument	Correct general argument
4	18	111	12	56	33	19	23
	Inductive Reasoning			Deductive Reasoning			

By way of contrast, 48% moved beyond just giving specific examples to support the claim by stating that they needed to make a general argument. However, almost half of these students (56 out of 131) also indicated the need for further verification by providing specific examples to "plug into" a given algebraic statement or saying that the classmate could "plug in" numbers of their choice. Only 8% of the 276 students created a mathematically sound argument and did not refer to the need for further verification. Another 7% attempted an algebraic argument, but were unsuccessful. In attempts to create equations, their lack of algebraic skills seemed to get in the way for these students as seen in the following excerpts:

$$n(5) = (x+12) = (y-n) = (z \div 4) = (n+3)$$

Not sure. I would figure out exactly how this formula works and then give examples as more proof.

$$\cancel{n} + (n \cdot 5) \cancel{+n} + 12 + (n \div 4) = n + 3$$

$$2n + 8 = -n \qquad 3n + 5 + n \cdot 5 + n \div 4 = n + 3$$

$$\frac{7n \div 2}{1} = -\frac{n}{4} \qquad 2n + 2 + n \cdot 5 + \frac{n}{4} = 0$$

Based on students' responses to Item 2, we categorized 131 (48%) of the students as *deductive thinkers*. When considering the data according to the course in which students were enrolled, there are significant differences between the percentages of *deductive thinkers* versus *inductive thinkers*. For example, when comparing students in the introductory course with those in the fourth course, there was more than a complete reversal of the distributions. Specifically, we categorized 33% of the students in the first course as deductive thinkers, whereas, we categorized 81% of the students in the fourth course as deductive thinkers. Consequently, we conducted chi-Square tests to make comparisons between the numbers of students categorized as deductive thinkers in these courses. We found a statistically significant difference between the students in each course-by-course comparison, but only report one example due to space limitations. (See Table 6.)

Table 6. Chi-square test between courses.

Item 2	Inductive	Deductive	p-value
Math 107	71	45	
Math 305	5	21	0.0011

We could interpret the significant difference as evidence that the students get better at evaluating and generating mathematical arguments as they move through the course sequence. However, we hesitate to do so without further investigation.

Implications & Conclusions

Analysis of these data leads us to question how much prospective teachers rely on examples to justify mathematical claims. Findings from Item 1 suggest that students recognize the weakness in using examples to justify claims, whereas findings from Item 2 suggest that students rely to some extent on examples to verify a mathematical argument. The fact that 20% used both examples and a general argument raises questions about students' understanding for the need to move beyond using examples to utilize the power of deductive reasoning. On the surface, this finding seems to be consistent with the findings of Martin and Harel (1989). Like their students, these prospective teachers felt the need for further empirical verification, for either themselves or others. However, students in the Martin and Harel study tended to employ both inductive and deductive frames to evaluate arguments, whereas students in this study tended to do so when generating arguments.

In short, the students seemed to be relatively adept at evaluating arguments; however, many had difficulty with generating an argument. The following student sums it up well: "*I'm not sure. I was never good at making my own equations. Usually I can recognize when other people make them and then it makes sense, but it's hard to come up with them on my own.*"

These findings have caused us to wonder if there is some kind of predictive relationship between evaluation and generation of an argument. Must someone be skillful in evaluating prior to becoming skillful in generating arguments? Perhaps these skills develop in a reflexive manner, where growth in one precipitates growth in another.

To better understand students' perceptions about making mathematical arguments, one-on-one interviews with students must be conducted. In doing so, we can hope to determine the extent to which they understand the utility of using both examples and making general arguments. More generally we hope to better understand what, for the students, constitutes evidence and justification in mathematics.

Endnotes

1. We did not attempt to categorize the 4 students who indicated that they would use a visual explanation to convince a classmate because there was some indication from Item 1 that some students were interpreting Jamie's visual explanation as a general argument while others did not.

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IMPACT OF RESEARCH ON STUDENT TEACHERS' CONCEPTIONS ABOUT IMPROVING MATHEMATICS WRITTEN TESTS

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Despite their important role in school mathematics assessment, current calls for reform-based assessment have paid little attention to written tests. This paper reports on a pilot study designed to find out the impact, if any, of the reading and discussion of some selected research studies and reform texts on student teachers' conceptions about classroom assessment in general, and about the role and value of written tests in the assessment of student learning in particular. The activities proposed during the study impacted the participants' conceptions differently: their dispositions towards reform-based teaching, together with passion and excitement for the teaching profession seemed crucial for improving written tests following reform recommendations. The role of cooperating teachers regarding the improvement of written tests was questioned, and some issues for future research were raised.

Despite current calls for diversifying classroom assessment tools, teachers still rely on typical written tests for assessing student learning (e.g., Associação de Professores de Matemática [APM], 1998; Romberg, 2001). As in other countries, many Portuguese teachers believe that written tests are the most objective and rigorous means of assessing student learning, typically ignoring other educational goals of affective nature (e.g., APM, Rafael, 1998). Relying solely on one assessment tool is bound to lead teachers to construct incomplete or incorrect pictures of what their students know and are able to do mathematically. More and richer information on student thinking, beliefs, and knowledge can be gathered using more than one assessment tool (National Council of Teachers of Mathematics [NCTM], 1999).

In fact, typical written tests suffer from a number of drawbacks. For example, they only provide the perspective of students' individual work on time-limited written tasks, and they reveal the results but not the processes of student thinking (e.g., van den Heuvel-Panhuizen, 1996). Typical written tests are inadequate for assessing students' ability to investigate and discuss mathematical ideas, or their perseverance and creativity, because students are not typically asked to construct their own answers, explain their thinking, justify their responses, or look for alternative solution paths (e.g., de Lange, 1993). Short-answer and multiple-choice questions – focused on application of algorithms or procedures, manipulation of symbols, and computation – are the norm of written tests, thereby reinforcing the already deeply rooted idea of disconnectedness of mathematics, and stressing the importance of getting the right answer disregarding the processes used in reaching that solution (e.g., APM, 1998; Romberg, 2001).

Nonetheless, written tests do have a role to play in school assessment, and teachers are very likely to continue emphasizing short-answer, in-class written test questions. In addition, teachers are familiar with the design and grading processes of such tests, and students, parents, and the general society also value those tests (Thompson, Beckmann, & Senk 1997). Moreover, written tests allow the screening of a whole class, and their improvement can be a way to start changing the quality of school assessment. Thus, written tests should be changed and improved rather than rejected (van den Heuvel-Panhuizen & Gravemeijer, 1993). In fact, "relatively minor changes in an item can have a major impact on the nature of a test" (Thompson et al., p. 59). For example, multiple-choice items can be transformed into more open-ended questions whose answers will offer teachers much richer information about their students' thinking.

Alternative Written Tests

Several alternative written tests, such as 2-phase tests, written essays, and group tests, have been suggested (de Lange, 1987) and implemented in several countries (e.g., Abrantes, Leal, Teixeira, & Veloso, 1997; de Lange). De Lange's five *Basic Principles of Assessment* provide the theoretical grounds for the use of those tests: 1) "tests should be an integral part of the learning process" (de Lange, 1993, p. 199) so that they may improve teaching and learning; 2) tests should themselves generate learning situations; 3) tests "should enable students to show what they know rather than what they do not know" (p. 199); 4) tests should consistently address all educational and curricular goals of school mathematics; and 5) "the quality of the test … [should] not be dictated by its possibilities for objective scoring" (p. 199).

The *2-phase tests* have two different phases, each one with its own goal. The first phase is similar to a typical written test, and it is aimed at finding out what students do not know or have difficulties in. The teacher's written comments on students' mistakes, suggestions, or requests for clarification provide students with an opportunity to deepen, elaborate, and improve their responses afterwards. In the second phase, students reflect on and complete or redo their answers at home within a few weeks. This phase is more focused on conceptual understanding, including more higher than lower level questions. Both *written essays and written reports* can be completed individually or in groups, inside or outside the classroom. In a written essay, students, with or without helping guidelines, are asked to elaborate on a certain topic or problem and allowed to produce, integrate, and express their ideas. In a written report, students describe their work on a certain learning activity, and critically analyze it (de Lange, 1987).

Student-generated tests, covering a certain content topic for instance, may stimulate students to look for data, charts, graphs, etc. in order to design exercises, essay questions, or investigations to include in the test, encouraging students' metacognition (de Lange, 1987). *Group tests* are group assessment tasks in which student-student communication and ability to express others' ideas and insights into personal words are the focus. Many curricular themes are suitable for *practical tests* which have a practical emphasis. For example, probability and statistics or geometry topics can be adequately assessed through practical tests, making use of manipulatives or technological tools. *Take-home tests* are to be completed at home. They usually encompass essay-like tasks and can be accomplished individually or in groups. Take-home tests are aimed at getting "a reasonable picture of the possibilities and capabilities of the students when confronted with tasks at a somewhat higher level" (de Lange, p. 233).

Purpose of The Study

The student teaching phase of teacher education programs is ideal for eliciting feelings of need for change as student teachers begin to explore classroom teaching and assessment, and they may benefit from some kind of support. An increasing awareness of (student) teachers' own classroom assessment practices and opportunities for assessment during instruction may lead to their progressive de-emphasis of written tests and quizzes as the privileged means of assessing their students. In addition, (student) teachers' increasing valorization of observational data is crucial for constructing a better picture of their students' mathematical understanding (e.g., Borko, Mayfield, Marion, Flexer, & Cumbo, 1997). As part of a larger study (Ferreira, 2003), I investigated the impact, if any, of the reading and discussion of selected research studies and reform texts on student teachers' conceptions of classroom assessment in general, and their conceptions of the role and value of written tests in the assessment of student learning in particular.

Methodology

The participants in this study were enrolled in a 5-year mathematics teacher education program for grades 7 through 12, offered by a public university in a large urban community in northern Portugal. This teacher education program is characterized by an emphasis on mathematics content coursework with no current mathematics education course offerings. The student teaching phase lasts for a whole school year and is designed as a group experience. Each group of student teachers works with a cooperating teacher and with a university supervisor (supervising a monograph, addressing topics hardly related to mathematics education). Student teachers work as full-time teachers with a reduced teaching load. Six randomly selected groups of student teachers completed a survey which sought information on their beliefs and practices of classroom assessment and written tests. Based on variability of responses and placement schools, and on availability, 9 student teachers were selected to further participate in this study. The participants were placed in 3 schools. Groups A, B, and C had 3, 4, and 2 members, respectively.

Data were collected in 3 time periods: October 2002, and January and March 2003. Besides the survey, written reflections upon 2 packets of selected readings (including reform texts and research articles), and group interviews were used to collect data. The selected readings were written in English and in Portuguese, and accompanied by an abstract in Portuguese written by me. The first reading packet included guidelines to facilitate written reflection but the second packet purposefully excluded reading guidelines. The semi-structured interview protocols (mostly based on reading guidelines and written reflections) were audiotaped for further analysis. The interviews, each lasting between 60 to 90 minutes, were conducted in my university office.

Results and Discussion

Many survey items (dealing, for instance, with students' reactions to their tests) were left blank because the respondents had not yet given any written tests to their students. Most respondents saw written tests as comfortable and rigorous means of quantifying student learning and teaching effectiveness. This is not surprising given the results of previous research conducted in Portugal (e.g., APM, 1998; Rafael, 1998). With a few exceptions (namely Group C), the participants' initial conceptions about school mathematics seemed consistent with the typical society perspective "of mathematics as a set of discrete hierarchically arranged facts and skills; … of learning mathematics as replication and repetition; … of teaching mathematics as exposition and practice; and … of assessing mathematics as paper and pencil testing for the sole purpose of grading and ranking" (Herrington, Herrington, & Glazer, 2002, p. 1105).

Group A never completed written reflections, and only one student teacher seemed to have read some of the texts. Group B's reflections were generally superficial, reduced to brief responses to reading guidelines (first packet) or to a poor summary of the readings (second packet). These student teachers (as well as those from Group A) failed to identify key ideas in the texts, and to relate them to their own classroom practice. Yet, towards the end of the study, two student teachers from Group B were starting to question their assessment practices, including the quality of their written tests. Student teachers' reflections from Group C were always deep and extensive, expanding on the texts' ideas that had caught these student teachers' attention the most and going beyond reading guidelines. They tended to relate the readings to their classroom practice and personal ideas, showing great concern for fairness in testing.

The group interviews served mainly as debriefing sessions on the selected readings and completed reflections. One theme that emerged from the interviews dealt with the role and use of quizzes. While student teachers from Group A never used quizzes because their cooperating teacher did not like this form of assessment, it was never clear whether they really agreed with him. Group B made frequent use of quizzes, right before the *real* tests, to identify students' learning difficulties. However, these student teachers never elaborated on what they would do after learning about those difficulties. Since quizzes were assigned so close to the *real* tests, most likely they served to indicate the learners what questions they would encounter in the *real* tests. This practice was related to the student teachers' own schooling experiences. In fact, quizzes are typically called *formative tests* in Portuguese schools, and, contrarily to the goals of *formative assessment* (e.g., Gipps & Murphy, 1994), such tests encourage students to memorize the teacher's responses, ensuring success in the *real* tests.

Group C gave several quizzes, never too close to the *real* tests. These student teachers' goals were to give their students enough time to identify and overcome their difficulties, and to use quiz information to revise lesson plans and classroom teaching in order to accommodate for students' needs. In addition, these student teachers explained to their learners what their goals were when assigning a quiz or a written test: to have students detecting their own learning difficulties in order to overcome them, and this significantly diminished students' resistance to quizzes and test anxiety. The student teachers in Group C were conceptualizing quizzes as forms of formative assessment, whereas those from the other groups misunderstood the true meanings of *formative assessment* and *formative tests* (or quizzes). The interviews were helpful to explain and discuss the differences between formative and summative assessment, distinguishing the regulatory role of the former from the certification role of the latter (Martins, 1996).

The interviews provided more opportunities for reflection than the survey or the written reflections. For example, for all groups, the reflective and conversation-like interviews were helpful in clarifying and broadening many of the issues addressed in the readings. Two main themes that emerged from both sets of group interviews: *alternative written tests* and *views about the cooperating teacher*. Concerning the first theme, the main ideas of the readings were not entirely new to student teachers in Group A because they had already been encountered during their numerous meetings with their cooperating teacher; however, those ideas had not been fully understood until later in the study. Alternative written tests were too difficult to do with misbehaved students. Despite being very knowledgeable about alternative written tests and actually using some of them, the cooperating teacher did not encourage his student teachers towards using alternative written tests.

In general, student teachers in Group B lacked interest about using alternative written tests. Students' lack of motivation, underachievement, and bad behavior were the excuses for viewing those tests as non-practicable. Despite recognizing their overemphasis on typical written tests, only two student teachers considered possibly changing their current views in the future. The cooperating teacher of Group B gave her student teachers relative autonomy in test construction and support as far as materials and help in lesson planning are concerned. However, all this support was rooted on her own perspectives which resonated with the student teachers' and with the general low expectations for academic life of the school community.

This study offered student teachers from Group C the first contact with alternative forms of assessment but they already had some intuitive and personal ideas for classroom assessment. However, they were afraid of implementing those ideas, as well as some of those offered in the readings, because of the obstacles posed by their cooperating teacher. Her extreme concern about curriculum coverage had already prevented the student teachers from using more reform-based instructional and assessment practices in several instances. Nevertheless the student teachers maintained their excitement about using alternative written tests, and they were finding out ways of attaining their goals without displeasing their cooperating teacher.

"Alternative assessment creates a climate of unpredictability because we can never be sure what students are going to say when we ask about their mathematical thinking" (Cooney, Bell, Fisher-Cauble, & Sanchez, 1996, p. 485). This certainly represents a challenge for teachers who prefer a classroom where students' and teachers' expectations and demands are well known by all. This seemed to be the case in Group A. Student teachers from Group B had low expectations for their students and they complained about student underachievement, misbehavior, and lack of motivation. Also, parents valued typical written tests, and were skeptical of other assessment instruments. These two factors seemed to account significantly for Group B's resistance to using alternative written tests, and to these student teachers' sense of helplessness about implementing new forms of classroom assessment. *Content coverage* also hinders teachers from implementing alternative forms of assessment, including alternative written tests (Cooney et al., 1996). This did seem to be the perspective of Group C's cooperating teacher, who even prohibited her student teachers from doing group activities and using technological aids for instruction, even not using class time. However, her student teachers were determined to overcome those barriers and implement alternative written tests.

One member of Group C seemed to be disappointed with the generalized lack of vocation for the teaching profession, and she related this issue with school assessment as follows: "The general unwillingness of teachers to change and their widespread lack of passion for the profession itself, largely contributes to their passive and accommodated attitude towards their mission as teachers" (Interview 2, March 2003; my translation). This student teacher suggested that younger teachers such as herself, being more willing to change current classroom practices, "should get together to discuss and reflect on issues of reform-based instruction and assessment as a first step to start changing the state of affairs in Portuguese junior high and secondary schools" (Int. 2). In this regard, "alternative written tests could work as ideal starting points since changes in these tests are not too difficult to do and can make a huge difference regarding the quality of information about student thinking that can be collected through such tests" (Int. 2).

Conclusions

The participants in this study held very different conceptions of classroom assessment in general and of written tests in particular, ranging from traditional perspectives of assessment based almost exclusively on written tests to more reform-oriented standpoints of assessment based on a plethora of instruments and practices that are integrated and aligned with curriculum and instruction. Group C seemed more homogeneous in their reform-based conceptions of classroom assessment, while the other groups were more heterogeneous, especially group B.

Besides providing the participants with useful information on alternative assessment and alternative written tests, this study, especially its interview component, constituted an opportunity for all participants to understand and/or clarify some of the ideas addressed in the readings, and to broaden their perspectives on classroom assessment and the role of written tests. For example, student teachers from groups A and B were able to understand the role and value of formative assessment. Interestingly, student teachers from Group C had intuitive ideas about formative assessment that largely resonated with its true sense.

This study seemed to have impacted differently the participants' conceptions about assessment and written tests. The study had a minor impact on Group A; the student teachers in this group lacked engagement and interest in the study's activities and showed no willingness to use alternative written tests in a near future. The impact of this study on Group B seemed to be relatively minor. Although student teachers were provided with interesting information about alternative assessment and written tests, they evidenced different degrees of willingness to use that information. Finally, this study seemed to have had a relatively major impact on Group C.

These student teachers were always seriously engaged in the study's activities, and showed a significant excitement about reforming assessment practices and improving written tests, fighting against some serious obstacles posed by the cooperating teacher.

The student teachers' dispositions towards reform-based teaching and their excitement about the teaching profession seemed crucial for embracing a reform-oriented practice of classroom assessment and a wiser and fairer use of alternative written tests. The participants whose perspectives on classroom teaching and learning (e.g., Group C) were more reform-oriented were the ones who engaged more seriously in this project and reflected more deeply on the topics suggested, going much beyond the reading guidelines. Also, they did not let the barriers posed by the cooperating teacher or school department policies prevent them from trying new practices nor from keeping their perspectives and ideas, at least for a near future. The student teachers with less reform-based perspectives on classroom teaching and learning were concerned with covering the curriculum and used this goal, together with lack of time, and students' misbehavior and underachievement, as excuses for not using alternative forms of assessment.

Limitations and Implications for Future Research

The limited opportunities for data collection prevented me from making classroom observations, interviewing the three cooperating teachers, and analyzing student work on written tests. Legal constraints prevented me from asking the participants to try out alternative written tests in their classrooms either, thus failing to situate "the change process in the actual teaching and learning contexts where the new ideas will be implemented [which] is an effective strategy for helping teachers change their practices" (Borko et al., 1997, p. 267). In addition, many texts were written in English, which may have caused some participants to feel discouraged about reading them. Future research should address these issues.

Based on the data collected, the three cooperating teachers differed greatly in terms of perspectives about alternative written tests and in terms of the constraints posed concerning test construction. Surprisingly, the cooperating teacher who posed more serious obstacles to alternative assessment did not seem to have affected her student teachers' dispositions to actually implement their non-conventional ideas for assessment. On the contrary, the cooperating teacher who seemed more knowledgeable about alternative written tests and who actually used some of these tests did not encourage his student teachers in using alternative written tests. Future research should take into account the preparation and contribution of cooperating teachers in order to facilitate changes in the student teachers' assessment practices, especially regarding the improvement of typical written tests.

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PRESERVICE TEACHERS' USE AND UNDERSTANDING OF PLACE VALUE

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This report describes the findings of a study crafted to identify critical elements preservice elementary teachers (interns) use to define place value and assess a child's understanding of place value. At the conclusion of the second semester of a teacher education program, interns developed interview protocols and used them to collect evidence of a child's understanding of place value. Each intern then developed a report describing the understanding the child's understanding. Findings from an analysis of a sample of the reports produced a list of critical elements, such as unitizing, interns associate with an understanding of place value. Analysis of evidence drawn from the reports of children's work with decimals suggests that interns' understanding of place value may still be developing as they enter the final year of their program of study.

Research suggests teachers' content knowledge impacts their practice (Putnam, Heaton, Prawat, & Remillard, 1992; Sowder, Philipp, Armstrong, & Schappelle, 1998). Teacher education programs are thus charged with providing experiences for the development of this and other knowledge for the teaching of mathematics. The objective of the study discussed in this report was to provide evidence of preservice elementary teachers' (interns) understanding of place value at a critical point in their teacher education program. Specifically, what elements do interns, who have completed their mathematics content and methods courses, identify as important in children's understanding of place value? What do interns' interview based assessments of children's understanding of place value reveal about the interns' understanding?

Perspectives

Comparison of research (Fuson, 1990; Hart, 1989; McClain, 2003) illustrates that understanding place value involves developing a system of units, powers of ten, and comparative relationships between them (e. g. tens are less than hundreds and ten tens are one hundred). Piaget and Garcia's (1989) view of psychogenesis provides a framework for investigating learners' understanding of place value. Piaget and Garcia describe three stages of development that apply to the mental construction of mathematical systems: inter, intra, and trans. These stages can be used to characterize the development of an understanding of place value. *Inter* refers to the development of and activity with a single unit. *Intra* refers to the development of relationships between units. The building of a cohesive framework for these relationships is what Piaget and Garcia refer to as the *trans* stage. This description suggests that the development of an understanding of place value may take much longer than time allotted in school mathematics curriculum maps affords. This perspective provides a lens through which interns' assessments of children's understanding of place value is viewed.

Research and interns' understanding of place value

Several research studies provide evidence of interns' understanding of place value. Schifter and Fosnot (1993) suggested that the internalization of rules associated with the Hindu-Arabic numeration system may impede the development of an adult understanding of place value. To overcome this barrier, they developed a novel investigation using a collection of artifacts in base-five. Spieser and Walters (2000) used this investigation and observed interns engaging in debates about the use of zero and the meaning of digits in various positions.

Other studies provide evidence of interns' difficulties with decimals that may stem from their understanding of place value. Stacey et al. (2001) noted that a surprising number of interns identified zero as greater than 0.6. The interns reasoned that zero is in the ones place while six is in a position to the right of the ones place. Putt (1995) found that many interns ordered decimals between zero and one by counting digits. Some interns appeared to reason that fewer digits in the decimal meant a larger value. Others reasoned that more digits indicated a larger value. These rules have been identified in the literature as shorter makes larger and longer makes larger respectively (Resnick et al., 1989). Thipkong and Davis (1991) noted that interns often used division to solve word problems with decimals that appeared to call for the use of multiplication. For example, some interns used division to solve the problem "Carol waters plants for .25 hours every day, how many minutes is that?" (p. 95). Interns justified their approach noting that "division makes smaller" (p. 97). These research findings suggest that interns are still developing their system of units and relationships for decimals.

This interpretation of research is supported by studies of interns' actions with novel numeration systems. Zazkis and Khoury (1993) asked interns to convert numerals in bases other than ten to base 10 representations (e. g. Convert 12.34_{five} to its equivalent in base 10). Consistent strategies used by the 59 participants included eight incorrect ones used by 47 interns. Among these were the assignment of incorrect values for the positions to the right of the decimal point or ignoring the digits to the right of the decimal (e.g. $12.34_{\text{five}} = 7.34$ since $12_{\text{five}} = 7$).

Method

I define understanding as a collection of beliefs (Cobb, 1986). Some connected, some realized, and others disconnected, or implicit. This view is the result of an integration of research literature (Hiebert & Carpenter, 1992; Pirie & Kieren, 1994; Sierpinska, 1994; Skemp, 1987) with the idea that affect and metacognition impact the development of understanding (Kastberg, 2002). Evidence of understanding as I have described it can be effectively gained through an analysis of qualitative data provided by participants. A careful analysis of archival data provided by interns, with a focus on identifying and describing themes, was used in this study to gather evidence of understanding.

Data

The elementary teacher education program at Indiana University Purdue University Indianapolis (IUPUI) consists of four semesters of study, Block I, II, III, and IV. Blocks I and II include 3-hour methods courses in mathematics teaching and learning for grades K-3 and 4-6 respectively. Near the end of Block II, interns create and collect data with a self-generated interview protocol designed to gather evidence of place value understanding of a child (usually in upper elementary school). As a program assessment, interns submit reports containing an evaluation of a child's understanding of place value as supported by collected evidence. A simple random sample of ten of the 111 spring 2003 reports was the data for this study.

Analysis

Three categories of evidence were gathered from the reports: components of place value, task descriptions and rationales, child's response to tasks as described in intern's report or transcript, and the intern's analysis of this evidence. Summaries of evidence for each category were generated. The summaries were used to identify critical elements in each intern's understanding of place value and were then used to hypothesize intern's meanings for these critical elements. For example, one intern's report identified unitizing as a critical element in the development of children's understanding of place value. The intern then defined the term. "Unitizing is used when children understand they can count each group of objects one by one" (Intern 012). In her description of the child's action the intern refered to groups of tens and ones. This evidence indicates that the intern is aware of grouping as a critical element in place value understanding. However it is unclear if she has made a distinction between grouping and grouping by ten.

The summaries and conjectures about definitions were then compared. This comparison resulted in collection of categories. For example references to unitizing and grouping formed one category. These terms were identified as a single category since interns used them to refer to the idea that making and counting groups was a significant landmark in the development of place value understanding. A final check of the explanatory power of the categories was made by rereading the interns' reports to be certain that common critical elements were accounted for.

Results

Elements of Place Value

Critical terms for four or more of the interns were *unitizing*, *decimals*, *reading and writing numbers*, and *algorithms*. Nine interns identified unitizing as an essential element in their assessments. Meaning for the term unitizing varied. Five interns associated the term with groupings of ten or a power of ten. Four interns did not distinguish between grouping in general (four groups of five) and groups of ten. This finding suggests that the interns attempted to use what they were taught in methods, that unitizing is significant in the learning of mathematics in general and of place value in particular. However, some interns struggled with the idea of unitizing as it is used in developing an understanding of place value. These interns focused on the global definition of unitizing as grouping, and did not differentiate between grouping and grouping by ten.

Seven interns identified decimals as an essential element in their assessment of place value. Interns' interpretation of evidence associated with decimals is fairly consistent: a child's success on comparison tasks (e. g. select the larger decimal) was indicative of his or her understanding. This limited interpretation may be based on novice readings of source documents (*Principles and Standards for School Mathematics* (2000) and course readings (Fosnot & Dolk, 2001a, 2001b, 2002)). These documents suggest that place value understanding plays a role in students' performance on decimal comparison tasks. This association may be taken as a "rule" by the interns: if the child cannot correctly compare decimals, then he or she does not have place value understanding associated with decimals.

Five interns identified reading and writing of numbers as evidence of understanding of place value, however the source of this reasoning is hard to trace. One source, cited by three interns may be the Indiana standards (Indiana Department of Education, 2000). This document identifies reading and writing numbers as a goal for grade K-6 children. Another possible link between reading and writing numbers and place value may be the interns' own experiences with children, teachers, peers, or mathematics.

The data contained evidence that interns are aware that the use of algorithms without understanding may impede understanding. However, four interns identified the use of algorithms as indicative of a limited understanding of place value. Associations of this type may have their roots in literal interpretations of readings cited by the interns (Nagel & Swingen, 1998; Ross, 2002). As in the case of decimals, interns may try to use references to develop "rules" for interpreting children's responses.

Interns' Understanding of Place Value

Interns' understanding of place value can be drawn from several pieces of data. First, there is evidence of misconceptions associated with decimals, including the idea that zero is larger than a decimal and that dividing by a decimal always results in a larger number. More significant however is the use of different approaches to assess children's understanding of place value for whole and decimal numbers. While, interns used a variety of tasks to investigate unitizing as associated with whole number quantities, they did not investigate children's understanding or use of units less than one. Instead they generally asked children to compare decimals (Which is larger 0.3 or 0.21?). Interns' analysis of evidence gathered from these tasks was focused on accuracy. If the child could correctly compare decimals, then he or she understood place value. This apparent disconnect between tasks used to assess place value understanding associated with whole numbers versus those used to assess understanding associated with decimals suggests that interns' own place value understanding may still be developing.

The second source of evidence of interns' understanding is the diverse list of operations (multiplication, division, addition, and subtractions), quantities (fractions, percents, zero), and or actions (counting, regrouping, exchanging, arranging numbers in numerical order, rounding) interns identified as associated with an understanding of place value. This breadth suggests there are more differences in the interns' understandings of place value than there are similarities. One explanation for this finding may be interns' beliefs about the teaching and learning of mathematics derived from their experiences. Research suggests these experiences can have a significant impact on interns (Borko et al., 1992; Gellert, 1999).

Conclusion

Performance based interviews can provide information about concepts interns associate with an understanding of place value and about their understanding of place value. Critical elements in interns' descriptions of place value included unitizing, a central focus of investigations in their methods courses. Although interns used the term, there is evidence that some associated counting groups of objects, in quantities other than ten, as evidence of a child's understanding of place value. Also significant was the number of interns who suggested that an understanding of place value should include decimals. Children's understanding of place value associated with decimals, as described by the interns, provides evidence of their difficulty with decimals consistent with existing research. In addition, differential approaches to an analysis of a child's understanding of place value of whole numbers and decimals suggests interns themselves may still be developing a system of units and relationships to be used in reasoning with and about quantities.

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"ALL TRIANGLES ARE SIMILAR" TRACING ROSE'S UNDERSTANDING OF SIMILARITY USING LESSON PLAN STUDY

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Focus of Study

The investigation of prospective teachers' (PST) knowledge of similarity was part of a larger study that investigated PST knowledge of *what* and *how* to teach concepts dealing with proportional reasoning, while engaged in Lesson Plan Study (LPS). The LPS involved a combination of teaching and research methods. The research method was based on the idea of Japanese lesson study and looked at ways PST developed an introductory lesson on a topic concerning proportional reasoning. However, since the lesson plan was seen as part of their methods course, it was imperative that the PST had an idea of some of the content and substantive structures involved in the proportional reasoning topics. In some instances it was necessary for the researchers to guide the participants towards these ideas. The researchers were a team of graduate students and faculty members that conducted interviews and were participant observers in various phases of the studies.

Conceptual Framework

The growth in similarity was assessed within the Pirie-Kieren (1994) model of growth in understanding as adapted by Berenson, Cavey, Clark and Staley (2001) to teacher preparation, while noting instances of folding back and collecting. Within each level of the teacher preparation model, Even's (1990) aspects of subject matter knowledge were looked at for potential growth in the context of *what* and *how* to teach. The transcripts and artifacts were examined for instances of *primitive knowledge, making an image*, *having an image*, *noticing properties*, and *formalizing*, while also looking for occurrences of *folding back* and *collecting*. The coding of the data used the categories from the Berenson, Cavey, Clark and Staley (2001) model for teacher preparation along with Even's (1990) aspects of subject matter knowledge.

The teacher preparation model is a framework for studying PST understanding of *what* and *how* to teach. The *what* of teaching includes the essential features of the concept, representations, and the knowledge and understanding of the concept to be taught. The *how* of teaching incorporates the ways of approaching the topic, the basic repertoire in teaching the topic, and the prospective teachers' knowledge about mathematics. It is within the discussions and presentations of a lesson to introduce the topic of similarity, that images and growth of PST knowledge of similarity can be explored.

Methodology

Rose (pseudonym) was a junior double majoring in Mathematics and Mathematics Education. She participated in the LPS on similarity occurring over a six-week period of time, within a 16-week semester, during her first mathematics education course.

The LPS contained four distinct stages (see Figure 1).

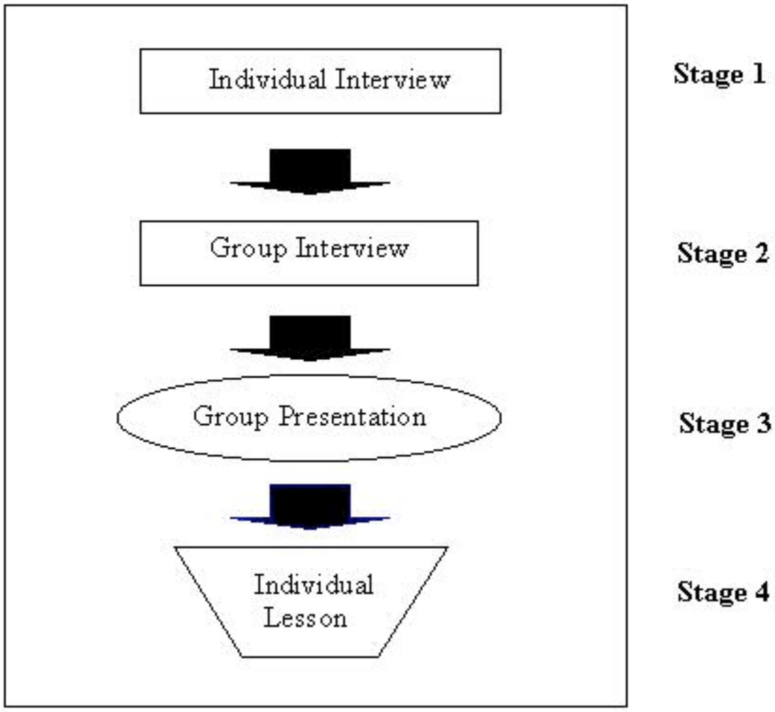

Figure 1. The structure of the lesson plan study cycle.

The first stage was an individual interview in which a researcher tried to get an understanding of what Rose knew about similarity and how she might teach similarity. The protocol for the individual interview consisted three major components: preinterview, lesson planning, and post-interview. The second stage was a group interview. Rose was grouped with four other individuals and asked to construct a group presentation on similarity and discuss their ideas. The third stage was the presentation of the group lesson to the methods class. In the last stage of the LPS, Rose produced a reconstructed view of her individual lesson plan (Berenson, 2002).

The data sources included videotapes of an individual interview, group interview, and a group presentation along with transcripts of these videotapes. Written artifacts such as the individual lesson, group lesson, and any written materials were also analyzed. Rose's autobiography and teaching philosophies were also examined for any background information for the research study.

The conceptual frameworks used were instrumental in the analysis of the data. Analysis of the data for the research involved coding, sorting into categories of significance, and establishing patterns among the categories. Data from interviews and the lesson plans were coded, extracted, and examined.

The issues of credibility are of utmost concern and steps were taken to assure that the findings are accurate and credible. Different techniques that helped in assessing the credibility of this qualitative research were the integrity of the observations, peer debriefing, and member checks. In looking at the credibility of the observations, several precautions were taken in dealing with the observed data. All the observations were recorded on videotape and the data from each episode was compared to previous episodes when analyzing the data. The triangulation protocol used data sources from all three video transcripts, written artifacts, and instructor feedback to make interpretations about the PST knowledge of similarity. By employing the use of peer debriefing, an outside, uninterested third party was asked to review the present research report. The use of a member check gave the participants of the study the opportunity to review any materials, and the report on their knowledge of similarity. Lastly, the theories chosen in the conceptual framework lend credibility to the research on the growth of PST knowledge of similarity when planning a lesson to introduce the topic.

Presentation of Findings

Rose began her discussion about similarity using an everyday definition of what similar means. When the interviewer (Int.) tried to probe her understanding of congruence it became evident that this was her initial idea of similarity.

Rose: If I am doing anything with similarity, I'll find some base concept like area, or perimeter or something. Say well okay, this is similar to this maybe by half or by twice or whatever, that's how I do anything that is similar… like people….

Int.: How you do anything that is similar, you try and find relationships like area and perimeter?

Rose: Yeah, like if I am doing a person I will be like this person is similar to this person, like likes and dislikes, or something. I'll have to find some root thing.

Rose's idea of the use of everyday similarity caused confusion during her individual interview. She eventually replaced this image with one that reflected mathematical similarity, but it was not until she after she had expressed other visual representations, she felt exemplified similarity.

Rose had a formalized illustration that she associated with *what* to teach about similarity.

Rose: Lets just use 3, 4, 5, and then if you had a littler triangle, and this was half of that

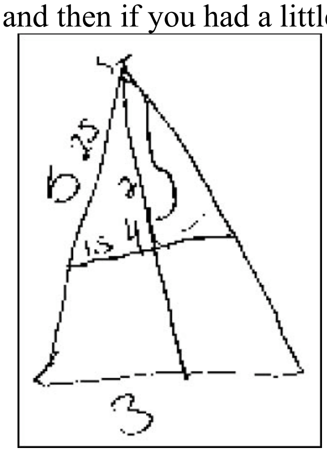

Figure 2. Rose's representation of similarity.

than this would be 2.5 and then 2, and then 1.5. Then find out the area of that one, and those would be similar because the area is twice the other one.

This illustration with the parallel bases through the midpoints of two remaining sides was seen as necessary for Rose to fold back and collect in order to do similar triangles using ratio and proportion. However, this image was replaced when she was realized that her understanding of mathematically similar triangles had been confused with her the understanding congruent triangles.

Int.: What does congruence mean to you?

Rose: Isn't that what this is kind of? Like, congruence is kind of like equal, in a way.

Int.: Equal in what way? Rose: Like one triangle would be… [points to Figure 2] Like this one, this triangle is congruent to this one because they both have like the same shape and almost the same size.

Int.: What are you talking about? This big one congruent to…

Rose: Yeah, well like if you had this triangle say *a*, *b*, *c*, would it kind of be congruent to this smaller version of it. Considering they would have the same lengths and stuff, well not the same lengths. They would both be isosceles, except maybe this one would have like 3 and this length would be a third smaller.

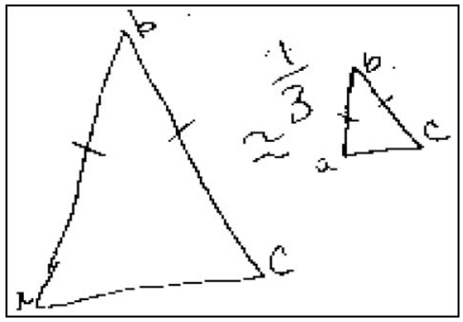

Figure 3. Rose's congruent triangles

As the LPS proceeded, Rose made images of the substantive structures in *what* to teach, such as indirect measurement and the similarity postulates.

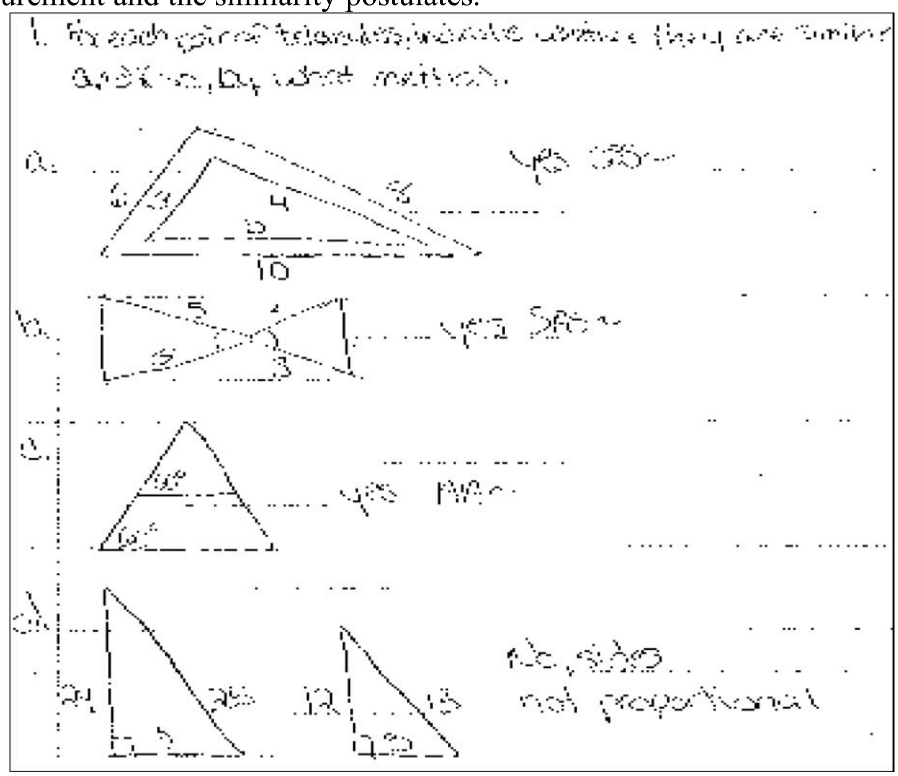

Figure 4. Problems Rose used to help reinforce similarity

Throughout the LPS, Rose made images of similarity that reflected either isosceles or right triangles. She focused on these two types of triangles because, as she stated, "teachers usually use isosceles triangles." She seldom included any representations other than right or isosceles triangles.

In trying to help come to a more precise meaning of *what* to teach about mathematical similarity, Rose made an image during the interviewers discussion about a figure he drew. Rose related to this figure and she began to change her images of similarity.

Rose: [pointing at a figure of a large triangle] But that would still be 40 right there, and that would still be 40 and so they would have like similar… the angles would be the same, only the side lengths changes. Oh, yeah we do angles too.

Int.: When we drew a smaller triangle that was similar and you said that the angles would be the same, it's just the side lengths are different.

Rose: Yeah.

Here Rose shifted her images of *what* to teach in similarity from only proportional sides to including angle measurements. At this point she considers both angle measurements and proportional sides to determine whether two triangles were similar.

Rose's formalized knowledge of *how* to teach involved a teacher-led lecture. In discussing the method of instruction her teacher used, Rose commented that this is a method that she will employ. These same sentiments reflect her teaching strategies in her lessons throughout the LPS and in her teaching philosophies:

"During instruction, I believe that the classroom needs to be of lecture style"

While wrestling with the image of how to write a scale factor in discussing *what* to teach, Rose collected previous images that she had from classroom lectures. She folded back to an example that happened earlier in the methods class and collected information to help her make an image how to write a scale factor.

Rose: …This is what I'm saying, because if you are going to ask us a question and say well you need to know the proportion of rock to ice then you can't switch them back, because the numbers can't actually like, like the numbers need to stay in the same order as what you are saying. That is what I was talking about in class and that would mess me up if they were doing ratios on these two triangles, and they said instead of … If I am asking them like from *A* to *B*, like I did right here *A* is to *B*, they would have to do that right here instead of switching it around.

Int.: Right, because we talked about it in class having 1-7 versus 7-1…

Discussion

Rose was an active participant in the similarity LPS and her vocalizations helped her to make and have images. Rose's images at the beginning of the LPS were eventually replaced as she folded back to collect on her understanding. Rose's mathematical knowledge of similarity was fragmented and effected her images of *what* to teach. Her initial images of *what* and *how* to teach changed during the LPS and apparently she noticed properties about the importance of indirect measurement, the similarity postulates, and the use of a conceptual based activity in helping her students understand mathematical similarity. Rose formalized her images of proportional sides, congruent angles, and the use of teacher-led lecture as a teaching strategy. There is evidence to suggest that Rose did not anticipate the students' responses during her lesson because of her beliefs in the use of teacher-led lecture as a teaching strategy.

Results from the LPS support Towers' (2001) conjecture that students' understanding is partly determined by teacher interventions. Lesson-planning activities may allow students to reflect on their knowledge of topics and build new images (Berenson, 2002; Cavey, Berenson, Clark, & Staley, 2001; Davis & Staley, 2002; Staley & Davis, 2001). Through reflections and discussions, it may be possible for students to realize their limitations and misconceptions about the knowledge needed to teach mathematics.

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ARISTOTLE AS SECONDARY MATHEMATICS TEACHER EDUCATOR: METAPHORS, STRENGTHS, AND EPISTEMOLOGICAL UNDERSTANDING

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Abstract

We describe a research experiment in which secondary mathematics teacher candidates studied, in a required methods class, Aristotle's argument that a line is not composed of points. The conceptual framework for this research is a blend of a fallibilist epistemology of mathematics; an insistence on approaching students' conceptualizations as cognitive strengths rather than replete with misconceptions; and "epistemological understanding," an inchoate construct that we employ to extend relational understanding/conceptual knowledge into the realm of mathematical foundations. We employ an analytical framework deriving from Lakoff and Núñez's Where Mathematics Comes From to tease out students' conceptualizations of foundational issues – most urgently, the point-line relationship and the nature of numbers – in terms of the metaphorical structure of their expressed thinking. We close with implications and challenges for mathematics teacher education.

Introduction

This study involves an experiment in secondary mathematics teacher education in which preservice teachers analyzed a chapter from Aristotle's *Metaphysics* (Calinger, 1995, pp. 85-87). In his text, Aristotle advances an argument for the theory that a line is not composed of points – a belief that had great currency to the ancients but is now considered mathematically incorrect. Directly contradicting what they were taught as students and what they know they will soon have to teach, Aristotle's argument is a fascinating and unsettling one for future mathematics teachers.

The first author led the seminar in which the Aristotle chapter was discussed and collected the data on her own. The second author is a close colleague who has accompanied her throughout all phases of the project. In our presentation, we will present the rationale behind the study, the gist of its enactment, and some of our findings thus far. We will highlight what we have learned about the mathematical metaphors (explained below) carrying the preservice teachers' understandings of the ancient point-line dilemma.

Rationale for and Origin of the Study

The study's deep roots lie in the profoundly unsatisfying experiences of the first author (hereafter referred to in the first person) as an undergraduate mathematics major and master's student. My initial forays into secondary mathematics teacher education raised strong suspicions that my students' experiences were sadly similar to my own – intellectually impoverished, conceptually barren, and devoid of any opportunity for their teachers to address their own questions, especially those that challenged canonical disciplinary knowledge. I developed the theory that unless preservice teachers can come to believe that mathematics has room for idiosyncratic ideas, they will teach their students as they were taught, and the cycle of passive student experiences of the mathematics curriculum and learned mathematical helplessness will remain unbroken. A radically different mathematical experience was called for to create a sense of possibility (at the very least). My own doubts about the taken-for-granted relationship between the real-number system and the real line, stoked by a doctoral seminar in which I first encountered the Aristotle reading, provided me with a specific area of mathematics on which to focus in the seminar.

Conceptual Framework

A host of intertwined conceptual influences underlies this study. The foremost theoretical underpinning is a fallibilist epistemology of mathematics (Ernest, 1997, 1998) – to wit, a theory of mathematical knowledge in which both mathematical truths and the means used to ascertain them vary according to just who is doing the ascertaining, along with where, when, with whom else, and for what purposes the ascertaining is being done. This element of the conceptual framework became actualized in the seminar as the epistemologically heretical principle that students' beliefs contravening those of the discipline can and in many instances should be counted as legitimate knowledge (Rosenthal, Sandow, & Schnepp, 2001).

The small body of literature on the value – the necessity – of identifying and attending to students' strengths was a lodestar in the formulation and enactment of the seminar; this "strengths" literature (Delpit, 1998; Howes, 2002) inspirits data analysis as well, most concretely when, by attempting to understand students' conceptions from their cognitive points of view, we strive to recast alleged misunderstandings and other academic misdemeanors as true positives. Our stance here should be counterpoised to that of the much larger school generally known as the "misconceptions" genre. Also immensely influential upon the seminar's development was the work on the pedagogical possibilities inherent in conversation (Haroutunian-Gordon & Tartakoff, 1996; Sfard, Nesher, Streefland, Cobb, & Mason, 1998). The demonstrable potential of the history, historiography, and philosophy of mathematics as media for and mediators of deep conceptual understandings of mathematics (Rickey, 1996; Swetz, Fauvel, Bekken, Johansson, & Katz, 1995) has both directly and implicitly informed this project.

Finally, no conceptual construct has played a greater role in recent mathematics-education research than the binary (albeit not dualistic) typology of mathematical thought known as procedural/conceptual knowledge (Hiebert, 1986) – or, more pertinent to this study, cast as instrumental/relational understanding (Skemp, 1978). We have posited a third dimension of mathematical cognition we term *epistemological understanding*, which encompasses students' theories and questions concerning the foundations and origins of mathematical knowing (and being). A simple example of epistemological understanding is knowing that the standard order of operations is a convention, not a mathematical necessity; we will develop a more extensive example in the next paragraph. A guiding hypothesis of this study is, simply, that epistemological understanding exists, and that curricular experiences in which we deliberately and explicitly attend to this type of mathematical understanding are worthwhile for us to test out and to study.

For a longer instance: Being able to apply the formula $1 + r + r^2 + r^3 + \ldots = 1/(1 - r)$ to obtain the result that 0.999… equals 1 constitutes a piece of instrumental understanding. Knowing that 0.999… is a highly condensed nickname for the limit of a convergent infinite series is an atom of relational understanding. Realizing that one has heretofore interpreted 0.999… as denoting a sequence rather than its limit; questioning why orthodox mathematics has dictated that a symbol appearing to stand for a process should instead stand for the result of that process; being cognizant of the fact although infinite processes and ultimate products thereof do not exist in the material world, certain assumptions that certain people make in a world we call mathematics enable the existence of these processes and products in this world – here is an element of an epistemological understanding.

When we compare and contrast our mindset to the well-developed work on epistemological obstacles (Nardi, 1996; Sierpinska, 1994), the "strengths" literature intersects our nascent account of epistemological understanding. For instance, an epistemological-obstacle spin on 0.999… = 1 has it – quite rightly – that it is anything but an epistemological slam-dunk to expect a 15-year-old student to overcome her embodied experiences of the physical world and believe that an infinite sequence of numbers can be added to yield a finite answer. Epistemological-obstacle theory then maintains that the misconceptions underlying the student's inability and/or unwillingness to accept the truth of 0.999… = 1 can be eliminated by good teaching and perhaps the passage of time. In contrast, our stance is that far from being misconceptions, reasons for not believing that 0.999… = 1 are likely to be sensible and logical; that someone who does not believe this purported fact is "thinking epistemologically" in ways none of us has been trained to hear; and that there is nothing cognitively pathological, and likely a great deal cognitively healthful, about never coming to believe that 0.999… = 1 , so long as one is aware that she is bumping up against mathematical orthodoxy.

Analytical Framework

The framework for data analysis derives from cognitive science, particularly the work of Lakoff and Núñez (2000) and (to a lesser degree) Lakoff and Johnson (1980). It is rooted in mathematical metaphors. Lakoff and Núñez write, "Mathematics…layers metaphor upon metaphor. When a single mathematical idea incorporates a dozen or so metaphors, it is the job of the cognitive scientist to tease them apart so as to reveal their underlying cognitive structure" (2000, p. 7). We are fortunate that Lakoff and Núñez devote an entire chapter to unpacking in painstaking detail cognitive metaphors involving "points and the continuum" (pp. 259-291) – precisely the dilemma that Aristotle addresses in his chapter and with which the seniors wrestled in their seminar. This "mathematical idea analysis" provides the basis for analyzing the seniors' conceptualizations of the point-line relationship in terms of the metaphors they use. We will interpret these metaphors to make claims about the seniors' mathematical knowledge that might not be supportable on other grounds.

For Lakoff and Núñez, "conceptual metaphor is a cognitive mechanism for allowing us to reason about one kind of thing as if it were another…[it is] a neural mechanism that allows us to use the inferential structure of one conceptual domain (say geometry) to reason about another (say arithmetic)" (2000, p. 6). In its simplest form – believe us, it gets a good deal worse – a conceptual metaphor comprises a *target domain*, i.e., that which is being metaphorized so we can reason about it; and a *source domain*, that which we use as a means of reasoning about the target. Lakoff and Núñez state, "[A metaphor] introduces elements into the target domain that are not inherent to the target domain" (2000, p. 46). Their headline for this scheme – the template they use as headings for their metaphors – is TARGET IS SOURCE. We find it more comprehensible and useful to expand this telegraphic phrase into **The source is a way to think about the target**. Thus a conceptual metaphor effects a transfer or borrowing of meaning from the source to the target. The target domain takes on meaning that might only make literal, technical, or formal sense in the source domain.

For example, the following is the first of the sequence of metaphors Lakoff and Núñez (2000, p. 263) develop to get to the heart of understanding what is often referred to as the real-number line. The target domain is "naturally continuous space" – the one-dimensional paths we walk, the two-dimensional flattened dough of the pizzas we eat, the three-dimensional air we breathe – and, metaphorically, they are asking us to think of any spatial object as a set with elements. Hence a set with elements becomes the source domain, the site from which meaning is transferred for the purpose of conceptualizing naturally continuous space. The points that are used as locators in a naturally continuous spatial object are associated with the elements of the sets. Properties of the spatial object are associated with the relationships between the elements of the set. One can expect that the new way of thinking about a naturally continuous spatial object will begin to include elements that derive from the relationships between the elements of the set from which it is being mapped.

A SPACE IS A SET OF POINTS

Source Domain A SET WITH ELEMENTS	*Target Domain* NATURALLY CONTINUOUS SPACE WITH POINT LOCATIONS
A set	→ An n-dimensional space– for example, a line, a plane, a 3-dimensional space
Elements are members of the set	→ Points are locations in the space
Members exist independently of the sets they are members of	→ Point-locations are not inherent to the space they are located in
Two set members are distinct if they are different entities	→ Two point-locations are distinct if they are different locations
Relations among members of the set	→ Properties of space

To us, A SPACE IS A SET OF POINTS encodes the cognitive mechanism, **A set of points is a way to think about a space**. In Lakoff and Núñez's terms, this is "a central metaphor at the heart of the discretization program" – i.e., the comprehensive reconceptualization of the continuous in terms of the discrete that (in Lakoff and Núñez's account) began with Descartes's creation of analytic geometry and has occupied a good deal of modern mathematics. Since "the continuous and the discrete are conceptual opposites" (p. 262), when the metaphors underlying discretization in general and the real-number line in particular are left unaddressed, confusion and miscommunication are bound to reign supreme.

Aristotle's proof (such as it is) that a line is not composed of points is an *a priori* dismissal of discretization. The teacher candidates who are the human subjects of this study – all senior mathematics majors – grew up with discretized mathematics as the pervasive, underlying, and unquestionable foundation of their studies. Thus we find Lakoff and Núñez's A SPACE IS A SET OF POINTS metaphor, along with the more complex metaphors and blends thereof that follow in their mathematical idea analysis of "points and the continuum," to be an invaluable analytical tool for understanding the seniors' understanding of Aristotle's argument.[1]

Research Methods

I collected the data for this study on three iterations of the same preservice secondary mathematics class (1997-98, 1998-99, and 1999-2000) offered at a huge land-grant institution in the midwestern United States well known for its innovative teacher-education programs. This class is a full-year 11-credit seminar taken by teacher candidates in their senior year. (A subsequent yearlong internship is required for certification.) I read the Aristotle chapter with each cohort of "the seniors." Each year, there was one class period solely dedicated to the chapter. However, the discussion of Aristotle continued alongside other activities for the remainder of the year.

The data used for this presentation come solely from the 1999-2000 academic year. I audiotaped all class sessions in which Aristotle was discussed; then either I or a paid assistant transcribed the tapes. A first read-through of the transcripts clearly indicated a highly complex conversational flow and a thick morass of student conceptions. Thus before undertaking systematic data analysis, I reread the transcripts numerous times, not so that themes might emerge, but to obtain a better overall sense of my subjects' cogitations plus determine the most salient portions of the hundreds of pages on which to conduct more focused analysis. I then sent these portions to the second author and another colleague (my dissertation director) with specific questions intended to help me make better sense of what the students were saying, particularly with respect to their possible metaphors. After extended discussions of our perceptions, I conducted one more "combing" of the reduced data corpus, this time making an analytic summary of each transcript. Again working closely with the second author and my dissertation director, I am currently in the thick of studying these summaries for evidence of the metaphors and blends thereof described by Lakoff and Núñez.

Findings

Thus far, analysis has shown the seniors' conceptualizations of the point-line relationship to be substantially related to the Lakoff-Núñez metaphors. (It remains an open question whether this mode of analysis is capable of identifying metaphorical elements different from those of Lakoff and Núñez.) Furthermore, although I have not systematically studied students' subjective experiences of the seminar (Holt, Gann, Gordon-Walinsky, Klinger, Toliver, & Wolff, 2001), there are indications that their engagement was generally high, that some seniors were able to reclaim a bit of the passion for mathematics that had been anesthetized by their coursework, and that some began to question the foundations of the mathematics they had been taught.

The imposed brevity of this paper permits only a tiny taste of the fruits of our analysis. We bring you *in media res* into a conversation in which the seniors' have been trying to apply relations in naturally continuous space from the Aristotle reading to their current thinking about sets of numbers. In reading the piece of text from the *Metaphysics* in which Aristotle argues that a naturally continuous line cannot be composed of points, my students very early on in their conversation introduce sets of numbers as their primary example. By so doing they are involved in what Lakoff and Núñez refer to as *conceptual blending*, which is "conceptually combining two distinct cognitive structures with fixed correspondences between them" (2000, p. 48). The essence of conceptual blending is that the source and target domains are simultaneously active in a metaphor. Near the end of the conversation on the first day, the seniors seem to realize what Lakoff and Núñez state as imperative to learning mathematics: To understand mathematics is to understand and be cognizant of the various metaphors of which it is composed, along with their limitations.

As we moved through Aristotle's argument and his definitions, we arrive at the final word to discuss, continuous. For Aristotle, two things are contiguous if they are in succession to one another and touch one another. Continuous is one step further in that the extremities that are touching become one.

Jim: I was thinking you got a real number line, you got any real number and any other real number, there's that whole bunch of stuff in between 'em, so –

?: No matter what your numbers are –

Jim: Yeah, there's never two numbers that, touch, because there's always numbers in between'em, no matter what number you pick.

Baron: And they're the same class, they're the same succession.

Jim: So why does he even bother defining contiguous, I guess. There's no application to what we're doing.

Baron: There is geometrically. There might not be numerically, but he's not talking about a geo-he's talking about a geometric principle, not a numerical principle.

Joe: He's building up continuous, he says, as a species on contiguous.

…

Baron: No, you can't think of a number system that's contiguous, but that's not, he's doesn't care about numbers. He's talking about geometric things.

In this small piece of transcript, Baron clearly states the dilemma that Jim does not appear to grasp. The students have been blending the ASPACE IS A SET OF POINTS metaphor along with a metaphor particularizing it to sets of numbers. When Baron says, "There is geometrically. There might not be but he's not talking about a geo- he's talking about a geometric principle, not a numerical principle," he is separating out geometrical ideas from numerical ideas and set-theoretic ideas. For Jim, this is all combined. Even though Aristotle does not refer to numbers or sets of numbers, Jim sees no problem with these entities being an essential part of the conversation. He is conceptually blending to beat the band; for Jim, all pertinent metaphors are in play simultaneously.

Baron's thinking is very different. Using Lakoff and Núñez's terminology, he is making clear that there are principles and relations that make sense in naturally continuous space and there are different principles that make sense in set theory or with sets of numbers. He is not at this juncture cognitively willing to have these distinct domains meet, much less communicate with each other.

Significance to Education

Helene Alpert Furani (2003) has recently written,

Lakoff and Núñez (2000) offer a detailed analysis of the conceptual metaphors underlying many pieces of expert mathematical knowledge, but this work does not yet provide insight into how conceptual metaphors are attained by students or the transformations conceptual metaphors may undergo on their way to resembling those that comprise the mathematical canon. This is largely unexplored territory.

Our study explores a small tract in this territory. We maintain that students know a great deal more than either the mathematical canon or educational research typically gives them credit for. What would it mean for preservice mathematics teachers to believe – to know – that their alleged misconceptions are, instead, legitimate knowledge? What could result from eliciting teacher candidates' epistemological understandings through conversation and constituting these understandings as mathematical strengths? How might they then perceive and respond to their own students' incorrect answers and inventive reasoning? Our study cannot provide definitive answers to these questions. But it may well be of service to mathematics teacher educators in using the mathematical understandings and experiences of teacher candidates to help them relate to their students in a fashion that is cognitively compassionate and pragmatic, intellectually respectful, and, quite plainly, humane.

Endnotes

1. We are well aware that a number of eminent scholars have found fault with the Lakoff and Núñez (most notably, Henderson, 2002), and we give great credence to these critiques. In the final stages of data analysis and interpretation, we will attend fully to all aspects of the Lakoff-Núñez program that these critics find suspect on either cognitive or epistemological grounds.

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LESSON STUDY IN PRESERVICE EDUCATION

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This report presents the research of two mathematics educators each using a Lesson Study approach with prospective teachers. One educator worked with prospective secondary teachers in a micro-teaching environment and the other worked with prospective elementary teachers in their field experiences in K-6 schools. Following the Lesson Study cycle of collaborative planning, lesson implementation with observation by classmates and guests, analysis of the teaching and learning, and lesson revision, the preservice teachers exposed their knowledge, beliefs and practices to the scrutiny of peers and then reconsider those beliefs and practices based on group discussions. The Lesson Study experience for both elementary and secondary prospective teachers helped them understand and begin implementing teaching practices that are consistent with reform-oriented teaching.

Introduction

As teacher educators we were interested in identifying contexts and activities that provide prospective teachers with common experiences to help them develop positive images of reformed mathematics teaching. The *Lesson Study* approach, as an adaptation of a professional development process often cited as highly valued among Japanese teachers, was the context used for helping prospective teachers trial and analyze potential reform practices. This report presents the research of two mathematics educators each using a similar Lesson Study approach with prospective teachers. One researcher worked in a micro-teaching environment with prospective secondary teachers. The other researcher worked with prospective elementary teachers as they implemented lessons in their field experiences in K-6 schools. Though each researcher worked in a different geographical region and at different classroom levels, their use of Lesson Study led to a collaboration and inquiry using similar research questions. Their common purpose was to investigate the strength of the Lesson Study model in influencing prospective teachers' professional development and dispositions as mathematics teachers, independent of the level and location at which the model was implemented.

Perspectives and Framework

Lesson Study offers a "structure by which teachers transmit, reformulate, and share craft knowledge through practice and collaboration with peers" (Shimahara, 2002). Also known as research lessons, the process follows a cycle of identifying a goal or problem for the lesson, collaboratively developing a lesson plan, implementing the lesson with observation by colleagues and other experts, analytically reflecting on the teaching and learning that occurred, and revising the lesson (Curcio, 2002; Lewis, 2000; Stigler & Hiebert, 1999). This approach provided a context for prospective teachers to develop pedagogical content knowledge, knowledge of mathematics content, teaching and learning, and images of reform-oriented teaching.

The theoretical perspective guiding this research has its basis in both psychology and sociology. The two perspectives treat learning to teach as a process involving individual active construction and individuals' adaptation and assimilation of cultural norms within their learning community. This perspective is consistent with Cobb's "emergent perspective" (2000) of uniting the cognitive and the social. According to Cobb, learning is a cognitive process of individual construction but simultaneously a sociological process of participation in a group. Mathematical, cultural, and social norms are constructed as prospective teachers examine their individual mathematical and pedagogical knowledge through interactions within teacher education learning environments. These constructions enable and constrain prospective teachers' reorganization of knowledge and beliefs.

Effective learning environments need to be learner centered, knowledge centered, assessment centered, and community centered (Bransford, Brown & Cocking, 2000). The Lesson Study approach used in this investigation was a collaborative task. Prospective teachers worked in small groups to develop, implement, and analyze the teaching of mathematics lessons. It was knowledge centered focusing on helping prospective teachers understand and develop images of reform-oriented teaching as envisioned by the National Council of Teachers of Mathematics (1991, 2000). It was assessment centered in that opportunities for feedback and revision were structured in the teaching cycles. Small learning communities were created that provided prospective teachers with opportunities to experiment, make mistakes, discuss, and negotiate among peers. Often a mentor classroom teacher or university instructor provided additional feedback.

We approached the inquiry guided by these questions;

- To what extent does the Lesson Study approach help prospective teachers develop positive images of reform-oriented teaching when so much of the practice they have experienced as learners or observed in schools is still very traditional?
- How does the Lesson Study approach influence prospective teachers' understanding of subject matter knowledge and pedagogical practices? What kind of content and pedagogical changes do prospective teachers propose in their interactions? How does this teaching experience influence their thinking on practice?
- What are prospective teachers' views on the collaborative discourse and analysis of teaching and learning that took place in their Lesson Study context? What kind of feedback do they provide each other?
- What were the similarities and differences in the secondary and elementary experiences and outcomes?

Modes of Inquiry

The Lesson Study research under discussion was conducted with both elementary and secondary prospective mathematics teachers. Thirty-six secondary prospective mathematics teachers participated in the investigation. The data from the secondary prospective teachers was collected during two semesters in a course on teaching secondary school mathematics. Each semester, the course consisted of 18 preservice mathematics teachers. The prospective secondary teachers typically completed this course during their second semester of their enrollment in a two-year upper division teacher education program. During each semester, the 18 students were placed heterogeneously into groups of three. The groupings were based on their responses to a questionnaire on mathematics and pedagogy related to ideas to be explored during the Lesson Study and on prior performance in the course. Each group was assigned a mathematical relationship or concept to teach to their peers during their Lesson Study. From the questionnaires, concepts or relationships that lacked familiarity or understanding on the part of the prospective teachers were selected for the research lessons.

The Lesson Study cycles completed by each secondary group were as follows: (1) The groups analyzed each others' prior videotaped lessons; (2) The groups of three researched and planned a 25 minute lesson on their specified topic to teach to different small groups of their peers (5 or 6 students); (3) After the planning, one member was videotaped teaching the lesson to a small group of their peers. Each of the three teachers watched the videotape, analyzed aspects of the lesson, discussed the lesson, and made revisions for the re-teaching. (4) Then a second member taught the lesson to a new group of peers and the analysis and revision process was repeated. (5) Finally, the lesson was taught by the third member to a third group of peers and once again revised, producing a lesson plan that was shared with the entire class. In Japanese Lesson Study, the lesson is typically shared with the teaching community. Throughout the experience, the instructor was available as a resource, observing lessons, watching videotapes, and providing questions and feedback for the groups to consider throughout the planning, teaching and reflecting phases. At the end, each group submitted a notebook documenting the cycles of their Lesson Study.

The elementary group consisted of 48 preservice teachers enrolled in a four semester field-based undergraduate program offering initial certification. For each of the first three semesters of the program the preservice teachers completed two full days a week of supervised work in an elementary classroom and two days of educational coursework. The fourth semester of the program consist of traditional student teaching. Those participating in this project were in either their second or third semester of the program, so they had varying degrees of experience in classrooms and were enrolled in either one of two required elementary mathematics methods courses.

Each elementary Lesson Study group consisted of three or four prospective teachers. The groups were formed based on the grade level and location of their field placement. School locations for elementary groups were purposely mixed to give prospective teachers opportunities to consider if and how differences in location influenced their choices of teaching practices. Each group was involved in three cycles of planning, teaching and observing, analyzing, and revising a mathematics lesson. Individual lessons typically lasted 45 minutes to one hour. The time between consecutive iterations of the cycle varied among the groups, with all groups completing the three cycles in about three weeks. Mathematics content topics varied from group to group, but the guidelines for their research lessons required that the lessons be consistent with the practices promoted in the methods course. The lessons needed to involve hands-on activities that required the children to do more than just listen to the teacher, and encouraged student discourse and group work. Content topics were selected in collaboration with the regular classroom teachers.

The research design involved qualitative data collection and analysis. Data sources included: questionnaires of potential Lesson Study content topics; observations of planning, implementing, and analysis of lessons; group documentation of three cycles of planning, teaching, analysis and revisions; individual reflective writings on the process and outcomes; videotapes of lessons; videotapes of end-of-semester group presentations; and surveys of feedback, analysis, and collaboration. Field notes of these observations and the group interactions were kept. Data from the elementary and secondary groups were analyzed separately. At both levels analysis involved the coding of the prospective teachers' development, insights, and struggles within individual Lesson Study groups. As patterns emerged they were compared across groups. Then outcomes from secondary groups were compared with data from elementary groups to identify common characteristics and differences between these contexts.

Findings

Data analysis the research lessons across the secondary prospective teacher groups revealed that the lessons became less teacher-centered throughout the Lesson Study cycles, incorporating more student experimentation, analysis and reasoning than that found in their first lessons. For example, a group teaching about Euler's Formula initially gave their student peers the formulas or definitions for their topic and focused on applying that information. After receiving feedback from the instructor and analyzing the videotape of their own lessons, the following lessons engaged their student peers in experimenting with a variety of polyhedrons and in looking for patterns to determine a relationship among the edges, vertices and faces. The prospective teachers noticed the difference in their own lessons:

> The improvement from our first to second lesson was dramatic. Our first lesson was far more teacher-centered and did not really center around the idea of constructing a concept or discovering a relationship. This time, we kept to the idea of constructing definitions, not stating, and then justifying them. This time it seems as though we had a better understanding of how to do this.

In a Lesson Study group teaching about the equation of an ellipse, the preservice teachers also became less teacher-centered after their first lessons. Initially, they presented a formula and gave their student peers examples in which they used the formulas. But as they implemented, analyzed and revised their lessons, they developed tasks involving explorations with physical models and graphing calculators through which their peers could gather data about the ellipses, make observations, and construct the general equation for ellipses. This group recognized that teaching aligned with recent reforms is challenging:

> As a group we realized how important it was to make each lesson more student led rather than teacher led. Getting away from a teacher-centered classroom is hard, since that is how most of us learned. After seeing how a student-led lesson can improve a student's performance, we see how valuable it is in our teaching.

Similar to the secondary teachers, the written lesson plans of prospective elementary teachers showed that their lessons became less teacher-centered and showed evidence of more student engagement in experimenting, analyzing, discovering, and reasoning. An elementary Lesson Study group planned an activity for first grade children on identifying and classifying shapes. They used commercially produced attribute blocks that could be sorted by size, shape, color, and thickness. Observations and collaborations from the first implementation noted that the children spent too much time listening to the teacher talk about attributes and model the student activities, without enough time for the children to handle the materials. Despite the teacher explanation and modeling of attributes, the children still had some difficulties in sorting and classifying the blocks. In the revision of the lesson, children were given the materials earlier in the lesson with less time given to teacher talk. The preservice teachers noted in the post-lesson discussion that as a result of the revisions, the children were more familiar with the characteristics of the blocks and had an easier time sorting the materials and creating their own patterns.

Through the Lesson Study experience, in addition to developing their understanding of teaching strategies aligned with recent reforms, the prospective secondary teachers enhanced their mathematics subject matter knowledge that in turn influenced their teaching. For example, after the first lesson, a few groups struggled with understanding the relationships they were teaching and the need to understand more in-depth arose from questions raised during the lessons. Towards the end of an initial lesson on traceable paths, the students being taught asked why a traceable path can have at most two odd vertices (the teacher could not figure it out before time ran out) and did not pose the question for the student peers to explore. After the lesson, when the teacher for that lesson got together with her Lesson Study group members they explored the relationship in order to understand why this occurred and incorporated this into their next lesson. The next time the lesson was taught, however, the teacher posed the question and then answered it, rather than giving the students an opportunity to engage in the deductive reasoning for themselves. The third time the lesson was taught the student peers were asked to explore why paths are traceable with at most 0 or 2 odd vertices; however, time ran out before arriving at a response.

Many of the lesson plans taught as part of the Lesson Study improved as the groups moved through the three cycles. In describing a lesson in its third revision a preservice elementary teacher who had taught the first lesson explained:

> I must say – it was terrific! It was organized, it was well planned, it was clear, and it made sense. [The third teacher] is a great teacher, but I cannot give him all the credit. Without [my] trail blazing and my experimentation gone bad, [the third] lesson could not been as smooth as it was. …The changes to the lesson presentation that our group talked about proved to be valuable, and I saw how simple changes could make a big difference in a lesson.

For some groups their lesson plans improved; however, the implementation of the plans did not necessarily improve. This is not surprising given that this was one of the first experiences these students had teaching. For a few groups, their lack of group collaboration and concern for others feelings when providing feedback was related to a lack of improvement in the lesson implementation cycles. One of these groups was teaching about traceable paths during the first semester of the investigation in the secondary education course. During this research lesson, the member teaching the third lesson did not fully develop her understanding of traceable paths. Although she tried to engage the students in reasoning about traceable paths and non-traceable paths, she quickly gave up when the students began to struggle with discovering the relationship. She moved on to provide the definition for them and then explained how to use paths to represent maps. Her Lesson Study group members' assessment of her lesson lacked depth due in part to concern for her feelings. However through the lesson analysis process, she recognized that she had given up and did not achieve the overarching goal of developing students' reasoning and ability to study patterns. She learned from the experience that she needed to listen more closely to the students and be better prepared mathematically, "I believe that I could have been a little more prepared with the definitions and paid a little more attention to the students constructing their ideas of the concepts but I don't think that John and Tami [her Lesson Study group members] would say that to me."

Prospective teachers considered a broader range of influences on their choices of methods and content when working in small groups. Due to the influence of their Lesson Study peers, some tried instructional approaches different from their usual choices or from those used by the classroom teacher. Through group discourse, they encountered and confronted philosophical perspectives different from their own and were forced to negotiate and compromise on the lesson design and implementation. As one student said:

> I feel I learned a lot about math and teaching math through the Lesson Study process but what I got most out of this was learning to work with others. I learned to work with different personalities along with other teaching beliefs and styles. I felt this Lesson Study was difficult at times but made me realize that this is just the beginning because in the future I will be collaborating with others who may not have the same beliefs as me. Therefore, this whole Lesson Study was an eye opener to me and has taught me to keep an open mind and listen to what others have to contribute to the team.

In some instances at the elementary level, the negotiation process resulted in an iteration of a lesson that was weaker in the depth of the mathematical ideas presented and in the level of student engagement. In one case when children struggled to complete a task with fractions where they had to fold a paper strip into various equal portions, the Lesson Study group revised the lesson by placing inked fold lines along the paper strip. The group claimed that the task became easier for the children, but failed to understand that it was now mostly a folding activity that lost much of the part-whole nature of the previous task. Even when a group member was aware of this situation, the dynamic of the group prevented this individual from exerting an altering influence.

Discussion

There were several commonalities between the elementary and secondary groups. Many of our preservice teachers had considered teaching as a rather isolated practice where content and pedagogical decisions often resided in commercial materials or school district curriculum guides. The Lesson Study experience placed decision-making about content and methods more in their hands and decision-making was seen more as a negotiated process than an isolated endeavor. Also, as would be expected of prospective teachers, issues of time management and transitions during lessons were common concerns and common targets for revisions for both elementary and secondary groups. Additionally, at both levels issues of group dynamics arose with the main issues being one member of a group not "pulling their weight" in contributing to the lesson development and revision.

One difference between the experiences of the elementary groups and the secondary groups was the focus on the mathematics content. While some elementary groups proposed changes related to content understanding across the cycles of their research lesson, most of their recommended changes focused on issues of classroom management and general pedagogy. Activities were often reorganized to allow for such changes as easier distribution of materials, clearer activity sheets, and better control of student behavior.

As teacher educators our goal was not to develop a set of "best" lessons, but to invite our students to participate in the professional development process of analyzing and providing feedback on lessons. The Lesson Study experience for both elementary and secondary prospective teachers helped them understand and begin implementing teaching practices that are consistent with reform-oriented teaching. It provided them an opportunity to expose their beliefs and practices to the scrutiny of peers and then to reconsider those beliefs and practices based on group discourse. Lesson Study can provide prospective teachers with opportunities to develop and teach lessons differently than they were taught or than they see in field experiences. The process clearly provided preservice teachers with a greater sense of ownership and control over a lesson and the mandate, or at least license, to make and trial changes. As one of our preservice teachers remarked, "Overall our lesson developed into a lesson to be proud of. It would be wonderful if every lesson we make could be as good the first time."

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MATHEMATICAL AUTHORITY IN PRESERVICE ELEMENTARY TEACHERS' EXPERIENCES WITH STANDARDS-BASED CURRICULUM MATERIALS

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This report provides illustrations of the role of mathematical authority in preservice elementary teachers' experiences using Standards-based middle school curriculum materials in an undergraduate mathematics course. Preservice teachers communicated a desire for their instructor to play a more direct, evaluative role in class discussions about the mathematics problems of the curriculum materials. In addition, teachers suggested that the curriculum materials were, in effect, missing important textbook features such as explanations, examples, and summary sections – each of which would have reduced their struggles to develop mathematical ideas for themselves. These findings suggest that teachers' orientations toward authority can act as inhibitors to their learning from and about the innovative approaches to mathematics and pedagogy presented in Standards-based curriculum materials.

The present reform movement in mathematics education is propelled largely by the recommendations of the *Standards* (National Council of Teachers of Mathematics, 1989, 2000). To support the vision of mathematics instruction introduced by the 1989 *Standards* document, a dozen or more federally-funded projects produced reform-oriented curriculum materials during the 1990s. Although each curriculum series has its own unique emphases (e.g. technology, cooperation, or real-world contexts), all the materials share a commitment to student exploration and discussion of mathematical problem situations. Evidence about the positive impact of reform-oriented curriculum materials on students' learning of mathematics has been extensively documented (Senk & Thompson, 2003). Despite the promise of *Standards*-based curriculum materials for student learning, case studies of teachers using new curriculum materials consistently present the considerable challenges of teaching unfamiliar content in unfamiliar ways (Frykholm, 2004; Lambdin & Preston, 1995; Lloyd, 1999; Lloyd & Wilson, 1998; Manouchehri & Goodman, 1998; Wilson & Lloyd, 2000). Through this research, we have become increasingly aware that teachers' implementations of new curriculum materials are also influenced by their conceptions of mathematics teaching and learning. Because many teachers' mathematical and pedagogical conceptions are deeply tied to traditional curriculum and instruction, making change – even with the support of innovative curriculum materials – can be very difficult. On the other hand, classroom instruction with novel curriculum materials may compel teachers to alter their conceptions on the basis of new types of classroom experiences with students and content (Collopy, 2003; Heaton, 2000; Lloyd, 2002; Remillard, 2000; Russell et al., 1995). These reports point to the influential role that experiences with curriculum materials can play in the professional development of inservice teachers.

What are the implications of these issues for the preparation of future teachers? In an attempt to offer *preservice* teachers greater familiarity with reform-oriented materials, mathematical subject matter, and pedagogical practices, the project described in this report engages preservice teachers in the use of *Standards*-based curriculum materials during teacher education coursework.[1] Part of this ongoing teacher education project involves documenting various components of preservice teachers' experiences using the curriculum materials for their learning.

The purpose of the present report is to offer several illustrations of the role of *authority* in preservice elementary teachers' experiences with *Standards*-based curriculum materials. Numerous studies have indicated ways that teachers' orientations toward authority relate closely to their tendency to teach in innovative ways and to be reflective in their thinking about teaching and learning (Cooney, Shealy, Arvold, 1998; Mewborn, 1999; Wilson & Goldenberg, 1998; Wilson & Lloyd, 2000). It is also worthwhile to consider how teachers' orientations toward authority contribute to their experiences with reform-oriented curriculum materials. Due to the predominance of student-centered and inquiry-oriented lessons in *Standards*-based curriculum materials, engagement with the materials brings into play issues of *mathematical authority*, as Wilson and Lloyd (2000) discuss:

> When students are expected to explore and discuss mathematical ideas, they must accept much of the responsibility for learning. As they determine appropriate procedures and methods to solve and verify problems, students are using mathematical authority to make sense of situations, issues, and questions. (p. 151)

If textbooks and teachers cease to serve as the primary sources of mathematical authority, students must play a much greater role in the development and testing of mathematical ideas. The present report discusses ways in which issues related to mathematical authority arise in teacher education activities in which preservice elementary teachers are required to engage with *Standards*-based middle school curriculum materials for their own learning of mathematics.

Selected Methodological Information

The preservice teachers discussed in this report are the 34 sophomore-level students (all female) in a Fall 2002 mathematics course for preservice elementary teachers at a large public university in the Mid-Atlantic region of the United States. Preservice teachers' names used in this report are pseudonyms. The author of this paper was the instructor of the course.

In this course, selected units from *Standards*-based middle school curriculum materials (Mathematics in Context [MiC] and Connected Mathematics Project [CMP]) were used as the primary text materials for preservice teachers' mathematical learning. These units were chosen to correspond to the mathematical emphases typical to college mathematics textbooks for teachers.

This report draws on data from multiple sources. Each class session was observed and videotaped, and fieldnotes were taken by two graduate assistants. Selected segments of recorded class sessions were transcribed. At the beginning and end of the semester, preservice teachers completed surveys, composed of open-ended, multiple choice, and scaled items, that included questions about beliefs about teaching, learning, and curriculum. Five teachers participated in two audio-taped interviews conducted by the graduate assistants. Due to the space constraints of this paper, interview data for only two of the preservice teachers (Jessica and Meg) are used.

Results

Mathematical Authority in the Teacher Education Classroom

This section of the report illustrates ways that preservice teachers' participation in the mathematics course was impacted by their expectations that the instructor should indicate the validity of the teachers' mathematical ideas during whole-class discussions. Consider, for example, a whole-class discussion conducted during the fourth class session of the *Mathematics for Elementary Teachers* course. Prior to this conversation about why the lattice method of multiplication makes sense, the preservice teachers had been asked to consider this method of multiplication in their work with the MiC (1998) *Reflections on Number* unit. Regarding the practice of finding partial products by adding the diagonal lattice entries, the instructor posed the question, "Why can you do this? It just seems really random to be adding diagonally. Why does that work?” Following Nicole’s explanation in which she explained how lattice multiplication related to more traditional methods of multiplication, almost every preservice teacher shifted her gaze from Beth to the instructor. The following interaction then took place:

> *Instructor*: Why is everyone looking at me?
> *Nicole*: Because you weren’t like, “Oh you’re right!” (Class laughs)
> *Instructor*: Is that what I’m supposed to do?
> *Many preservice teachers*: Yes. Yeah. (Some teachers nodding)
> *One teacher:* You’re the teacher!

Whereas the instructor had hoped to encourage ongoing interaction between the preservice teachers themselves by not speaking immediately after Nicole’s explanation, teachers clearly expected some sort of evaluation from the instructor.

The instructor rarely offered the sort of evaluation expected by the preservice teachers in the episode above, but did frequently ask, “What do you think about that?” after a preservice teacher’s comment. As Meg described, the instructor “does not say, ‘This is the right way to do it,’ or ‘This is the wrong way to do it’ but instead asks ‘What do you think?’ and lets us battle it out.” Similarly, in an interview early in the semester, Jessica described how, during class discussions, the instructor

> . . . sometimes cuts in and asks “Why?” or “Why isn’t it this way?” and then kind of leaves you hanging. She doesn’t give an answer. Once she asks a question, she doesn’t say, “Okay, yeah, that’s right.” She just leaves that in your mind to kind of let you think about what you feel is the right answer.

During the same interview, Jessica described this practice as

> . . . kind of neat because it makes you think of your own way. As a teacher you need to have your own methods and your own techniques, so it’s kind of cool to have a bunch of different things in your mind about how to solve problems or how to think about problems or how to teach problems.

Clearly Jessica was able to appreciate some reasons that the instructor was conducting class discussions in this manner. However, as the semester progressed, Jessica – as well as many of her classmates – became increasingly frustrated that the instructor directed the discussions toward *preservice teachers’* development of consensus about correct problem solutions. Following a discussion about fraction division, which carried over more than one class session, Jessica expressed the following:

> I would like it if when there is kind of an argument or a strong disagreement on something that maybe [the instructor] stepped in and pointed out which one is the better direction so that people can be more comfortable with what’s right. . . . Sometimes we walk out of here and no one has any idea how something should properly be solved.

Similarly, Meg suggested, towards the end of the semester, that the instructor “is not just trying to completely frustrate us, even though sometimes that’s what happens.”

Despite the frustration described in these quotes, over the course of the semester, most of the teachers appeared to become more comfortable with relying on other preservice teachers’ questions and comments for determining the correctness of their own mathematical ideas. As Jessica herself noted at the end of the semester,

> Sometimes when I don't understand the homework, I get really aggravated and I want to have a right answer, but then I have to stop and tell myself- If it's only one or two problems, that's fine. Just wait until class, and then someone will probably clarify any questions I have. I can ask someone in my group or whoever. I would feel very

> uncomfortable turning my homework in just knowing that I'm the only one that had any say in it, and I didn't have any outside opinions on how to do it a different way or better way, which is nice.

Jessica's sense that her mathematical understandings were importantly influenced by those of her classmates is consistent with an interesting survey result (reported previously by Spielman and Lloyd, 2004). When asked at the end of the semester whether the instructor, the other preservice teachers, or the *Standards*-based curriculum materials had been most beneficial to their learning, 28 of the 34 teachers (82.4%) chose "other preservice teachers." The other two choices were each selected by three of the teachers. A natural question emerging from this result is why so few teachers credited their curriculum materials as being most beneficial to their learning.

Mathematical Authority in Teachers' Engagement with and Views of the Curriculum Materials

Given teachers' reactions to alterations to traditional authority roles in the classroom, it is interesting to consider how teachers reacted to the representations of teachers and teaching in the curriculum materials used in the mathematics course. Consider the following example. In one particular investigation in the MiC (1998) unit, *Reflections on Number*, the teachers had to assume the role of a tutor who aims to help a child, Harvey, with a mathematics problem. The following situation is presented:

> During one tutoring session you say to Harvey, "For every multiplication problem, there are related division problems. For example, you can take 3 x 7 = 21 and write 21 ÷ 3 = 7." . . . Next you ask Harvey to write a division statement for the following situation; "You have six stickers, and you share them with no one." Harvey smiles, "I know that! I have six stickers and I share with nobody. The division statement is 6 ÷ 0 = 6. I am sharing with nobody, so I have all six stickers." (p. 28)

After this situation, numerous questions are posed, and then the text goes on to explain, "After thinking about it, you realize that Harvey's answer cannot be right. To help Harvey see that something is wrong with his answer, you ask him to divide 6 by 0 with his calculator. Following Harvey's surprise at the calculator's output, "You [the tutor] now ask Harvey to write division statements for 7 x 0 = 0 and 8 x 0 = 0. He writes 0 ÷ 0 = 7 and 0 ÷ 0 = 8" (p. 29). Next the student is asked to explain "what is odd about Harvey's division statements" (p. 29). The aim is for students to notice that, problematically, 0 ÷ 0 could equal any number.

The preservice teachers worked on this investigation in small groups, approximately six weeks into the semester. Following the teachers' small group work, a whole class discussion was conducted. The aim of the discussion, from the instructor's perspective, was to engage the teachers in thinking about division involving zero. In the course of this discussion, various comments were made by preservice teachers regarding the role of the tutor (and to a lesser extent, the text) in "teaching" about this idea. Consider Beth and Nicole's remarks about the lack of direct explanation by the tutor in the investigation:

Beth: I just want to say that I was personally offended that I was being a tutor but they didn't explain the problem to the kid right. Wasn't it something about sharing something with only himself? And he says if you divided it with no one or whatever? It's like if you didn't share it with anyone, then I, the tutor, would have been like, "Well, don't forget yourself – you're 1." And then there would have been none of this confusion. Right, you're not dividing by 0 because you're 1, so it's 6 divided by 1 which is 6. Isn't that the easiest way? Why didn't he [the tutor] just tell him [the student]?

(Sighs, class laughs)

Instructor: Well, what do you think of the tutor in the problem?

Several preservice teachers: Bad. Weak. (Some preservice teachers shrug their shoulders.)

Nicole: The tutor didn't explain. One of our homework problems was to compare the explanation in that book [Reflections on Number] with the explanation in [another book being used in the course], but there was no explanation at all that I could find in that (pointing at the Reflections on Number book on her table).

After the class had shifted attention away from the tutor and back to the mathematical idea of "6 divided by 0," Jessica made some comments that turned the discussion again to Beth's idea that the tutor should have told Harvey that he had forgotten to include himself.

Jessica: To put myself in a kid's point of view, I would think that 6 divided by 0 is 6. You have 6, you divide it by nothing, you have 6. I think Harvey's explanation for his age made perfect sense and I was like, well how do you say it's not? And the tutor used a calculator. That's not a great learning tool. It still didn't tell him [Harvey] *why*. He [the tutor] is just like, "Oh, it doesn't work. The calculator says it doesn't work so it doesn't work."

Cathy: 6 times 0 would be 0 and not 6. I mean, it only checks one way, which may confuse them, like she [Jessica] said. Explaining would help!

Beth: She [the tutor] should have said, "Well, you're splitting it by no one but you have it yourself, so it's really 6 divided by 1."

These teachers' comments indicate concern about the lack of direct explanation by the tutor in the investigation.

The comments made above are consistent with teachers' more general views of the curriculum materials they were using in the course. Nicole's description of the lack of explanation in the *Reflections on Number* investigation is suggestive of many more extensive comments and complaints about the lack of explanations in the curriculum materials. For instance, Meg described *Reflections on Number* as "more of a pamphlet than a textbook" because

> . . . it goes through different topics as a textbook would, like the number zero or Pascal's triangle, but a typical textbook has a lesson and a group of questions. What we have is similar but it doesn't necessarily teach what we're learning.

In Meg's view, the curriculum materials "show different methods and problems, but not as much of what you would see in an instructional textbook. It's more questions and a lot of explanations required. They'll ask questions and then 'Why? Why this? or What could be different?'" In other words, although the curriculum materials sometimes presented mathematical information, that information was usually followed by questions and problems that demanded extensive analysis or use of the information.

For many of the preservice teachers, the unfamiliar format of the *Standards*-based materials was problematic. As Jessica explained, examples and explanations are important elements of textbooks for her and for children learning mathematics:

> I like to have an explanation and then maybe an example or two and then some problems.
> And I think that's beneficial for elementary school children, or any student, because if there's any confusion when they get to the problems, they can always convert back to the examples and see how the example was done, which gives them an idea of how to do it.

For Jessica, the materials' lack of explanation contributed to feelings of frustration at times: "Some of the books just say 'do this problem' and you don't really have any basis for what you're doing, or where to start." Her desire for the curriculum materials to provide the explanations and examples to which she was accustomed was a determining factor in her general preference for CMP materials over MiC materials. In her view, the CMP materials provided more information to the student. For instance, comparing CMP's *Prime Time* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998) and MiC's (1998) *Reflections on Number*, Jessica explained:

> If I were a teacher, I would use *Prime Time*. This book is a more beneficial book for children to learn from. It uses great explanations with important words bolded so the students can easily understand what it is they will be doing in each lesson. After explanations there are problems the students solve using the method explained at the beginning. *Prime Time* also has the students explain their methods of acquiring their answers, but the book does not concentrate mainly on explanations like *Reflections on Number* does. *Prime Time* gives the students examples of problems they will be working on in their problems. *Reflections on Number* doesn't do a very good job of giving examples for the students to follow.

These comments suggest that Jessica (and likely other teachers in the class) held views about authority (namely that the instructor and/or the textbook should tell students the information that they need to know) that shaped her experiences using the curriculum materials as a learner and will likely impact her use of curriculum as a future teacher.

Concluding Remarks and Questions

Many preservice teachers approach their teacher education courses expecting to be told the mathematical subject matter they should know as well as how to teach that subject matter to children (Cooney, Shealy, Arvold, 1998). As the examples of this report illustrate, such expectations can influence preservice teachers' ability and inclination to take responsibility for mathematical sense-making in the teacher education classroom. These examples contribute to a better understanding of factors that may inhibit preservice teachers' appreciation of the innovative approaches to mathematics and pedagogy presented in *Standards*-based curriculum materials. An interesting question emerging from this report is: How might teacher educators *capitalize on* the opportunities created by curriculum materials' ability to bring authority issues to the forefront of preservice teachers' learning experiences? That is, how might preservice teachers' struggles and frustrations related to authority be used in an educative manner?

The illustrations of this report also raise questions about preservice teachers' development of *pedagogical authority*, namely "the strength of [teachers'] own voices and conceptions for determining classroom activities and content" (Wilson & Lloyd, 2000, p. 151). As preservice teachers use *Standards*-based curriculum materials for the learning of mathematics, what lessons are they learning about pedagogy? As they struggle with issues of mathematical authority, what issues of pedagogical authority are also stirred up? Because instructional materials of any sort – traditional or reform-based – offer limited pedagogical information, teachers must learn to make informed, context-dependent decisions about how to use curricular recommendations in the design of instruction. Does experiencing mathematics as a humanistic endeavor directed toward sense-making influence preservice teachers' inclination to view curriculum as a resource to be adapted to the specific situation of one's own classroom? Consideration of these questions about the interplay between authority and instructional materials has potential to greatly inform the work of researchers, curriculum developers, teacher educators, and those concerned with the professional development of teachers.

Endnote

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CASE STUDIES OF FUTURE SECONDARY LEVEL MATHEMATICS TEACHERS' MODE OF EXPLAINING

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This research report, part of a larger study (Proulx, 2003), elaborates on a particular framework construed to analyze mathematical oral explanations given in the mathematics classroom. Originating from a lack of information on the characteristics of future mathematics teachers' oral explanations, this study aimed to understand and make better sense of the oral explanations of five future mathematics teachers in their classroom practicum. Different types of explaining are highlighted in which some focused mostly on: (a) a technical and repetitive approach, (b) a logical and precise step-by-step approach, (c) an approach in which a conceptual gap was present between abstract and contextualized ways of explaining, (d) a 'revoicing' and clarifying of students' answers, and so on. The research shows that a rich sketch can be drawn from the analysis of teachers' ways of 'speaking mathematically'.

Questioning and Objectives

Numerous studies show the importance of teachers' oral explanations in classrooms with respect to the mathematical understanding of students and in the creation of links between notions (Ball, 1991; Mopondi, 1995). These explanations appear central in the establishment of a mathematical culture in the classroom and have an effect on the way in which errors are managed (Ball, 1991; Bauersfeld, 1994), the way of arguing and negotiating meanings (Voigt, 1994; Cobb & Yackel, 1998), and the way in which students make sense of mathematical activity (Ball, 1991). The analysis of the difficulties encountered by future mathematics teachers in 'speaking mathematically' (Bauersfeld, 1994; Bednarz, 2001) in class – for example, the difficulties in using and imposing a rigid, formal and technical language of mathematics, in contextualizing the mathematics in everyday life settings, in using everyday words to talk about and explain mathematical reasoning and notions, or the difficulty in using metaphors and analogies – have made us want to know more regarding the construing of those skills, in regards to oral explanations in future mathematics teachers.

This research aims to better understand the nature of oral mathematical explanations developed in action by future mathematics teachers: What are the characteristics of oral explanations of future mathematics teachers when they teach in a classroom?

Theoretical Clarifications

Mathematical discourse can be characterized as any attempt to communicate, interact, reflect, and render explicit content related to mathematics. This same discourse can be seen under two main aspects: public discourse and private discourse. Public discourse can be described as an interaction or the implied interaction of a minimum of two persons (that is, a speaker and a listener). As for private discourse, it is mostly personal reflections and internal mathematical thoughts (Sfard, 2001). Obviously, those different aspects of discourse however share the same intents: to clarify, explore, explicate, render explicit, reason, and make understandable the concepts under study. Our study, being situated at the level of future mathematics teachers and their classrooms, focuses on the first aspect, that is, the public mathematical discourse. This public mathematical discourse can also take many forms: gestures, words, images, writings, and so on. The research here will attempt to describe the public mathematical discourse at the level of the future mathematics teacher's oral explanations in the classroom, that is, the usage of words, sentences and phrasings used to communicate mathematics and to render explicit a notion, a particular reasoning, a mathematical phenomenon, and a mathematical activity to the students.

Analytical Framework

To analyse future mathematics teachers' oral explanations in the classroom, we have construed an analysis grid to account for many possible dimensions of practice with respect to the oral explanations given. Ten elements were highlighted in this analytical framework: The role and the place of students in the explanations given (e.g., Is the teacher articulating his or her explanations in relation to those of the students? Is he or she interacting with them or is he or she the only one speaking? Etc.); the sort of questioning (e.g., open questions, directive questions leading to predetermined answers, questions to test skills, questions to open discussions, etc.); the nature of the oral explanations (e.g., centered on the construal of meaning and reasoning, centered on techniques and know-how, centered on the connections between concepts, instrumental or relational (e.g., Skemp, 1978); openness to different answers and student strategies (e.g., explanations open to diverse strategies, explanations leading to a unique answer and way of doing, explanations focusing on the reasons underpinning a strategy, etc.); the creation of links between concepts; the type of language used (e.g., usage of metaphors, usage of everyday language, usage of a technical language and precise vocabulary, etc.); flexibility in oral explanations (e.g., repetition of the same explanation many times, usage of different examples and different levels of language, presence of 'revoicing' (Forman & Ansell, 2001), etc.); the presence of mathematical verbalization[1]; the status of the oral explanations in the teaching (e.g., words used seen as support for reasoning, as facilitators to understanding, as the precise objects of study, etc.); the mathematical validity of the oral explanations (presence of errors in explanations). It is to be noted that the last two categories will not be reported on in this research report, mostly because in this situation they do not bring crucial information to the cases analyzed.

Methodology

A multicase study (Karsenti & Demers, 2000) was conducted with five future teachers doing their second practicum in their second year of a four-year teacher education program for mathematics teachers at the Université du Québec à Montréal (UQÀM). The choice of participants was made on the basis of their subject of teaching. We tried to have subject-areas that had been seen by participants in their teacher education programs, some that had been partially seen, and others that had not yet been seen in their teacher education program[2]. To answer our research questions (as in p. 4) we were able to obtain three videotaped lessons for each future teacher from different moments in their classroom practicum. Those videotaped lessons were then precisely analyzed in relation to the analytical grid/framework, presented in the previous lines, to draw a general sketch of their professional and teaching tendencies with respect to their oral explanations. With this detailed grid, each part of the discourse was coded. It is obviously possible for the same part of the oral discourse to be coded in diverse categories (that is, giving information for different categories in the grid).

This study report is part of a wider study (Proulx, 2003) in which individual interviews were also conducted to better understand the rationales underpinning the decisions taken in their classrooms and their planning with regard to the oral explanations given in their classrooms. That part of the study will be reported on another occasion. Other information and intricacies regarding to the whole study with respect to the case studies and the interrelation of the oral explanations and the dynamics underpinning the rationales can be obtained in Proulx (2003, 2004a, 2004b).

Results

In an attempt to give a sense of the way in which the analysis was conducted, we describe in detail the analysis of one case (Bertrand) and present a synthesis of the other four cases.

Also, it is important to note that the intention here is not to judge or give a critique of the ways in which these future teachers spoke or taught. The main intent is to 'better understand' the practice of classroom oral explanations in future mathematics teachers. So, we attempt to describe their oral explanations; be they good or bad, it is the readers' responsibility to carry out that judgement and not ours as researchers.

The Case of Bertrand

Bertrand was teaching linear relations during his practicum, and this subject-area was partially worked on during his teacher education program.

The role and the place of students in the oral explanations given

Bertrand was in constant interaction with his students in the sense that he was always discussing with them, was always questioning them and was bringing them into his discourse. Here is an example:

Bertrand: So, we have Marielle who has a field of strawberries, ok? So, right away, if we link them in relation, here what would we make in relation? […] Nathalie? What would we make in relation here?

Student: [not understandable]

Bertrand: The number of persons and the time to pick up the strawberries. In your opinion, which one of the two variables will be the independent one here?

Student: [not understandable] the number of persons.

Bertrand: The number of persons. Is it logical that it is the number of persons?

Students: Yes.

The sort of questioning

We saw that Bertrand was mostly using a directive sort of questioning that was in some ways guiding the students towards the wanted/expected answer. He was often using a step by step questioning that was giving more and more clues to the students in relation to the expected answer. This strategy is closely linked to Brousseau's (1998) 'Topaze effect' or Bauersfeld's (1978) 'funnel approach' that describes a teacher's strategy in guiding the students. Here is an example:

[In the context of the construction of a house where it was asked how much time it would take to build 80 houses in relation to the number of construction workers. This context was represented by the formula y=80/x. The students' task was to find the type of relation that was represented by this context and that rule.]

Bertrand: Ginette, where will you write that? […] Do you have some idea?

Student: No.

Bertrand: No, you really do not have an idea. If you look at the formula, does it makes you think of a rule that you remember?

Student: [makes a sign with her head that says no.]

Bertrand: No. You would not have an example if for example you had the right to look at the marked homework of a moment ago? What did we had? Did we have the number of pick ups, the number of pickers, and the time to pick the strawberries? Does this one looks like this?

Student: Yes.

Bertrand: If we had the number of construction workers, and the time taken to build the house, does this has a particular meaning.

Student: Yes.

The nature of the oral explanations

We saw that Bertrand focused a great deal on the application of techniques and step-by-step procedures to be followed by students. He seldom ventured into a deep conceptual level of discussion regarding the understanding of mathematical notions; he mostly stayed at an *instrumental* level (Skemp, 1978), he seldom explained the 'why' of things. Here is an example:

Bertrand: [answering a student who said that she did not understand and gave him the answer she had obtained]. 0,625. Ok, what is your 'y' variation? What did you do?

Student: Well…

Bertrand: What seems to be your y2 minus your y1?

Student: Well, it gave me 5 over minus 8.

Bertrand: Ok, but your variation of 'y'? Have you really done your 25 minus your 17? Write it to yourself: y2 minus y1 over x2 minus x1. You really have to write it down, you know? You have to write down what it gives you. 25 minus 17 over 1995 minus 1990. Then, you write your answer.

The openness to different answers and student strategies

Bertrand was quite close-minded concerning the things to do and the way to answer: he had precise expectations. He was very insistent on the format and the type of answer to be given, what he called 'the good way to do it'. Here is an example:

[The question consisted of finding the value of the side of a square for which the area is 2500. The students were working with the formula $A=c^2$ (in which A was the value of the area and c was the value of side of the square).]

Student: 50.

Bertrand: What did you do to obtain that? What did you do? We always start, when we have a question like this [interrupted]

Student: I did the square root.

Bertrand: You did the square root, that's good. However, do not forget that you always have to start with your initial rule. Ok, you start with the rule that was $A=c^2$. Then [he wrote the formula on the board], that is the calculation that needs to be seen on your sheet. Ok? Always, when we ask you a question in which you have to find the independent value or the dependant value. You always have to write down your rule, the values that goes within, and show all the steps of the solution to get to the answer.

Creation of links between concepts

We did not find any particular passages on this issue.

Type of language used

Bertrand was quite rigid concerning the vocabulary he used. Bauersfeld (1994) calls that a celebration of the technical language. A very formal and 'mathematical' language is imposed by his oral explanations. Here is an example:

[It is a context describing a situation involving an inverse proportional relation. The question asked is to recall what type of variation it is. However, the discussion is now focused on its name.]

Bertrand:	Right away, does it makes you, it makes you think of what type of relation?
Student:	Not the same sense.
Bertrand:	Is that a variation's name 'not the same sense'?
Student1:	[laughs] In the contrary sense.
Student2:	Indirect! [not understood by Bertrand]
Bertrand:	Is that one also a name?
Student1:	Indirect!
Bertrand:	No. We have the direct one, the partial one, the null one, the second degree one. The one we are looking for is the inverse variation.
Student1:	Indirect and inverse is the same thing.
Student2:	It is the same thing!
Bertrand:	Ha, you have to take the inverse. You take the inverse [not understandable], we have to use the same names [in the interview for the whole study, Bertrand explained that he was insisting on the vocabulary because for him you have to know the vocabulary to really understand the concept related].

Flexibility in oral explanations

Bertrand mostly re-explained in the same way. He always had the same formula to follow, the same words and often the same examples and contexts to make sense of the concepts.

The presence of mathematical verbalizations

We did not really note specific mathematical verbalizations, the focus was much more on the vocabulary and the formulas.

Conclusions regarding the case of Bertrand

Bertrand interacted a great deal with the students in his classroom. However, he seemed to be rigid, precise, closed, and predetermined in his oral explanations. His technique of questioning was quite directive, and opted for procedural explanations based on repeated techniques to be followed step-by-step, insisting on the precise vocabulary to support it. As suggested by Skemp (1978), we would say that his teaching was mostly centered around instrumental understanding.

The Case of Donna

Donna was teaching algebraic operations, a subject-area that had been seen rather completely in her teacher education program. Before beginning a description of Donna's discourse, it is important to note the structure of her lessons. They were divided into three parts: a short part focused on the previous given homework or a short test; a second part centered on mental arithmetic (linked to algebraic operations, for example the distributivity law) and involving a series of rectangles to discover their area (with numbers or algebraic variables); and the third part, which was used as a conclusion, centered on the formula to be used to carry out algebraic operations (e.g., binomial multiplication). We will refer to those parts (1, 2, 3) in the descriptions that follow; we will also use the word 'rule' to describe a procedure to be followed in carrying out an operation – since it is Donna's own word.

The role and the place of students in the oral explanations given

Donna was in constant interaction with her students and was always asking them to interact in the classroom to give their ideas and answers. In addition to responding to different sorts of questions, students had to explain their solutions to problems and justify their strategies.

The sort of questioning

Donna used many open questions that were focused on students' explanations; those questions were aimed at the explications of strategies by the students.

The nature of the oral explanations

Donna's oral explanations were divided into two types. The first type was focused on mathematical reasoning, the concepts and the construal of meaning. That type was used in the second part of the lessons (mental arithmetic and areas). The second type of explaining was centered on the detailed explications of the rule and the procedures to follow to carryout the algebraic operation (parts 1 and 3 of the lesson). There was a radical change in her way of explaining, depending on the part of the lesson: Part 2 which was aimed at the construal of reasoning and conceptual meaning (we call this part 'BEFORE-the-rule') versus Parts 1 and 3 which were aimed at the establishment of a rule to follow in finding the solutions to those same problems (we call this part 'AFTER-the-rule'). In the interview, she explained that one of her goals was to automatize those procedures.

The openness to different answers and student strategies

In the BEFORE-the-rule part, Donna was creating an environment open to different answers and solutions. In that part, students were asked to propose solutions, explain their thoughts and judge each other's answers and suggest alternatives, if needed. Concerning the AFTER-the-rule part, we do not have clear clues concerning her openness to answers, except that she was always using and asking for the rule to be followed.

Creation of links between concepts

Here, since she was starting from mental arithmetic linked to specific algebraic operations, then going to the calculation of areas still linked to algebraic operations and then ending with a rule to solve those (normally a sequential step-by-step rule explaining which terms to multiply or divide – a procedure), she was making links between those concepts to get to the next one. However, whereas in the beginning those links were not always clear, she made them more and more clear as the practicum advanced.

Type of language used

Donna used everyday language in her explanations. Also, she varied her explanations, and so she had different ways of talking about the mathematics.

Flexibility in oral explanations

When she had to re-explain to a student who did not understand, she used different ways to attempt to help the student make sense of the mathematics: she changed and adapted her explanations. She also often used 'revoicing' (Forman & Ansell, 2001) of students' explanations. However, those tendencies were only present in the BEFORE-the-rule part.

The presence of mathematical verbalizations

We witnessed mathematical verbalizations in her way of talking about algebraic operations (in the BEFORE-the-rule part), for example: [A student offered the solution "$9 \cdot 12 - 1(12 - x)$" to find the area of the rectangle shown in figure 1. Donna offered to expand that answer to make it equivalent to another answer given by another student: "$8 \cdot 12 + x$". She arrived at "$9 \cdot 12 - 1 \cdot 12 + x$".] Donna: Look, we take 9 twelve times, it means that we repeat 12 nine times and then we subtract 12. So, it is, we repeat nine times minus 1. We repeat it eight times.

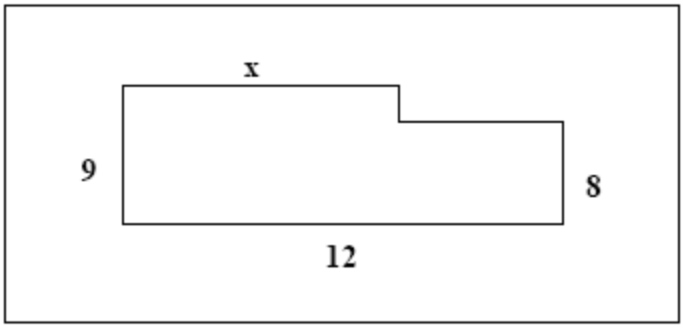

Figure 1 : Area problem given by Donna in her class.

Conclusions regarding the case of Donna

Donna's students were really active and they frequently asked to participate. Concerning her oral explanations, it is important to note the rupture between the two parts of her lessons (BEFORE and AFTER the rule). Whereas openness and construing of sense and mathematical meaning were central in the former, rigidity and a procedural-step-by-step approach dominated the latter. A focus on the rule became central in the latter – the application dominated[3].

The Case of Albert

Albert was teaching algebraic factorization, a subject-area partially worked on in his teacher education program.

The role and the place of students in the oral explanations given and The sort of questioning

Students in Albert's classroom were often questioned and solicited, however it was mostly to answer procedural and technical know-how questions.

The nature of the oral explanations

Albert's oral explanations were mainly focused on techniques and the procedures to be followed in a step-by-step logical patterning of deduction. The explanations were mostly instrumental (Skemp, 1978) in that they focused on know-how and not on 'why'. This explanations were, however, precise and clear and, as mentioned, followed a clear logical sequence based on deduction (architectural model).

The openness to different answers and student strategies

Albert clearly asked for and expected a particular type and model of answer. He even asked for precise chronological steps in factorizing (e.g., take out a common factor was mandatory as the first step, whenever possible). He also rejected adequate mathematical answers that were not based on his proposed model of doing.

Creation of links between concepts

Albert created a lot of links between the different possible factorizing situations that could occur. He then linked the different strategies in an attempt to create a deductive link between them (taking out a common factor, factoring by grouping that involves taking out a common factor, difference of two squares using those previous ones, etc.). This was his way of simplifying the newly learned method – by showing that they were only some steps further than the strategies they already knew.

Type of language used

The fact that he was using the particular mathematical language meant that he was not imposing it on his students.

Flexibility in oral explanations

Some of Albert's explanations seemed to be pre-planned in advance and those were repeated and repeated. However, some other explanations seemed more natural and on-the-spot; here, different ways of explaining were used to make sense of certain notions. He also did some 'revoicing' of students' answers (however, those were revoiced following the expected format that he was imposing).

The presence of mathematical verbalizations

No particular verbalizations were observed.

Conclusions regarding the case of Albert

Albert's explanations were mainly centered on know-how concerning procedures to follow in factorizing certain algebraic expressions. That pattern was however based on 'deductive-logical' explanations that followed each other stepwise. Albert also showed some flexibility when he had to answer on-the-spot to students' questions, but also framed his answers in a particular way[4].

The Case of Carl

Carl was teaching analytical geometry, a subject-area not worked on during his teacher education program.

The role and the place of students in the oral explanations given and The sort of questioning

Carl frequently solicited students' input in his classroom to answer technical questions, however, mostly, when he explained something, he did all the talking – he "took a lot of space" in his classroom.

The nature of the oral explanations

Carl's oral explanations were mostly focused on the precise algebraic explication of the procedures to be followed in solving problems. Everything was mostly centered around the algebraic solution to the analytic geometry problems and focused on algebraic manipulation.

The openness to different answers and student strategies

Carl seemed to have precise expectations in regard to the procedures to be followed in solving problems and exercises (with algebraic manipulations). He was also rigid concerning the way to represent the answers to problems (e.g., he would impose a fractional representation in opposition to a decimal one); however, as the practicum advanced, he became much more flexible with respect to answers.

Creation of links between concepts

Carl created many links between previously seen notions and the ones he was teaching. However interested in the links, he would not go into detail to re-explain those concepts if they were not understood by the students (he would tell them that they were supposed to be mastered).

Type of language used

Carl was not really rigid concerning technical language, but he would on occasion remind them of the proper mathematical words.

Flexibility in oral explanations

Since most of his explanations were centered on algebraic manipulations, when students had questions he would go slower and in more precise detail to describe the steps used to solve. However, on some occasions, he would use counter-examples, numerical examples and visual examples to give a sense and to explain what he was describing.

The presence of mathematical verbalizations

We did not really note specific mathematical verbalizations.

Conclusions regarding the case of Carl

Situated in a setting where Carl took most of the place, and mostly focused on algebraic manipulations, Carl's oral explanations were mainly technical, but precise. He also showed that he was expecting a specific format of answer (but softened as the practicum went on). It is also important to note that he explained precisely the algebraic manipulations but he explained quite rapidly as long as nobody was asking questions or clarifications – in which case he would slowdown the pace to explain. In the same way, he refused to re-explain notions that were supposed to be already known.

The Case of Enrico

Enrico was teaching integers, decimals and fractions, subject-area that had been worked on in his teacher education program. Before describing Enrico's oral explanations, it is important to situate them in the climate of the classroom. Enrico's classroom was rooted in a culture of sharing and co-construing meaning. Enrico did not talk much in his class; it was mostly the students that did all the talking: they offered, validated, explained, convinced and questioned answers. What Enrico did most was 'revoicing' his students' ideas and clarifying them. In a very generalized way, in Enrico's classroom, if the students did not talk, nothing happened. In the same way, since this climate and culture was established by Enrico's actions and open questions, if Enrico was not there to make it happen, nothing happened. Finally, central to his teaching strategy was Enrico's way of trying to perturbate or desequilibrate his students in their understanding by questioning them to force a reflection to make them sure of and question their assumptions and knowledge.

The role and the place of students in the oral explanations given and The sort of questioning

Enrico's students were constantly solicited and asked to give answers. They were constantly triggered by Enrico to probe and answer open questions and conceptual problems; they were asked to reflect on problems and answers. When they gave an answer to a problem, Enrico would always ask: 'Why?'. He would also ask questions that would make the students doubt their answers, so that they would try to convince others of the validity of their answer or to explain if a particular answer was good (he would sometimes offer wrong answers). Students would also have to answer other students' questions that were first aimed at Enrico, but that he re-asked of students instead of responding directly to them. As already mentioned, Enrico did not talk much in his class; the students were doing all the talking.

The nature of the oral explanations

Two types of explanations existed in Enrico's teaching. The first type concerns the reformulation of students' answers to problems. This represented most of Enrico's oral explanations. He would repeat them, clarify them, correct them if needed, adapt them and make them accessible to the other students and to what was being taught at the moment, contextualize them, make them more precise, add important aspects to complete them, and make them legitimate in the eyes of everybody.

The openness to different answers and student strategies

Enrico was open to all possibilities for approaches, right or wrong. However, the students would have to explain them. He tried to make the possible solutions emerge in an attempt that the students would validate them. He would sometimes even offer wrong answers to create that situation.

Creation of links between concepts

Enrico did not explicitly create links between notions; however he would sometimes link notions that had already been seen (or that would be seen in the future) to the new ones.

Type of language used

Enrico used many different ways of talking and passed from one to the other: he worked with mathematical terms, linked them with everyday language and placed them in different contexts. We also felt that he discussed with students in some kind of negotiation. He used a language that was adapted to them, and spoke slowly.

Flexibility in oral explanations

By using different ways of speaking, and working with many different solutions to problems, he showed great flexibility in his oral explanations. It was not rare that he would use up to three different ways to explain something.

The presence of mathematical verbalizations

By his constant contextualization and usage of different ways of talking about the mathematics to make it clearer, Enrico interlinked mathematical verbalizations to his oral explanations. His flexibility and his variable sort of language created those verbalizations.

Conclusions regarding the case of Enrico

Enrico's oral explanations took all their meaning in the culture that he had established in his classroom with his students. There was a climate that brought students to negotiate, justify, explicate, render explicit, convince, reflect and analyze the mathematics. Enrico's oral explanations were mainly reformulations of his students' answers and reflections, and those took their source in Enrico's comments and questions that brought students to reflect.

Conclusions

This analysis brought forth many different ways of 'speaking mathematics' in the classroom by future teachers: some oral explanations were seen to be very technical and repetitive by focusing on procedural explanations, techniques, and precise vocabulary; some oral explanations were mostly characterized by a rupture or an opposition between the construal of meaning and mathematical reasoning, and the application and generalization of a specific rule; some other were structured around a logical-deductive pattern that followed clear mandatory steps; others implied mostly precise technical algebraic manipulations to explain how to get to the answers; and the last implied mainly reformulations and 'revoicing' in diverse flexible ways to establish a climate and a culture of mathematical reflection in which students shared their thoughts and understandings. Those five different cases, that seemed on some aspects quite different from each other and not at all aimed toward a generalization of the exhaustive possibilities, give us a lot of information on the possible characteristics of future mathematics teachers' oral explanations in classroom situations.

It is also interesting to note that even though these future teachers took the same courses and followed the same program (except Donna on some aspects), they ended up with very different ways of 'speaking mathematics'. This can however be something we already knew. It suggests nonetheless that a lot of other factors are implied in the process of creating a teacher's identity, and this is mostly what was analyzed and rendered explicit in the larger study of Proulx (2003) that informed this research report.

Finally, the study gave us much more information than only the part concerning oral explanations. We were also able to learn a great deal on aspects and features concerning their whole practice as teachers (culture, mathematical difficulties, structure of lessons, choices, attitudes, conceptual gaps created, etc.). For us, focusing on oral explanations – their ways to ‘speak’ their subject-area – represented probably the richest way to grasp fundamental information on and to make better sense of the way in which teachers’ practices are manifested. As Stubbs (1976) clearly said: “There is a sense in which, in our culture, teaching *is* talking” (p. 17, italics in the original).

Endnotes

1. Mathematical verbalizations could be described as the expression, the elaboration and the explication, by means usage of everyday language, of notions and mathematical activities, in an attempt to give them meaning [bring forth, make emerge, construe meaning] – meaning that can be external to it – to contextualize, render concrete and accessible the concepts under study, and/or to bring forth and make emerge a key reasoning and its underpinnings.
2. At the beginning, we were aiming for six future teachers, and that would have given us two participants for each category (subject-areas seen, partially seen, and not seen). Unfortunately, some problems concerning the videos occurred with the sixth participant and we had to delete this participant from the study. Therefore, two participants were in the ‘seen’ category, two in the ‘partially seen’ and one in the ‘not seen’ category.
3. For more precise information on Donna’s case, see Proulx (2004b).
4. For more precise information on Albert’s case, see Proulx (2004a).

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PRESERVICE ELEMENTARY TEACHERS' CONCEPTIONS OF MULTIDIGIT WHOLE NUMBERS: WHY IS UNDERSTANDING REGROUPING SO DIFFICULT?

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To understand *why* algorithms work, PSTs must have good understandings of both multidigit whole number and ways we treat multidigit whole numbers in algorithmic situations. In, for example, 527, the reference for the entire number is understood to be ones, so 527 is thought of as 527 *ones*. Each digit name also refers to ones: 500 ones, 20 ones, and 7 ones. However, when working with algorithms, we view a number in terms of the reference units for each place value (ones, tens, hundreds, etc.); 527 is seen as 5 hundreds, 2 tens, and 7 ones. We can, however, behave as if each column is ones, because within a column, we operate on units of the same type. Because of the underlying structure of the base-ten power sequence on which our numbers are built, each reference unit is 10 times as large as the next lower reference unit; thus, we can regroup one unit of larger size into 10 units of the next smaller size and vice versa.

Few researchers (for example: Ball, 1988; Ma, 1999) have examined PSTs' understanding of multidigit whole numbers. Most in-depth research on understanding multidigit whole numbers has been conducted with children (for example: Fuson et al., 1997; Kamii, 1994) and has been focused on 2-digit numbers and the relationship between tens and ones. Understanding 2-digit numbers requires one to rapidly shift between seeing a ten as 1 unit (a ten) and as a collection of 10 single units (10 ones).

Mode of inquiry

The data analyzed are drawn from two 75-minute semistructured interviews with each of 15 PSTs at a large, urban, state university. Each interview was transcribed and analyzed, and Interview 1 served as a basis for Interview 2.

Results and discussion

In the context of 3-digit numbers (527 – 135, see Figure 1a), all 15 PSTs in the study knew that regrouping did not change the value of the number, but 66.6 % were unable to explain the reason. Two of these 10 PSTs considered each digit as representing ones only (concatenated digits) and applied rules they learned in school. Carmen, unable to connect the 1 *one* she "borrowed" to the 10 *ones* she added in the next place when regrouping, said, "I don't get how the 1 can become a 10. … Where did the other 9 come from?" The other 8 of these 10 PSTs were aware that a digit in the hundred's place represented hundreds but viewed the regrouped digits that now resided in the ten's place as representing ones instead of tens. In the context of solving 527 – 135 (thus regrouping 527 to 4[^{1}2]7), these PSTs knew that they took 100 from the hundred's place but saw the digits in the ten's place (12) as representing 12 *ones* instead of 12 *tens* or 120. Thus they struggled to explain how the regrouped and original numbers had the same values. Delia realized that the 1 borrowed from the hundred's place was 100, but she said, "Once you put it into the number, it becomes a ten." All PSTs in the study knew that 10 tens make 100, but those who saw the digit in the ten's place as representing ones were unable to draw on that fact. To understand number, one must see each digit as representing a different unit type (ones, tens, hundreds, …), and, in the context of the algorithm, these 8 PSTs did not see the digit in the ten's place as representing tens.

Of the 5 PSTs who could explain why the regrouped and original numbers had the same value, 3 said that they regrouped 1 hundred into 10 tens. Rachel, for example, stated that she was “taking a hundred and making it into … 10 groups of ten.” The remaining 2 PSTs explained that they regrouped 100 ones, which they combined with the 20 ones to get 120 in the ten’s place. Although this mathematically correct explanation suffices to explain the subtraction (120 – 30), these PSTs struggled to explain the 12 in the ten’s place in the algorithm. Melissa explained that it “becomes 120, but I don’t know why it is that you don’t see the 0 [in 120].” To explain the 12 in the ten’s place requires changing the reference unit from (120) ones to (12) tens, a shift that is not obvious. The term *hundred* refers to both a collection of 100 ones and the reference unit *hundred,* but to connect hundreds to tens, the PST must change the reference unit *explicitly* to tens.

Recall that flexibility in seeing a ten as 10 ones and as 1 ten was important for understanding 2-digit numbers (see Figure 1b). In this special case of regrouping, the next lower reference unit is *ones*. Extension to 3-digit numbers adds another level of complexity. In addition to seeing a unit in terms of itself (i.e., 1 hundred) and in terms of ones (i.e., 100 ones), one now needs to add a third component, namely seeing the unit in terms of the next lower reference unit (i.e., 10 tens for 1 hundred, see Figure 1c).

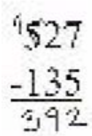

Figure 1a. Standard algorithm solution for 527 – 135

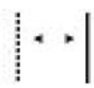

Figure 1b. Base-ten relationships for *ten*

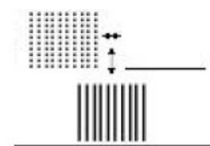

Figure 1c. Base-ten relationships for a hundred

Conclusions and Implications

To help a child make sense of algorithms with regrouping, teachers themselves must understand regrouping, being aware of each digit’s reference unit and the relationships among reference units to reconceive one reference unit in terms of another. In this study, only 3 of the 15 PSTs interviewed spelled out a shift in reference unit to explain regrouping between hundreds and tens. The other 12 showed no evidence of reconceiving the hundred as 10 tens, or vice versa, in these contexts, even though all demonstrated knowing that 10 tens make 1 hundred.

Regrouping between tens and ones is a simplified case of general regrouping, because the next lower reference unit for tens is ones and we are accustomed to thinking in terms of ones. When we discuss regrouping in the classroom, we should be aware of the ideas underlying regrouping and discuss regroupings between adjacent units other than ones and tens.

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EXAMINING THE PERCEPTIONS AND QUALITY OF ALTERNATIVELY PREPARED TEACHERS

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Purpose

An ongoing debate in this country is whether or not non-traditional approaches to teacher education are producing high quality teachers (Darling-Hammond, 2003). Investigations into the quality of teacher education programs have led to the listing of criteria for high quality preparation programs (national commission of teaching and America's Future, 2003). Additionally, research has shown that teachers' perceptions' of their preparation are connected to teacher quality (Darling-Hammond, Chung, & Frelow, 2002). This study is one component within a longitudinal study that is designed to investigate the quality of teachers prepared through an alternative program known as the teacher education environment in mathematics and science (TEEMS) program.

The TEEMS program began in 1994 as a non-traditional approach to mathematics and science teacher education for grades seven through 12. The program is benchmarked by the ten Principles of the Interstate New Teacher Assessment and Support Consortium (INTASC). Embedded throughout the program are the standards of the National Council of Teachers of Mathematics (NCTM) and the National Educational Technology Standards (NETS). The design of the program is based on constructivism (von Glasersfeld, 1989) and the work of Shulman on the Stanford Knowledge Growth in Teaching Project (Shulman, 1987).

In this study we share the results from the data collected during the teachers' first year of teaching and discuss the current progress of the study. Our research question is: How do first-year TEEMS teachers perceive the impact of their practice on student achievement?

Theoretical Framework

The major focus of this study was to investigate the differences and commonalities in the perceptions of TEEMS teachers and how these characteristics affected their perceptions about their impact on student achievement. For this study we used a storytelling approach to examine first year TEEMS teachers' perceptions of their impact on student achievement. The researchers examined the perceptions of student teachers through a storytelling approach. Hansen and Kahnweiler (1993) state that stories are unique and powerful in that they contain implicit morals, which in turn reflect the belief systems of the storyteller(s) and that stories are a natural vehicle for telling one's perceptions of past events. The authors also state that "…stories permit researchers to examine perceptions that are often filtered, denied, or not in subjects' consciousness during traditional interviews" (p. 1394).

Methods of Inquiry

Storytelling

We developed an interview protocol that was used to prompt the teachers to tell a story. The research team conducted individual interviews. During the interview the teachers were informed that we were seeking to understand their perceptions of their impact on student achievement in mathematics and the way in which they define success in the mathematics classroom. We asked the teachers to think of their stories as simply relating an incident that was interesting about mathematics teaching and learning. The emphasis of the storytelling interview was to obtain the teachers' narratives or accounts in the teachers' own terms. The stories were audiotaped, transcribed, and coded for analysis.

Participants

Primary participants for the study were three TEEMS teachers who completed their certification requirements in the spring of 2003. The storytelling data was collected during the spring semester of 2004. The primary participants represented a mix of genders (one woman and two men), content area backgrounds (degrees in business, engineering, and mathematics), age groups (from 23 – 35), and ethnic backgrounds (two European Americans, one African American). All participants were in their first year of teaching since completing the certification requirements of TEEMS.

Results and Conclusions

Significant statements were taken from the stories (words, phrases and sentences that pertain to perceptions) and meanings were formed. The teachers, after completing the TEEMS program, were found to be confident in their ability to impact student achievement by focusing on motivation, learning styles, and attitude. The three TEEMS teachers not only think, believe, or have opinions, but they ***know*** they have made an impact on their students' achievement. A common theme across the teachers' stories was the focus on affective dimension of teaching. Each of the teachers described a story in which she or he was concerned with the student's disposition in the classroom. Another significant finding in the two of the stories was inconsistencies between teacher perceptions and teacher actions. For example, one of the teachers stated that he worked with a student to change the student's learning style for the purpose of improving the student's performance in Algebra II. However, the teacher's action in the story revealed that the teacher invested time in working with the student to identify the student's optimal learning style and to maximize the student's opportunities to apply his learning style in Algebra II class. Therefore, the teacher made accommodations for the student's learning style as opposed to trying to change the student's learning style. Common themes with respect to the principles of INTASC emerged through further analysis and interpretation of the data. Across the three teachers' stories, we found several INTASC principles that were more prevalent than others. The principles that were prevalent across the three stories focused on: student development; diverse learners; reflective practice and professional growth; and school and community development.

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USING STANDARDS-BASED MATHEMATICS CURRICULA TO BETTER PREPARE ELEMENTARY SCHOOL MATHEMATICS TEACHERS

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Done within the framework of the "scholarship of teaching" (Shulman, 1999), the action research project presented in this paper involved a mathematics education methods instructor examining the effectiveness of using standards-based mathematics curricula within her elementary school mathematics methods course. Specifically, the author analyzes the impact of using these curricula on the quality of preservice teachers' work and reflections.

Introduction

The work described in this paper is part of the research domain known as "the scholarship of teaching." According to Shulman (1999), "A scholarship of teaching… requires… faculty [to] frame and systematically investigate questions related to student learning—the conditions under which it occurs, what it looks like, how to deepen it…" (p. 13). Thus, in this paper, I examine my own attempts at preparing elementary school teachers to teach mathematics "for understanding" (Ball, 1994) as a means to investigate my students' learning.

One of the major goals of my mathematics methods course is for preservice teachers to develop an understanding of how to teach conceptually-based mathematics. Traditional mathematics instruction has typically emphasized memorization, drill, and skills. Because preservice teachers have generally experienced this traditional instruction, they have difficulty conceptualizing what "reformed" mathematics instruction is (Lampert & Ball, 1998).

Methods

In the past, I had tried a variety of instructional techniques (modeling lessons, researching "best practices", observing master teachers) to help my students build an understanding of "reformed" mathematics teaching. However, few of these techniques had much impact on my students' learning. I therefore decided to have my students explore standards-based elementary school mathematics curricula (Reys, Robinson, Sconiers, & Mark, 1999) as a means of "seeing" examples of reformed mathematics teaching. These curricula, developed in the last decade with funding from the National Science Foundation, were created to help support teaching to new national mathematics standards (NCTM, 1989). As the standards call for much greater instructional emphasis on mathematical problem solving, reasoning and communicating, the standards-based curricula were written to reflect this new focus. Thus, in spring of 2004, the preservice teachers in my course worked extensively with standards-based curricula.

The culminating activity of my students' coursework was a written lesson plan, which preservice teachers had to adapt from one of the elementary school standards-based curricula in our school library. Once they developed the lesson plan, they had to implement the activity with a small group of children and then complete a written reflection on their implementation.

In the past, I have used this same requirement (lesson plan and written reflection), with the same grading rubric, without the stipulation that the lesson plan be based on standards-based curricula. Once the student work was completed, I compared the quality of lesson plans and reflections based on standards-based curricula from this semester (Spring 2004) with those from the previous semester (Fall 2003), which had not been grounded in standards-based curricula.

Results

When I began reviewing the work of the Spring 2004 students, I was impressed with their efforts. My "qualitative" perception was that the quality of the activities they chose for their lesson plans was much higher than activities chosen by students in past semesters. In addition, I felt student reflections were stronger when using standards-based curricula.

My perception was supported by a comparison of the distribution of grades on these assignments from one semester to the next. The percentage of top grades (A's) was significantly higher for the spring group on both assignments (Table 1).

Table 1: Distribution of Grades on Assignments for Fall '03 and Spring '04

Grades	Lesson Plan Grades		Reflection Paper Grades	
	Fall 2003 N=48	Spring 2004 N = 21	Fall 2003 N = 48	Spring 2004 N = 21
A range	44%	72%	29%	43%
B range	46%	19%	52%	48%
C range or below	10%	9%	19%	9%

In addition, I found that my students' level of reflection on children's understanding of the mathematics in their activities was substantially greater. In the past, I had rated only 25% of my students as providing evidence of substantial understanding of children's thinking in the implementation of their lessons. However, with the Spring 2004 group, I found 52% provided evidence that they understood children's mathematical constructions during the activity.

One interesting finding was that my students preferred the TERC curriculum, *Investigations in Data, Space and Number.* to the University of Chicago's *Everyday Mathematics* curriculum. I believe this is because students were looking for relatively short, but substantive activities that they could do as a "stand-alone" lessons. Students appeared to find the TERC activities easier to adapt to this context as 16 of 21 students (76%) used the TERC materials.

Conclusions

My incorporation of standards-based mathematics curricula in my course certainly seemed to improve my students' choices of conceptual mathematics activities. I did not get any lesson plans involving addition bingo or M&M sorting. In addition, preservice teacher reflections on what mathematics children learned through these activities seemed to be more substantial and revealing, most likely because of the quality of these materials. I will thus continue to develop activities and assignments connected to these standards-based materials in hopes of further improving my students' understanding of what it means to "teach math for understanding."

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MATH AND EDUCATION FACULTY WORKING TO IMPROVE CONTENT KNOWLEDGE FOR PRE-SERVICE ELEMENTARY TEACHERS

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Math and education faculty worked together to quantify math content knowledge gained in a pre-service education program. Selected items from the TIMSS 8th grade test were chosen to construct a pre-test and post-test in a control group longitudinal design. An analysis was performed on the results. Correlations between SAT scores, TIMSS scores, and math grades were also calculated. Programmatic changes were made and the pre-tests and post-tests were given again. Analysis was repeated and comparisons were made.

Theoretical Background

Recent studies in education continue to point out that mathematics knowledge of elementary teacher candidates across the nation is lacking. In 1999, Ma found that 43% of U.S. teachers were unable to divide two fractions (Ma, 1999). Teachers with weak mathematical backgrounds often create ineffective classrooms centered on memorization, rules, and rote learning (Ball 1988). It is vital that elementary teachers have a conceptual understanding of the content they will ultimately be teaching. Ma referred to this knowledge as PUFM, profound understanding of fundamental mathematics. Without sufficient content knowledge, teachers will not be able to promote mathematical exploration in their own classrooms (Ball, 1988; Schifter 1993).

Purpose/Goals

We focused our longitudinal research on students enrolled in MTH 210, a required math content course for all elementary education students at Elon. We had three major goals: 1-To quantitatively measure a gain in math content knowledge of students over the course of a semester while enrolled in MTH 210 and make comparisons with other non-majors 2-To determine if there were correlations between SAT scores, TIMSS scores, and math grades 3-To employ programmatic changes to improve content knowledge gain for MTH 210 students.

Description of the Study

We employed a pre-test/post-test control model for our research design. Our experimental group consisted of 44 elementary and middle school education majors enrolled in MTH 210. MTH 210 is a content course that focuses on the conceptual development of logic, number systems, measurement, and geometry. All students completed or received AP credit for at least one mathematics course at the college level. The control group consisted of 67 students enrolled in SOC 111, an introductory sociology class taken during the sophomore year by a cross section of majors. The test was administered at the beginning and end of the fall 2002 semester to both groups. The examination contained 30 multiple choice questions taken from the released items of the eighth grade TIMSS (1995) exam. Calculators and computers were not allowed.

Results

	Control Group	Experimental Group
Pre-test mean	66%	68.6%
Pre-test SD	15.7 points	14.3 points
Post-test mean	66%	74%
Post- test SD	17.3 points	14.1 points

For the pre-test and post-test, we used a two sample t-test with α =0.05. Getting a p-value of .3698 for the pre-test, we had evidence to support the claim that the pre-test scores for our control and experimental groups were not significantly different. Getting a p-value of .0089 for our post-test, we had evidence to support the claim that there was a significant difference in the post-test mean scores for our two groups.

We also looked at correlations of variables of interest.

r-value	Grade	SAT-V	SAT-M	SAT	Pre	Post	MTH-210
Grade	1.00	.14	.33	.26	.369	.46	.73
SAT-V	.14	1.00	.50	.88	.43	.41	.17
SAT-M	.33	.50	1.00	.85	.66	.60	.31
SAT	.26	.88	.85	1.00	.63	.57	.27
Pre	.39	.43	.66	.63	1.00	.69	.44
Post	.46	.41	.60	.57	.69	1.00	.62
MTH-210	.73	.17	.31	.27	.44	.62	1.00

At the significance level of .05 with n=44, many of the correlation coefficients we found were significant. The data shows that there are many indicators for how well a student will perform in a math content course.

We continued implementing a pre-test and post-test in MTH 210 each semester through the winter 2004 semester. On the pre-test, the overall mean was 64%; on the post-test, the overall mean was 72.4%. Again, while the math content knowledge gained was significant, we felt dissatisfied with the 72.4% score. In the spring 2004 semester, programmatic changes were implemented in MTH 210 to determine if post-test scores could be raised. Since the NCTM Principles and Standards document emphasizes the importance of problem solving in developing and improving content knowledge, the instructor incorporated daily problem solving into the course (NCTM, 2000). Students were asked to solve daily word problems using Polya's model. Students were required to journal their responses and rank their perception of the difficulty of the problem. The mean pre-test score for the spring 2004 semester was 69.5% and the mean post-test score was 74.3%. The programmatic change was not significant in raising post-test scores.

Conclusion

While quantifying content knowledge gained in MTH 210, we concluded that the math content knowledge of our MTH 210 students was significantly higher than other students at Elon University. There were significant correlations between SAT scores, math grades, and TIMSS grades. While the programmatic change we made was ineffective in raising post-test scores, we plan to examine our curriculum more closely. While this study focused on Elon students, we as educators should feel confident that math and education departments can work together to provide much needed content knowledge for our pre-service teachers. As educators, we need to determine what level of content knowledge is reasonable. At Elon, we feel at 74% on an eighth grade exam is not reasonable. We will continue to explore other programmatic changes and ways to measure math content knowledge of pre-service elementary teachers.

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INTERNS' AND NOVICE TEACHERS' STRESSES WHILE ATTEMPTING NCTM STANDARDS-BASED TEACHING [SBT]

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Introduction

Teachers report work-related stress more frequently than most other human service occupations (Kyriacou, 1989). While studies on stresses of learning to teach are frequent, few explore how learning to teach by the National Council of Teachers of Mathematics [NCTM] *Professional Standards for Teaching Mathematics* (NCTM, 1991) affects stresses experienced.

Conceptual Framework

Kyriacou & Sutcliffe (1978) characterize teacher stress as a response syndrome of negative affect resulting from aspects of a teacher's job perceived as threats to self-esteem or well-being. Like Miller & Fraser (2000), I focus on factors in the environment, rather than on personality, in an attempt to identify ways to support teachers. Literature reviews (Phillips, 1993; Kyriacou, 1989) identify six recurring classes of teacher stress: poor student motivation, poor student discipline, poor working conditions, time pressure and work overload, low status and opportunities, and poor school ethos. Interns experience additional stresses including observation and evaluation by university supervisors and mentor teachers, university coursework concerns, and balancing professional and personal commitments (Murray-Harvey et al., 2000).

I define a novice as teaching less than 2 years. I judge SBT by six criteria from *Professional Standards for Teaching Mathematics* (NCTM, 1991): worthwhile mathematical tasks, teacher's role in discourse, students' role in discourse, enhancing discourse, analysis of teaching, and learning environment. I operationalize SBT as non-trivial examples meeting two or more criteria.

Research Questions

1. Which stresses do novice secondary teachers and interns experience while attempting SBT?
2. How do those stresses relate to novices' and interns' attempts to implement SBT?
3. How do surveys and interviews complement each other when gathering data on teachers attempting to implement SBT?

Method, Instruments, and Sample

The seven-page survey consisted of ranking, yes/no, and short answer questions. Participants rank ordered stresses in their current teaching situation, discussed how those stresses changed over time, and described their experiences with and stresses associated with attempts at SBT.The interviewer followed up on survey responses to understand whether participants attempted SBT, which stresses were related to attempts at SBT, and how stresses changed over time.I contacted graduates and teachers affiliated with the secondary teacher education program of a large, public, Midwestern university. Of 100 surveys, 19 were returned. Of those 19, all used either University of Chicago School Mathematics Project materials (Usiskin, et al., 1990), an NSF-funded curriculum (e.g. Connected Mathematics Project, Lappan, et al., 1998), or a teacher-written curriculum from a professional development school. From the group of 19 survey participants, a subgroup of 5 were interviewed because they met the criteria for attempting SBT.

Analysis and Results

For both surveys and interviews, I coded stressors using survey categories, rank ordered them by frequency and weight, and compared the measures within and across groups. A difference of four ranks appeared "relatively large," because it was less easily attributed to small sample size.

Using data from pilot interviews, I constructed the survey to capture stresses likely related to SBT and those predicted by literature. Survey participants experienced most stresses predicted by the literature; however, only 8 of 165 stressors, or 4.8%, were clearly related to SBT. To the contrary, all interviewees related at least half of their top five stressors to SBT. Overall, 21 of the 43 individually reported stressor categories from the interviews, or 48.8%, were related to SBT.

A comparison of interview to all survey rankings found "relatively large" differences for 12 of 22 stressors by frequency and 9 of 22 stressors by weight. Comparing survey and interview rankings for only interviewees showed a surprising number of "relatively large" differences in ranking, 16 of 22 stressors by frequency and 8 of 22 stressors by weight.

Several stresses related to SBT were predicted by the literature, such as poor student motivation. Some teachers said that it arose as student resistance to SBT practices, like reading and discussing mathematical books and whole discussions to build mathematical concepts.

There were several relatively common stressors for participants attempting SBT not salient in the existing literature. For example, participants reported stresses related to teachers' and interns' differences in beliefs about educational practice with students, parents, university personnel, school administrators, and other teachers who did not value or hindered their attempts at SBT.

Discussion and Conclusions

The literature predicted some top stressors, like role as classroom manager and time issues, and many of these were related to interns' and novices' attempts at SBT; however, three highly ranked stressors not predicted by the literature were also strongly related to SBT, as follows: differences in beliefs about teaching practice with authority figures (i.e. university faculty, mentor teachers, school administrators, etc.), colleagues, differences in beliefs about teaching practice with parents and students, and opportunities to learn a new pedagogy.

The evidence that interviews collect very different data from surveys on stressors related to SBT is overwhelming. This study provides evidence that teachers and interns struggle to engage with the construct of SBT. Until researchers can find better ways to communicate this construct, it will be difficult to improve the efficacy of surveys in studying this issue.

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MATHEMATICS VIRTUAL LEARNING VILLAGE: OCCASSIONING STUDENT TEACHERS LEARNING TO TEACH MATHEMATICS

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Research agenda

There is agreement in teacher education on the importance of learning from experience. Following this contention, student teaching among other things such as field experiences and school observations, remains a very important part of most teacher education programs. However, while many teacher education programs include student teaching in preservice teacher education, there is increasing evidence from the literature that practicing teachers as well as prospective teachers continue to experience and talk about a gap between theory (University courses) and the lived experiences (practice) of teaching (Grimmet & MacKinnon, 1992).

In mathematics education this problem is compounded by reforms in school mathematics such as those reflected in National Council of Teachers of Mathematics (NCTM) Curriculum and Evaluation standards and Principles and Standards (NCTM, 1989, 2000). In the light of the NCTM standards, the role of teacher education is to enable teachers to choose worthwhile tasks; orchestrate classroom discourse; create a learning environment that emphasizes problem solving, communication and reasoning; and develop the ability to analyze their teaching and student learning. While mathematics teacher educators are making changes in their programs to take account of the reforms there is no evidence of their impact on prospective teachers during their internships. There is an indication that student teachers revert to conventional forms of teaching during their internships. This is not surprising in mathematics education where it is noted that practicing teachers are also struggling with teaching reform mathematics (Boaler, 2003). This begs a question in mathematics teacher education: In what ways can student teaching experiences or teaching practices be supported to make them more educative for both student teachers and practicing teachers?

There is a growing literature in the use of technology as a tool for reforming and improving teacher education programs (Capper, 2001). The common theme running through this literature is that technology provides medium for new forms of pedagogical relations, new collaborative practices and new forms of learning communities.

The literature on the use of technology in teacher education provides some indication on how technology environments might be used to support learning in teacher education programs. However little attention has been paid to the relationship between the supportive technology environments and what preservice students or practicing teachers are learning. What is the nature of this learning? In addition very little is known in terms of the nature of structures of these technology environments and their relationship to worthwhile education tasks that may bridge the gap between theory and the teachers lived experiences of teaching.

This short oral representation will discuss preliminary results of an ongoing research project designed to address these questions. The project involves a number of preservice mathematics teachers as they participate in technology structured mathematics education tasks during their student teaching. As well the research involves some practicing teachers as they engage in these activities as part of their professional development. Specifically the study addresses the following questions:

1. In what ways can technology environments be structured to make student teachers and teachers learning from their practice more educative?

2. What kinds of worthwhile mathematics teaching tasks might these technology environments support?
3. What is the relationship between these mathematics teaching tasks and student teachers' and practicing teachers' classroom teaching experiences?
4. What are student teachers or teachers attending to in these technology supported mathematics-teaching tasks?
5. What is the relationship between what student teachers as well as practicing teachers are attending to in mathematics-teaching tasks and their teaching?

The research involves three sites. Each site provides an opportunity to explore particular aspects of the research questions.

1. My own experiences in this study. I keep a journal in which I record my reflections through out the study
2. Online Centre or Virtual Centre:
   This is major site in this study. Online prompts or categories are designed where:
   a) Teachers and student teachers plan mathematics teaching tasks such as lessons, activities, investigations and assessment tasks both individually and collaboratively.
   b) Teachers and student teachers reflect on their teaching using the plans above both individually and collaboratively
   c) Teachers and student teachers engage in the discussion around their experiences of teaching around the mathematics tasks
3. School mathematics classes This site involves teachers and student teachers teaching around mathematics tasks they have planned in the virtual centre. Also the site involves my observations of some of the classes in which these teachers are teaching.

Data is collected from all the records of practice (lesson plans, activities, investigations, reflections and discussions) posted on the online site. In addition data will be obtained from classroom video tapes. I intend to follow up with unstructured interview with some teachers and students.

In the field of mathematics education there is an agreement among educators that there is a great need for research that is focused on mathematics teaching practices. For example, in her plenary presentation at the annual meeting of the Psychology of Mathematics Education (PME), Boaler (2003) argued that, while the field of mathematics education has made great strides in understanding mathematics learning and the creation of rich learning environments for mathematics learning very little is known on what kind of teaching practices might bring about this learning. Boaler contends that what is needed in the field is research that will contribute to understanding of the work involved in the teaching mathematics. This research must start by understanding the act of teaching itself.

Another important point in mathematics education is the observation on the unique nature of mathematics that teachers are engaged with in teaching. Ball and Bass (2002) argue that the mathematics that teachers are engaged with in teaching is different from the kinds of mathematics that research mathematicians or physicists are involved with. Ball and Bass view the work of teaching mathematics as specialized form of mathematical problem solving. That to say as teachers respond to different ways students approach mathematics tasks or as they respond to student errors they are actually engaged in mathematics problem solving. In her plenary presentation at the Canadian Mathematics Education Study Group (CMESG) conference, Sierpinska (2003) puts another swing to this direction, by urging mathematics educators to focus their research on what is common to all mathematics stakeholders, mathematicians, mathematics educators, mathematics teachers, and mathematics students - mathematics. Sierpinska contends that any good mathematics education research should be structured around mathematics activities or tasks.

It is expected that this presentation will open a discussion among mathematics educators on the nature of teaching practices that might bring about learning as articulated by reformers. As well, the presentation will open discussion among mathematics educators not only on the nature of mathematics tasks teacher and students are engaged with but also in what ways teachers attend to these tasks and how does this translate in their teaching.

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TEACHER EFFICACY IN PRESERVICE MATHEMATICS: CHANGING PERCEPTION THROUGH EXPERIENCES

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In a qualitative study of preservice teachers, elementary teacher candidates struggled with their sense of efficacy related to the teaching of mathematics. The study identified obstacles preservice teachers faced and strategies that enhanced self efficacy.

Research in the area of teacher efficacy (Gibson & Dembo, 1984; Bandura, 1986; Tschannen-Moran, & Woolfolk Hoy, 2001) has produced a solid body of literature that focuses on how teachers judge their own capability to bring about student learning. Bandura (1997) suggests four key sources of efficacy development; the most powerful being mastery experiences. Personal teaching efficacy generally rises with experience, particularly following practice teaching opportunities (Housego, 1990). Elementary preservice teachers are at a critical juncture where their sense of efficacy teaching mathematics has the potential to increase. This is important because teachers with high self efficacy are more committed to the teaching profession and are more likely to seek further professional development (Coladarci, 1992). "The development of teacher efficacy beliefs among prospective teachers has generated a great deal of research interest because once efficacy beliefs are established, they appear to be somewhat resistant to change."(Woolfolk Hoy, 2000, p.5)

In reform-based elementary mathematics teaching, the process of enhancing efficacy is complex. Elementary teacher candidates learned mathematics from teacher-directed procedural approaches. Yet the reform movement in mathematics indicates that student-directed conceptual approaches and paradigms are more effective (Ross, McDougall & Hogaboam-Gray, 2002; Simon Tzur, Heinz, Kinzel, 2000; Spugin, 1996). Exploring this tension can be approached by asking: What obstacles do preservice teachers face in the teaching of mathematics and what interventions facilitate increased efficacy?

Data Sources, Analysis, and Findings

The site for this study was a newly established School of Education in an Ontario University. Participants in the study were 18 elementary preservice teachers enrolled in a mathematics methods course. Data sources included open-ended surveys, focus group interviews and math logs. Because this study focuses on participant experiences, qualitative methods of line by line open coding of transcripts and texts were used. Code notes and theory notes were combined with visual maps to further clarify and confirm understanding of the data. Key themes and rich descriptions emerged and intersected with theoretical frameworks of self-efficacy and mathematics education.

Obstacles

One of the main obstacles teacher candidates faced was past experience. Preservice teachers revisited their prior experiences as mathematics students. The stories they told revealed a series of dominantly negative experiences. These memories were vivid and left lasting impressions that lowered the confidence of teacher candidates entering the course.

> Kerry: … I had a D in my grade 12 math, and I went in for regular help, and at the end he literally said to me, "You know, I'm just going to pass you, because if you took it again, you still wouldn't get it"…I knew I failed and he just didn't think it would be productive for me to take it again, because he just didn't think I was getting it.

All participants experienced procedures driven methods of learning as math students. This presented two problems. First, the teacher candidates were not confident in their conceptual understanding of mathematics. Second, the teacher candidates had no models of active, engaging math programs. The fragility expressed by participants and the need to redefine how to teach for mathematical understanding posed challenges to preservice teachers.

Kerry: I learned totally transmission style - This is the way it is on the blackboard, and this is the way you do it. But I realize now that the kids need so much more and my whole insecurity is that I'm not going to be able to give them that.

Facilitators

In order to tackle these and other obstacles, participants were encouraged to acknowledge prior experiences and construct a way forward. Preservice teachers stated that they would only take risks within a safe, positive environment. David described the course environment as "something that evolved and started with trust and respect and openness." The preservice course offered a safe learning environment through the explicit development of a community of learners that emphasized asking questions, modelling strategies, and investigating theories. The principle strategies for building community were: Encouraging math discourse; soliciting multiple solutions; and exploration with manipulatives. For those preservice teachers who struggled with broad mathematical ideas such as infinity, zero, and algebra, manipulatives became an anchor for meaning-making. Throughout the focus group interviews, preservice teachers emphasized the value of making meaning for themselves as mathematicians so that they could then facilitate student understanding in their own classrooms.

Betsy: When I had to teach grade 6 geometric solids, I had a panic attack, because I have no idea what I'm doing. And then it was the worry, that, what if I can't learn this, what if this is too hard, 'cause if it didn't make sense to me - but then I started learning it through hands on stuff that I was going to do with the students and all of a sudden it clicked and I can tell you all of this wonderful stuff about polyhedra and platonic solids.

Opportunities to teach mathematics both through micro-teaching simulations and while on placement proved to be enormous confidence builders for preservice teachers.

Conclusion

This study contributes to the qualitative understanding of self efficacy in mathematics education. The rich descriptions of teacher candidate experiences provides insight into the obstacles faced when learning to teach mathematics. By acknowledging prior experience and engaging in a student-directed, community-based methods course with significant modelling, self efficacy increased for participants.

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USING SCENARIOS TO ENHANCE THE MATHEMATICAL UNDERSTANDING OF PRE-SERVICE SECONDARY TEACHERS

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Stigler and Hiebert (1999) found that American teachers tend not to be able to anticipate student responses as well as their Japanese counterparts. Teachers' knowledge about teaching mathematics should be grounded in what we know about how children construct mathematical ideas from prior experiences (Cobb & Steffe, 1983). Pre-service teachers, also, must connect the abstract mathematics learned in their content courses with the contextualized school mathematics that they will be teaching. Often, pre-service teachers experience a 'disconnect' between abstract mathematical ideas and secondary students' limited understanding (Bromme, 1994). To address this 'disconnect' between what they know and what they are being asked to teach, researchers (Philipp et al , 2004, Rubenstein, Beckman, & Thompson, 2004, and Schifter, 1996) have used actual classroom work in order to provide authentic situations for analysis. Brahier (2004) used teaching scenarios with his pre-service teachers to expose common misconceptions or incomplete understanding. This study explored the ways that scenarios involving questions from the classroom enhance pre-service teachers' understanding of mathematical concepts.

Theoretical Framework

A social constructivist perspective assumes that learning takes place in a social environment in which knowledge is shared between individuals under the tutelage of another so all can benefit and gain new insights (Vygotsky, 1978). In this study, pre-service teachers discussed their responses to the presented scenarios and the researcher asked questions to help the teachers clarify mathematical misconceptions. In this way, the pre-service teachers' understanding of mathematics was enhanced.

Methodology

Actual questions posed by secondary students were used to create instant-feedback scenarios. These scenarios were posed to the pre-service teachers and their responses were recorded. The responses were analyzed using constant comparative methods (Merriam, 1998) to characterize the cognitive decisions (Cooney, 1988) made by the pre-service teachers.

Results

Pre-service teachers were unable to use the abstract content knowledge gained in geometry and algebra courses to respond to the mathematical concepts addressed by the teaching scenarios. By discussing teaching scenarios in a methods class, pre-service teachers explored their conceptual understanding of mathematics and expanded their repertoire of explanations. Pre-service teachers report that the use of these scenarios was "one of the most beneficial parts" of the course; they are forced to focus and reflect upon their understanding of mathematical content.

Implications

To help pre-service teachers make appropriate cognitive decisions at the exact moment of interaction in the classroom, they must first reflect upon their own understanding of mathematical content and its application to school mathematics. Many pre-service teachers are successful in the mathematics classroom but struggle to explain mathematical concepts to secondary students. This study indicates that mathematics educators must pay attention to the development of pre-service teachers' understanding by providing opportunities for them to address students' misconceptions and incomplete understanding.

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EXPLORING THE CHALLENGES OF LEARNING TO TEACH REFORM-ORIENTED MATHEMATICS

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This paper looks at changes two preservice teachers made to lessons presented in a reform curricula. Both teachers tended to reduce the cognitive demand of the original tasks.

Over the last decade, a number of publishers have put out mathematics curricula designed to support elementary teachers' efforts to enact the NCTM standards. These curricula have asked teachers to promote reasoning and problem-solving, to rely on the conventions of mathematics to determine right answers and to allow students to explore their own conjectures (National Council of Teachers of Mathematics, 1991). Because learning to use these curricula in meaningful ways can be especially difficult for novices, I decided to explore the teaching choices of two elementary preservice teachers who were using Math Trailblazers, a reform curriculum.

To better understand the practices of two preservice teachers I supervised, I decided to use Stein, Grover and Henningsen's (1996) framework for looking at the cognitive demand of a task as it progresses through various phases of implementation: as it is presented by the teacher, implemented in the classroom, and interpreted by students. However, for preservice teachers, who are often required to do thorough written plans, I felt it was important to understand the ways in which these teachers changed tasks during planning. So, I modified the framework to analyze changes in tasks as they moved from the curriculum intended by the publishers, to the curriculum planned by the preservice teachers, to the curriculum enacted in the classroom.

As a field instructor in a year-long internship program, I observed four elementary preservice teachers weekly in a variety of subjects. In the fall semester, these interns gradually took over instructional tasks in the classroom and began "lead teaching" in February. For this study, I focused on two of the four interns: Anna, who taught first grade, and Stephanie, who taught second. I observed Anna and Stephanie each teach two math lessons early in their lead teaching. I audio taped these observations and the post-observation conferences, collected copies of their plans and made copies of the Math Trailblazers lessons on which they based their plans.

I observed Stephanie teach one second-grade lesson on volume and one on arrays. The Trailblazers curriculum allotted an hour for each lesson; in her plans Stephanie cut that time in half and in the classroom, she further reduced each lesson's length. While teaching, Stephanie stood at the overhead and controlled most of the conversation, instead of allowing the student exploration called for by the teachers guide. For example, in the volume lesson Stephanie led the children through the questions on a handout that had pictures of three containers: D with 62 beans, E with 43 beans and F with 29 beans. The class agreed that Container D had the largest volume. Stephanie then asked the next question on the handout: Would D would still have the largest volume if water were used to measure instead of beans? Many children thought the result would be different. When Stephanie asked how that could be possible, a child said that the beans in Container D might have been smaller than those in Container F. Stephanie agreed that this would affect the measuring of volume, but said "I don't think the book would do that." When another student said he thought Container F would be the smallest whether measured with water or beans, Stephanie validated his response. Another student disagreed. Stephanie repeated the number of beans in each container and told the class to write "Container D" in the blank.

In her plans for both lessons, Anna increased the amount of modeling suggested by Trailblazers. In the hundred's chart lesson, she eliminated all independent and group work; instead, she planned to have the children follow her work on the overhead. During the interview, she said the children would not have understood the lesson if they had worked independently. In the grouping by tens lesson, Anna's plans revealed that she had intended to follow the Trailblazer suggestion to begin by asking children to experiment with various ways of grouping and counting; however, in the classroom, she eliminated this step. When a student suggested sorting her erasers by color, Anna rejected this. When another child suggested grouping by tens, she told him that was a good idea and modeled how to put the erasers in a tens frame. She then asked the children to count their erasers in the same way at their seats.

In analysis, I mapped out the activities as suggested by Trailblazers, as described by the interns in their plans and as taught in the classroom. I then looked for changes between stages and evaluated each change to see if it was likely to have increased, maintained or decreased the cognitive demand of the original task. Virtually all of the changes made by interns in both planning and teaching reduced the cognitive demand of the original tasks. Initially, I coded each of these changes using factors that Henningsen and Stein (1997) describe as contributing to cognitive decline, including removal of challenging aspects of the task, allocation of too much or too little time, emphasis on correct or complete answers, selection of inappropriate tasks, and problems with classroom management. However, I found I needed a slightly different set of categories to describe the factors at play for the preservice teachers I observed. These were: too little time, determination by teacher of right and wrong answers, elimination of discussion after independent work (or no independent work), and extensive modeling.

When I asked the interns in post-lesson interviews why they had reduced the lesson length or eliminated independent work, both interns expressed a concern that students would not be able to successfully accomplish the original tasks, either because students couldn't sustain attention for so long or because the tasks were too difficult. In one post-conference, Stephanie said she chose not to use a more complicated problem because "there are kids who are going to struggle figuring that out." Comments like these seemed to speak to a confusion about standards-based instruction. They both said they wanted to bring open-ended questions and multiple solutions into their classrooms, but they also feared making students struggle with the mathematics. This beginning investigation suggests that researchers need to do more to understand the ways that teachers new to standards-based teaching adapt reform curricula. Focusing on the fears novices may have of asking students to perform difficult tasks may help to inform this work.

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STANDARDS-BASED CURRICULUM MATERIALS AND TEACHERS

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Using ideas generated by a working group at a recent conference, this report proposes research questions related to the role of Standards*-based curriculum materials in preservice and inservice teacher learning and development, with particular attention to the contexts and ways in which engagement with curriculum materials might influence teachers' knowledge and beliefs.*

This report draws on a series of focused discussions that occurred over a three-day period as part of the Second Annual Show-Me Researchers' Workshop, held at the University of Missouri-Columbia in May 2004. The authors of this report were the facilitators of the *Instructional Materials and Teachers* Working Group composed of 12 mathematics education doctoral students and faculty members from universities across the United States.[1] The working group focused on the influence of curriculum materials on teachers' beliefs and pedagogical content knowledge, and the implications of this influence for professional development and teacher preparation. In this report, we offer a brief summary of our working group discussions and a set of research questions related to the role of *Standards*-based curriculum materials in preservice and inservice teacher learning.

Curriculum Materials as an Influence on Teachers' Conceptions: Potential and Limitations

To generate research questions related to the role of curriculum materials on teacher learning and development, the working group consulted specific *Standards*-based curriculum materials and worked together to identify instances in which the teachers' editions of the materials seemed to contain opportunities to influence a teacher's subject-matter knowledge, pedagogical content knowledge, or beliefs about mathematics, teaching, and learning. Next, the group began to consider (1) various contexts and ways in which teachers might engage with curriculum materials, and (2) the possible limitations to the influence curriculum materials might have.

Although curriculum materials themselves may depict pedagogical situations or pose mathematical questions that are potentially educative for the teacher, it is only through some type of interaction between teachers *and* the materials that opportunities for the teacher to learn actually take shape. Our group identified a variety of contexts and ways that different teachers might engage with curriculum materials: teachers might solve and discuss the mathematical problems of the curriculum materials in small groups (in preservice university-based settings or school settings for inservice teachers); analyze design features of different materials (perhaps using the frameworks and instruments of researchers and curriculum developers); make and test predictions about students' experiences with a particular curriculum-based lesson or activity; examine actual students' experiences (including student work, class discussions, etc.) in one's own or another teacher's classroom; and collaboratively plan for instruction using materials in study groups, teach with the materials, and reflect (jointly and individually) on instruction.

As numerous research reports over the past decade have illustrated, engagement with curriculum materials does not necessarily bring about substantial change in instruction or conceptual understanding on the part of the teacher. Our group felt it would be helpful to identify possible limitations or constraints to the impact of curriculum materials on teachers' knowledge and beliefs. The following two examples are intended to suggest ways that the potential impact of teachers' experiences with curriculum materials may be limited by contextual and individual factors, as well as by particular curricular design features.

The student editions of the materials, in which mathematical problems and situations are posed, were written for K-12 students—not adults. When teachers use the student editions for their own learning of mathematics, the materials may not sufficiently or appropriately fit teachers' unique subject-matter knowledge needs. A related problem is that, when teachers attempt to use the curriculum materials for their learning of mathematics, their beliefs about the purpose of and appropriate structure for textbooks may limit their inclination to engage with open-ended, investigative problems and activities. In general, teachers' ability to engage with *Standards*-based curriculum materials, and learn from that engagement, may be limited by their own personal experiences as traditional learners and teachers of mathematics.

A second example of a possible limitation relates to the notion of authority. Although the teachers' editions of the materials encourage teachers to share authority (for the development and examination of mathematical ideas) with students, the teachers' editions tend to employ directive, authoritative language when presenting information to teachers. This tendency may limit the extent to which teachers can develop new pedagogical understandings when using the materials for planning and teaching. We also note that because teachers tend to read and use curriculum materials with their students' learning in mind and not their own, it may be difficult to promote teachers' reflection even when the materials do intend to foster teacher learning.

Research Questions

In light of such opportunities and limitations, our group developed the following set of general research questions intended to encourage members of the mathematics education community to investigate the complex role of curriculum materials in teacher development:

- How does classroom instruction with *Standards*-based curriculum materials influence teachers' knowledge and beliefs? Are some conceptions more or less likely to be influenced during curriculum implementation?
- How might experiences learning mathematics with *Standards*-based curriculum materials (e.g. in study groups, in mathematics courses for preservice teachers, etc.) influence teachers' subject matter knowledge and beliefs about the nature of mathematics?
- How do different forms of teacher engagement with curriculum materials (e.g. using the materials as learners of mathematics versus using the materials as a classroom teacher) relate to changes in different components of teachers' knowledge and beliefs?
- How are teachers' beliefs about their efficacy as mathematics teachers influenced by different experiences with *Standards*-based curriculum materials?
- How do particular design features of the curriculum materials relate to the learning of different groups of teachers (e.g. preservice, beginning, inservice)?

We recognize that the above questions are broad in scope and we invite researchers to use their own theoretical and methodological perspectives to shape more specific questions related to the ones we have proposed. Empirical work in the areas we have outlined will contribute valuable information to our understanding of the influence of curriculum materials on teachers and will greatly improve our ability to capitalize on curriculum materials as tools for teacher learning.

Endnote

1. A list of participants and further information about the working group can be accessed online at http://www.math.vt.edu/people/lloyd/show_me/materials_and_teachers.htm

AN APPROACH TO INCORPORATING LESSON STUDY IN ELEMENTARY PRESERVICE COURSES

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The initial stage of a project to develop an approach to lesson study in mathematics pedagogy courses for elementary teacher candidates involved the development, piloting, and revising of an assignment. Some findings, based on assignments of 12 candidate pairs, comments from 2 interviews and observations of the researcher/teacher are discussed.

Research has shown that the experience of researching, planning, teaching, analysing, and revising a lesson in collaboration with colleagues has positive effects on teaching practice (cf., Fernandez, 2002; Presmeg & Berrett, 2003; Stigler & Hiebert, 1999). Two valuable aspects of lesson study are: 1) It involves teachers working with each other – and research has consistently shown this to be beneficial (cf., Graven, 2003; McGraw, Arbaugh, Lynch, & Brown, 2003); and 2) It provides a flexible long-term model for teacher growth.

Participating in lesson study is also valuable to teacher candidates because it develops several key ideas: 1) that competency in teaching develops over time, 2) that sharing ideas and results with colleagues is important, and 3) that exemplary lessons are the result of analysis and deep knowledge (Hiebert, Morris, & Glass, 2003). Nevertheless, constraints (e.g., time, multiple grade levels) make it difficult to implement a traditional lesson study model in a preservice course. Hiebert et al. propose that education faculty collect lessons for candidates to analyse and revise. Another alternative is to develop an assignment that requires candidates to conduct mini lesson studies of their own lessons. The present study, is part of a project to investigate whether the latter approach is practicable and effective in elementary mathematics pedagogy courses. It involved developing, piloting and revising a lesson study assignment.

The Study

The participants were teacher candidates in two 36-hour elementary mathematics pedagogy courses taught by the researcher; 31 in a K-6 course, and 37 in a gr.4-8 course. The 12 sessions of each course were spread over fall and winter terms. Most candidates had taken no university mathematics. The course began with an introduction to the elements of lesson planning and to the framework of lesson study described by Stigler and Hiebert (1999). The assignment (30%) required that each candidate work with a partner with a practicum placement in the same grade; however, in 4 cases partners taught different grades (e.g., 3-4) and there were 2 groups of three. At the 5th session, candidate pairs handed in: a) Their preparation for the planning of the lesson, outlining - research into methods for teaching the concept, rationale for choice of problems, examination of possible student misconceptions, and questions to pose; b) A lesson plan. After receiving feedback, each member taught the prepared lesson to their respective class (and watched one another, if possible). The candidates then compared their experiences, examined samples of student work and revised their lesson plan. At the10th session they handed in: a) 4 student work samples each, and an analysis of whether that work provided evidence of student understanding of the concept(s) taught; b) An analysis of their teaching experiences, addressing what went well, and what did not go well in light of the original plan; c) A revised lesson plan, accompanied by the rationale for any changes.

Twelve assignments were resubmitted by candidates. They were coded to: a) gather evidence about timing/organization, (e.g., on time, needed extra time (reason), placement related problem); b) examine expectations more closely (e.g., too difficult, vague); c) gather information about collaboration (e.g., evidence of joint work; disconnects; contradictions); d) identify areas requiring more preparation/support (e.g., class management, trouble working with partner). The 2 interviewees were asked to talk about their experience, to comment on difficulties with the timing/organization, the expectations, and the collaboration, and to offer ideas for improvement.

Results

The researcher received many unsolicited positive comments about the value of the lesson study assignment; those who arranged to watch one another were especially enthusiastic about the experience. There is also anecdotal evidence that the assignment had a positive impact on the seminars – the lively, dialogue that arose in both classes about how to present particular concepts was motivated by the need to research teaching methods; and candidate-initiated discussions about individual understanding and how to help students make conceptual links were stimulated by the experience of examining student work samples.

The following is a brief presentation of some findings: 1) There were few timing/organizational problems except for difficulties with mixed grade pairs. The only pair in the study to "break up" was a 4-5 pair. One of the interviewees noted that working with someone in the same grade was one of the most beneficial aspects of the experience. She recommended groups of 3 rather than mixed grade pairs. 2) Most of the 12 assignments showed considerable evidence of collaboration on research, analysis and planning. However, disconnects between the research and the lesson plan, and contradictions between sections showed that members of 2 pairs had worked independently. At the next stage, the importance of joint work will be explicitly discussed. 3) Class management and time planning were coded frequently as sources of difficulty. This is in line with findings on the needs of beginning teachers (Artzt, 1999). Thus, the revised assignment will prompt candidates to explicitly consider these issues in their plans. 4) Candidates sometimes misinterpreted the level of understanding in student samples. Since being able to analyse student work is a critically important skill, seminars in the next stage of the project will provide additional preparation in this area.

The results indicate that mini lesson study is workable in a preservice environment. The next stage will investigate whether the (redesigned) assignment helps candidates plan more effective lessons, and will explore candidates' attitudes towards this collaborative experience.

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A FRAMEWORK FOR UNDERSTANDING THE RELATIONSHIP BETWEEN STUDENTS' CONCEPTUALIZATIONS OF MATHEMATICS AND THEIR DEVELOPING PEDAGOGY

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In work with pre-service teachers [PST], it became evident that "knowledge of mathematics for teaching," though valuable, was not "fine-grained" enough to be of use. In this paper, the construct of pedagogical content knowledge will be examined and re-conceptualized from a constructivist perspective. Pedagogical content knowledge has become a "catch phrase" in the research and policy regarding mathematics teaching and mathematics teacher education. It would be hard to question *the spirit* of the construct, however as currently conceptualized, however its utility can be questioned. It has been clear in the current literature on mathematics teacher education that research on teacher knowledge (of which research on PCK is one strand) reached its pinnacle in the late 1980's and early 1990's and a question of interest is *why* has there been a shift in the research on the teacher education away from teacher knowledge. Though there is no explicit mention of this in the literature, I believe that this is because of the fact that there were little useful results gained from the previous research. A careful read of the literature on teacher education and the contradicting results (Ashton & Crocker, 1987; Druva & Anderson, 1983; Grossman, 1989) suggests a possible reason for the lack of utility of the prior work: The constructs being used and classification (and nature) of knowledge being studied were insufficient.

This paper focuses on students' understandings of mathematics and the implications of their understandings on their developing pedagogy. I have documented elsewhere that though pre-service teachers may develop more advanced conceptions of mathematics, it is likely that these understandings are fragile and furthermore, it is unlikely that they (a) aware of the powerfulness of their conceptions and (b) are therefore unable or unwilling to conceive of these more advanced conceptions of mathematics as endpoints of instructional trajectories (Silverman, 2004a; Silverman, 2004b). This research has led me to attempt to reconceptualize pedagogical content knowledge not simply as knowledge of mathematics for teaching, but as particular conceptions of mathematics that are both powerful and enable teachers to see educative potential in particular situations that would otherwise be transparent. For reasons of space, I cannot describe the development of the proposed constructs in detail, but their genesis lies in a finer grained measure of teacher knowledge: not just the *amount* of knowledge a PSTs possess, but rather *how they come to understand* particular mathematical content, for it is this understanding that mediates both how they, as students, learn additional mathematics and how they, as teachers, conceive of the mathematics to be taught. The work of Ma (1999), Thompson & Thompson (1996), and Silverman (2004a) call for more attention to this aspect of teacher knowledge.

My current work draws on Simon (2002), who introduces the idea of a key developmental understanding in mathematics as a way to think about understandings that can be useful goals of mathematics instruction. He describes two characteristics of a key developmental understanding. First, a key developmental understanding involves a conceptual advance or a "change in the learner's ability to think about and/or perceive particular mathematical relationships" (Simon, 2002, p. 993). Students who possess a key developmental understanding tend to find questions related to their understanding, but traditionally harder to explain, as trivial. I call this understanding a *powerful* understanding of mathematics. Second, Simon claims that key developmental understandings tend not to be developed through explanation and/or demonstration of the concepts to be understood.

An interesting question one might ask is: *Is having key developmental understandings sufficient to plan to teach for understanding?* My tentative answer is no. There is nothing in a key developmental understanding that indicates that one who possesses it is aware of its utility. It is for this reason that I propose the concept of a *key pedagogical understanding,* which involves a conceptual advance which is developed through the transformation of a key developmental understanding. In mathematics teacher education classes, we aim to teach pre-service teachers mathematics in a way that the mathematics makes sense. In developing this knowledge, however, students do not conceive of the mathematics as something that makes sense – they think about it as *mathematics*. I am currently investigating the conjecture that in order to develop teachers that have the possibility of teaching mathematics for understanding, we as mathematics teacher educators, must focus not only on mathematics that makes sense, but presenting opportunities for the students to come to realize that their understanding makes sense and is powerful—powerful for the purpose of helping others develop coherent understandings of mathematical ideas, conventions, and methods. It might be that only when one is cognizant of the way he or she understands the mathematics that one can have a chance to develop instructional environments where school students, who understand the mathematics in a multitude of different ways, can develop powerful mathematical understandings.

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DIVISION WITH FRACTIONS: A STUDY OF PRE-SERVICE TEACHERS' MATHEMATICAL SUBJECT MATTER KNOWLEDGE

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In 1996 I implemented a reform-based math methods course for pre-service elementary school teachers. That same year, I conducted an action research project to analyze the effectiveness of the course. My second research project emerged as a Master's Thesis (Cianca, 2000), and the data I gathered two years later, I will report in my doctoral dissertation. The research population for this third project includes six pre-service teachers enrolled in a two-semester math method course at a small, private college in Ontario, Canada, and me, the teacher-researcher.

Objectives and Rationale

A call has gone forth for raising the standards for teacher education (e.g., Cochran-Smith, M., 2001; CFEE,1986). That call presents a challenge to those who train pre-service mathematics teachers (Lappan, 2000; Anderson & Piazza, 1996). In response to the challenge, Shulman (1986) advocates a teacher reform movement built on developing teachers' mathematical subject matter knowledge. Various avenues for advancing subject matter knowledge have emerged, including an emphasis on problem-solving (Van de Walle, 2000; Sowder, et. al, 1993), discourse and collaborative learning (Ball & Cohen, 1999), reflection (Hart, 2002; Anderson & Piazza, 1996), independent research (Lubinski & Fox, 1998) and maintaining a dynamic interaction between self-study and group interaction (Von Glasersfeld,1991; Cobb, 1994; Ernest, 1994).

Throughout my project, I gathered and analyzed data to determine the extent to which pre-service teachers advanced in mathematical subject matter knowledge and to gain insight into which elements of the reform-based pedagogy influenced mathematical acquisition.

Perspective and Theoretical Framework

In transforming my math methods course from a traditional-based to a reform-based format, I proposed that prospective teachers would not only learn math methods, but in the process, pre-service teachers' mathematical subject matter knowledge would increase. To establish an environment conducive to reform mathematics, I followed Ball's (1993) model where students investigate mathematical ideas as individuals and then discuss and debate those ideas in both small group and whole group settings.

I concentrated on pre-service teachers' subject matter knowledge because, as Philippou and Christou (1998) state, mathematical subject matter knowledge presents a dilemma for instructors who attempt to teach reform math methods to men and women who, as a result of their prior schooling, know only traditional mathematics. From among many possible topics, I chose division with fractions. The complexity of division with fractions requires the development of subject matter knowledge in a number of categories (Sinicrope, Mick & Kolb, 2002): e.g., making sense of units and subunits, discrete and continuous quantities, extensive and intensive quantities, scalar-operator quantities, partitioning, and proportional reasoning.

Division with fractions with all of its complexities and nuances of meaning is the most difficult mathematical topic studied at the elementary school level and the area where teachers show the least understanding (Ma, 1999). In addition to these challenges, I chose this area because fractions span the entire mathematical domain (Kieren, 1993) and because division with fractions presented the greatest difficulty to participants in my prior research projects (see for example, Cianca, 2000). To direct my research, I posed the following question:

> "To what extent will pre-service teachers develop subject matter knowledge of division with fractions in an environment that promotes independent research, problem-solving, peer discourse, and reflection?"

Mode of Inquiry

For my method of inquiry, I chose action research because it allowed me to investigate the marriage between theory and praxis (Greenwood & Levin, 2000), because action research cycles facilitate the development of course curriculum, and because ongoing analysis can improve pedagogy (McKernan, 1996).

Realizing that the specificity of action research carries with it potential for criticism (Charles, 1999), I took measures to strengthen the reliability and validity of my study. For example, I attempted to reduce the potential for students' over-identification with the teacher/researcher (Glesne, 1999), and I applied Eisner's (1979) three criteria for testing validity: structural corroboration, referential adequacy, and consensual validation.

To avoid over-identification I created a student-empowered environment where risk-taking was welcomed and where I openly and without condemnation accepted participants' actions and comments. In a written consent form I assured participants of confidentiality and that marks for the course would not be affected either by taking part or not taking part in the research endeavour.

To improve validity, I took steps to strengthen structural corroboration/triangulation and referential adequacy by collecting data from multiple sources (see data sources below).

In an effort to build credibility, instill confidence, and leave room for critical analysis and judgment of my interpretations, my preliminary report contains characteristic details and background features of the research environment. As well, the final report will contain both supporting and disconfirming evidence.

The strongest argument in support of the validity of action research may be multiplicative replication (Phillips, 1987). According to this argument, reputable researchers must agree that understanding certain elements within a domain constitutes understanding of that domain (Miles & Huberman, 1984). A number of researchers agree on the essential elements for communicating an understanding of division with fractions (e.g., Kieren, 1993; Ohlsson, 1988; Behr, Wachsmuth, Post, and Lesh, 1984). Evidence from my study will support the presence or absence of these agreed-upon elements, thus increasing the validity of my interpretations.

External validity or generalizability deals with the degree to which findings from one setting can be applied to other settings (Merriam, 2001). Rather than striving to present generalizable conclusions, I attempt to elaborate on rich details and provide ample content for readers to extract their own generalizations and connections from my setting to theirs. This practice is advocated by Mills (2000) and Erickson (1986).

Data Sources

In our first class period, participants took a pretest to determine their baseline knowledge of division with fractions. That same day, they completed a survey, engaged in an audio-taped interview with the teacher/researcher, and wrote a mathography on their mathematical background and general feelings toward mathematics. Throughout the project, participants recorded their emerging subject matter knowledge in the form of journal entries. Further data collection consisted of audio- and video-taped group sessions (small groups and whole group); results from participants' independent research; sketches, notes, and other materials produced in group sessions; teacher/researcher's methodological journal and observational notes; and finally, a post-test.

Results

My target date for finalizing conclusions is April 2004. Preliminary and on-going analyses suggest that the combination of independent research and group interaction helped participants grow in subject matter knowledge. Emerging findings also show that participants requested more whole group work where they could rely on the teacher to confirm or disconfirm their conclusions. Rather than acquiescing, I maintained a somewhat aloof position and trusted participants to work out confusions and validate understandings in small peer groups. I will determine whether distancing myself was the best course of action when I compare student learning with and without my intervention.

Preliminary, on-going analysis also suggests that participants used manipulatives, drawings, and words to articulate understanding and to verify their positions when differences occurred or when peers asked for further clarification. Findings from videotapes, transcribed audio-tapes, and other data will determine how accurately participants represented the various division of fractions domains.

For my upcoming analysis, I plan to use a procedural approach: moving from broad general observations, to gathering specific evidences, to extracting yet more specific evidences (Spradley, 1980). Analyzing the data for reoccurring themes and emerging patterns, I will position patterns or themes within the context of my thesis to determine the extent to which participants gained subject matter knowledge and to ascertain which factors contributed to this acquisition.

Not to ignore more subtle evidences, and as a means of organizing findings, I plan to search for relationships among data. I could, for example, look for similarities and differences among participants, types of concrete materials, and kinds of interactions, and I could examine diversities within participants' independent research activities. As mentioned earlier, most importantly, I will analyze if participants understood essential, domain-specific elements.

Once findings have emerged, I will test various organizational methods to appraise which most readily lend themselves to ease and completeness and to determine which make case study reporting most manageable. I could, for example, use a contrast chart to cluster similarities and differences, could categorize relevant data under research questions, or might arrange the findings in order of events.

The intention of a case study report –reporting in an explanatory way and focusing on particular situations with particular persons (Stake, 1994)– appears to be consistent with the purposes for my investigation. Organizing data into cases will allow me to track the progress of individuals, as well as groups of students.

Research Report

In reporting my findings, I am eager to share and discuss the following division with fractions subconstructs: measurement, partitive, and rate. My report will include a description of the elements necessary for an understanding of these subconstructs. Evidence will attest to participants' progress in each domain. As results reveal which factors contributed to or detracted from participants' acquisition of knowledge, I will disclose which assignments and pedagogy I found most instrumental in advancing pre-service elementary school teachers' mathematical subject matter knowledge.

EMPTYING THE BOWL: AN INVESTIGATION OF PROBABILITY AND MATHEMATICS THAT PRESERVICE ELEMENTARY SCHOOL TEACHERS NEED TO KNOW

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This article advances a vision for the training of preservice elementary school teachers that explicitly connects rigorous mathematical study to investigations of early grades mathematics. College instructors are encouraged to provide teachers with more opportunities to intensely study the content teachers will actually teach. The feasibility of wedding rigor to early grades mathematics through an in-depth exploration of "Empty the Bowl," a probability lesson intended for first-grade students.

When teaching preservice teachers, two instructional extremes are equally tempting to pursue: (1) In a quest for mathematical rigor and depth, engage teachers in mathematics far above that which they will experience as practitioners; (2) In a quest to connect instruction to practice, engage teachers primarily in mathematics activities intended for very young children. Needless to say, both extremes are fraught with instructional dangers. Separation of rigorous mathematical study from early grades content, however tempting, is artificial and ill-conceived. Rather than separating the two, instructors of preservice teachers should consider more *rigorous mathematical study of early grades mathematics*. "Empty the Bowl," a probability lesson intended for first-grade students, provides a good example of an exercise well-suited for rigorous mathematical study.

Introduction to the Problem

The "Empty the Bowl" activity is described by Tank (1996, pp. 22-31) in the following manner:

> "Each pair of students uses one die and a bowl of 12 Color Tiles. One partner rolls the die to determine how many tiles to remove from the bowl and then removes the tiles, while the other partner records the numbers rolled and keeps track of the number of rolls it takes to empty the bowl. Each pair plays the game at least five times, switching roles for each game and posting their results on a class chart." (p. 22)

Because the activity provides children with opportunities to collect data and practice basic whole number subtraction, preservice teachers recognize "Empty the Bowl" as a worthwhile activity. Preservice teachers explore the mathematics of "Empty the Bowl" in three distinct phases.

<u>Phase 1</u>: Teachers predict the number of rolls that will most likely empty the bowl. Classroom dialogue centers around discussions of "expected value." The following questions are considered: (a) Is emptying the bowl in one roll likely? Two rolls? Twelve rolls? Twenty rolls? (b) What number of rolls will empty the bowl most often if we play the game 20 times? Predict.

<u>Phase 2</u>: Using dice, teachers calculate experimental probabilities associated with rolls needed to empty the bowl. Phase 2 culminates with a calculator-based simulation of "Empty the Bowl."

<u>Phase 3</u>: Exploring tree diagrams and integer partitioning, teachers calculate theoretical probabilities associated with rolls to empty the bowl. Using a combination of by-hand and technology-based solution strategies, teachers compare theoretical probabilities with

experimental probabilities obtained during the exploration’s second phase.

While teachers typically agree that the bowl can be emptied in as few as two rolls or in as many as twelve rolls, they often disagree about the number of rolls that will empty the bowl most often. Some hypothesize that the bowl will be emptied most often with the median number of emptying rolls (i.e. 7). Others explore the notion of a "typical" die roll to build conjectures. Teachers note that the sum of three "typical" die rolls is 3.5 + 3.5 + 3.5 = 10.5. Likewise, the sum of four "typical" die rolls is 14. Some hypothesize that the bowl will be emptied most often with four rolls (14 > 12). Others conjecture that three rolls will empty the bowl more frequently since the "typical" sum for three rolls, 10.5, is closer to the minimal emptying sum (1.5<2).

After predicting the number of rolls that will most frequently empty the bowl, teachers conduct calculator-based simulations and die roll experiments to investigate the feasibility of initial conjectures. Reasonable, yet conflicting, experimental results generated by calculator and die experiments serve to motivate calculation of theoretical probabilities.

Because textbook definitions (Bassarear, 2001) of probability typically refer to "outcomes," it is natural for students to begin a theoretical analysis of the "Empty the Bowl" experiment by listing outcomes that remove all twelve tiles. If one could somehow count the total number of ways to "empty the bowl" in n rolls (with $2 \le n \le 12$), one could calculate the theoretical probability of removing all 12 tiles with n rolls of the die. Initially, teachers chart outcomes with tree diagrams (e.g. create a tree depicting all outcomes for exactly 3 die rolls with first roll 6). Ultimately, teachers use the computer algebra system *MuPAD* to list all ways to empty a bowl of 12 tiles with n die rolls ($2 \le n \le 12$). Raw data generated from MuPAD is shown in Table 1.

Table 1

Complete listing of ways to empty the bowl as generated by MuPAD computer algebra system

		Ways to Empty in n Rolls										
		n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10	n=11	n=12
Initial Roll	6	1	20	50	55	29	6	n/a	n/a	n/a	n/a	n/a
	5	0	21	70	105	84	35	6	n/a	n/a	n/a	n/a
	4	0	15	91	175	189	119	41	6	n/a	n/a	n/a
	3	0	10	100	266	364	308	160	47	6	n/a	n/a
	2	0	6	99	360	630	672	468	207	53	6	n/a
	1	0	3	90	442	984	1302	1140	675	260	59	6
Total		1	75	500	1403	2280	2442	1815	935	319	65	6

Even with *MuPAD*-generated data, teachers experience difficulty calculating theoretical probabilities. For instance, denoting "ways to empty the bowl in n rolls" as w_n, teachers commonly express the theoretical probability of emptying the bowl in 2 rolls as $w_2/(w_2+w_3+w_4+w_5+w_6+w_7+w_8+w_9+w_{10}+w_{11}+w_{12})$. This highlights a common misconception that, in statistical experiments, all outcomes are *always* equally likely. Unfortunately, outcomes of the "Empty the Bowl" experiment are *not* equally likely. For instance, the probability of generating a particular outcome with 3 die rolls - say [6,5,2] - is $(1/6)^3 = 1/216$. In general, the probability of generating an outcome with n die rolls is $(1/6)^n$. For n rolls, the probability of emptying the bowl is $w_n \bullet (1/6)^n$. Using the above data, one may determine that, theoretical speaking, one is most likely to empty the bowl in 4 rolls ($P(4) = 500\bullet(1/6)^4$). This is slightly higher than emptying the bowl in 3 rolls ($P(3) = 75\bullet(1/6)^3$).

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LISTENING TO STUDENT UNDERSTANDING: PRE-SERVICE TEACHERS' DIFFICULTIES

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Introduction

Research has found that pre-service teachers have difficulty attending to student thinking (Crespo, 2000). Unfortunately, when teachers makes assumptions about students' understanding, they may inadvertently ignore the student's reasoning and thinking (Thompson & Thompson, 1994). Mewborn (2003) argues that teachers must "have a propensity to listen to students' reasoning" as well as their own understandings of the mathematics to make sense of students' thinking. The research described in this poster investigated ways that pre-service teachers could be encouraged to attend more to issues of student understanding.

Method

Two pre-service teachers in a secondary mathematics method class volunteered to participate. Each solved a math problem before class and then engaged in a discussion about student understanding of this problem during the class. Each teacher met with students twice (one student before and one student after the methods class) and observed him/her solve this same problem. Of interest were the questions the pre-service teachers would ask, how well they listened to the students' ideas, and how they responded to perceived student difficulties. Interviews between researcher and teachers were audio-recorded; the researcher took detailed field notes during teachers' sessions with students.

Results

During each initial session with students, the pre-service teachers asked few questions. Each merely gave the student the problem to work on and only interacted with him/her when a mistake was made. The few questions asked tended to be close-ended. The teachers asked more questions in their second round of problem solving, with an increase in the number of open-ended questions posed with one teacher increasing from one to five, while the second increased from three to seven. The class discussion that fell between each pair of student interactions highlighted multiple solution possibilities and the need to listen to others, which seemed to have a large effect on the increase in pre-service teachers' use of open-ended questions.

This small intervention suggests that methods classes focused on attending to issues of student understanding, particularly through the use of open-ended questions, have the potential to affect change among pre-service teachers. Further research into how to promote pre-service teachers' consideration of student thinking is needed if their decision making about learning environments should build upon current student understandings.

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USING AN ACTION-IMPLICATIVE FRAMEWORK TO EXAMINE NOVICE TEACHERS' INTERACTIONS WITH STUDENTS IN SMALL GROUPS

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Action-implicative discourse analysis has been used to describe problems, interactional strategies, and ideals-in-use in conversational practices found in such venues as hospice meetings, domestic dispute calls, academic colloquia, and discussion in college classrooms. This study uses the framework to perform a preliminary investigation into the problems teachers face in their efforts to support students' mathematical thinking in their interactions with students in small groups, the strategies they use and their situated ideals. It examines the value and usefulness of framing classroom discourse analysis in this fashion. Data come from the Learning to Teach Secondary Mathematics project (LTSM), a five-year longitudinal study of how individuals learn to teach middle and secondary school mathematics.

Many mathematics teacher researchers who have examined the ways in which teachers struggle with the problems of classroom practice have found a need for a better language for teacher moves. The current emphasis on the social nature of learning places discursive practices in a key role in teaching and learning, giving us a better opportunity to observe teachers struggling to effect what they consider exemplary discourse practices. Action-implicative discourse analysis (AIDA), a version of discourse analysis developed by Tracy, seems particularly appropriate for this type of study in that it focuses on "attention to describing the problems, interactional strategies, and ideals-in-use within existing communicative practices…so that a practice's participants will be able to reflect more thoughtfully about how to act" (Tracy, p.2). AIDA derives from four main discourse analytic traditions—conversation analysis, speech act traditions, discursive psychology and critical discourse analysis—while also committing to grounded practical theory (Craig & Tracy). Using the AIDA framework, this study performs a preliminary investigation into the kind of problems teachers face and how they deal with them by looking at their conversations with students in small groups. It looks at the strategies they use as they work to support students' mathematical thinking and attempts to reconstruct their situated ideals. For example, formulations of responses may show choice of strategy and make visible the teacher's expectations for students' identities as mathematical problem solvers. Is this sort of analysis useful for practioners when they reflect about the problems of practice? Does this usefulness contribute to validity?

The data used in this research come from the Learning to Teach Secondary Mathematics project (LTSM), a five-year longitudinal study of how individuals learn to teach middle and secondary school mathematics. The data emphasis is on transcripts of the conversations between teacher and students working in small groups, but also used data from surrounding interviews.

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CURRICULUM NEGOTIATION AND THE IMPLEMENTATION OF CONCEPTUALLY DIFFERENT MATERIALS: PRESERVICE ELEMENTARY STUDENT TEACHERS' PLACE-BASED EXPERIENCES

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The proposed poster will present the results of research addressing the following question:

Within student teaching placements, how are school- and classroom-related factors associated with the use of conceptually different mathematics curriculum materials negotiated in the classroom?

Study Motivation

Preservice teachers' experiences in traditional mathematics classrooms are believed to play an important role in shaping their beliefs about teaching and their eventual classroom practices. Their personal lack of familiarity with generating and testing mathematical conjectures and with participating in discourse on mathematical concepts as learners has often been cited as a challenge to their ability to implement reform objectives. Numerous suggestions have been made that teacher education provide preservice teachers with these types of learning opportunities.

The proposed poster will present findings from a longitudinal research project in which preservice elementary teachers learned mathematics in a yearlong course sequence incorporating reform-oriented curriculum materials and instruction during the fall 2000 semester. This same group of students is currently completing their student-teaching internships during the spring 2004 semester. Their teacher preparation experiences were similar, but the student teaching settings for the two participants, Kate and Linda, are quite different. Kate's school is located in a rural area about two miles from a major university, is utilizing the reform-based curriculum materials, *Everyday Mathematics*, and is considered a very good school. Linda's school is in an urban area, has had some recent difficulties with national standardized student testing, and is utilizing more traditional text materials. Given that both of these student teachers have had personal experiences as learners with reform-based materials and instruction, the site- and curriculum-level differences evident here provide for interesting comparisons and contrasts of curriculum-related student teaching choices, constraints, and mathematics teaching practices.

Data Collection and Analysis

Data collection addressed classroom- and school-related factors associated with these student teachers' planning, teaching, and overall shaping of the mathematics curriculum. Linda was observed nine times during her student teaching and interviewed five times. Kate was visited ten times and interviewed five times. All classroom observations were audiotaped and field notes were taken during each observation, with more detailed field notes typed up immediately following each visit. All interviews were transcribed and reviewed before the next set of interviews.

Data for each preservice teacher is currently being analyzed as two case studies, with common themes considered across cases. Thus far, school and support issues associated with national standardized testing, the availability and nature of curriculum resources, school and classroom norms for behavior, and time have surfaced as several of a variety of factors contributing to the shaping of the curriculum in different ways at the two different sites.

Poster Details

The proposed poster will communicate emergent themes related to curriculum negotiation, with comparisons and contrasts drawn across the cases of Kate and Linda. Segments of field notes and quotes taken from interview transcripts will serve as illustrations of these comparisons and contrasts.

MATHEMATICAL FLEXIBILITY: A CASE FOR UNDERSTANDING CO-FUNCTIONS

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In Adding It Up, a major report from the National Research Council (NRC), the authors conclude that three major components of mathematics teachers knowledge are necessary for effective mathematics teaching: knowledge of mathematics, knowledge of students, and knowledge of pedagogy (Kilpatrick, Swafford, & Findell, 2001). This poster reports results of a study of pre-service secondary school mathematics teachers' knowledge of trigonometry. The study takes careful account of the accumulated data and theories of teacher knowledge that point to the complexity of knowing (Dossey, 1992; Fennema & Franke, 1992; Hiebert et al, 1997; Hiebert & carpenter, 1992; Koehler & Grouws, 1992; Leinhardt & Smith, 1985; Shulman, 1986, 1987; Glasersfeld, 1996). An important cornerstone for the present study is Liping Ma's elaboration of Shulman's characterization of teacher knowledge into what she calls "profound understanding of fundamental mathematics [PUFM]" (Ma, 1999).

In phase 1 of the study, 14 participants completed two concept maps from emic and etic perspectives, two card-sorting activities, and a test of trigonometric knowledge. In phase 2, five of the 14 participants were studied in-depth using case study methodology. Results indicated as a group the pre-service teachers' knowledge of trigonometry was uneven and that several fundamental ideas were poorly understood. In particular, their knowledge of periodicity, radian measure, co-functions, reciprocal functions, 1-1 functions, inverse trigonometric functions, identities, and sinusoids lack depth. The findings support a conclusion that pre-service teachers' knowledge of school mathematics may not be sufficiently robust to support meaningful instruction on some key trigonometric ideas.

The focus of this poster presentation is pre-service teachers' conceptions and misconceptions of co-functions, and the interferences presented by other notions such as inverse trigonometric functions and reciprocals of trigonometric functions. The aforementioned study revealed that, as a group, the 14 pre-service secondary school mathematics teachers' knowledge of co-functions was particularly limited. The concept maps and the interviews showed that the pre-service teachers possessed weak understanding of the meaning of the *co* in the following co-function pairs (sine – cosine; tangent – cotangent; secant – cosecant). For example, 10 of the 14 participants used connectives such as *inverse, reciprocal* and *co-functions* to relate the co-function pairs in their concept maps, indicating that they saw *inverse, reciprocal* and *co-functions* as equivalent.

Knowledge and understanding of co-functions allows one to meaningfully engage the relationships among the aforementioned co-function pairs, which in turn support versatility and adaptability in problem solving. Knowledge of co-functions is also helpful in simplifying expressions to yield equivalent, yet simpler, expressions that facilitate proofs and enhance the process of problem resolution. The pre-service teachers' limited understanding of co-functions inhibited their flexibility in resolving trigonometric questions involving analysis of inverse trigonometric functions and their properties, triangle resolution, and proofs. Therefore, the poster presentation raises questions about what can and should be done at the high school level and at the college/university level. The criticality of the high school experience cannot be overemphasized because pre-service teachers exposure to fundamental concepts of trigonometry occurs in high schools.

BUILDING COMMUNAL UNDERSTANDING OF PROPORTIONAL REASONING USING INSTRUCTIONAL REPRESENTATIONS

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Pre-service teachers' conceptual understanding of the mathematical content they will teach is an important area of research in mathematics education. Research has shown that in-service and pre-service teachers lack a deep conceptual understanding of the mathematics they teach or will teach (Ball, Lubienski, & Mewborn, 2001). This study aimed to prepare future middle grades and high school teachers by deepening their conceptual understanding of proportional reasoning. The framework guiding this research was Lave and Wenger's (1991) situated learning. By situating the learning of high school mathematics within the context of lesson planning and instructional representations we expected to increase the pre-service teachers' understanding of proportional reasoning topics in the high school mathematics curriculum. We define instructional representations as the words, pictures, graphs, tables, contexts, problems, and tasks teachers use to communicate mathematical ideas to their students.

This qualitative study was based on assignments given to pre-service teachers enrolled in their first mathematics education course. Five topics and the connections to proportional reasoning were explored in these assignments: ratio and proportion, slope, similarity, right triangle trigonometry, and the unit circle. Pre-service teachers created four different instructional representations for each of these five topics that became the basis for classroom discussions. Evidence was collected from the representational artifacts and the field notes taken from classroom presentations and interactions. Analysis indicated growth in the pre-service teachers' conceptual understanding while developing their sense of a professional community.

Engaging pre-service teachers in mathematical discourse is a difficult process because they do not want to reveal their lack of mathematical understanding. Focusing on what to show students, what to tell students, and how to engage students through the use of instructional representations places the focus on teaching rather than the mathematics. They more readily join in community having thought about the representations individually in the assignments and then collectively as a community in class. By selecting high school mathematics topics related to proportional reasoning, we connected the learning across topics in a meaningful and contextual way.

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Teacher Knowledge

EXAMINING THE TEACHER'S MODES OF PARTICIPATION WITHIN A COMMUNITY OF INQUIRY

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Current calls for reform in mathematics education suggest transforming classrooms in learning communities. The activity of teaching in such classroom is focused on group inquiry and discourse. In these classrooms, it is assumed that the teacher acts as facilitator of knowledge (Borasi 1992; Wood, Cobb & Yackel 1991; Yackel & Cobb 1996). In fact, reform documents such as Professional Standards for school mathematics (1992) explicitly call attention to the teacher's role as one who "facilitates" learning of "all" students in such learning communities. Indeed, the metaphor of teacher as "facilitator" of learning is now widely used to characterize the role of the teacher within learning communities. References to this metaphor have been widespread within the educational circle. However, the notion of teacher as facilitator is inherently too general to highlight the multidimensional and complex modes and levels of participation demanded of teachers within such environment. More importantly, it is unclear how these multiple modalities of intervention and discourse change over time from the first attempts at initiating-establishing, to the later phases during which the teacher sustains the functioning of a learning community. In order to increase knowledge about ways in which the teacher adjusts his or her participation in whole class interaction within a learning community, detailed contextual and long term analyses are needed. Moreover, needed are more reports of studies that focus on the day-to-day events in the real world of the classroom life. Every day events are often left to the imagination of the teacher ending in frustration from attempting reform based strategies. This study serves to provide a detailed view of the multidimensional roles the teacher had to play to initiate and sustain a learning community.

The primary concern of the research study we report here was to develop a deeper understanding of the role of the teacher within a learning community. Of particular interest was to prepare a profile of teacher's actions, her modes of participation and discourse, and her contextual strategies when orchestrating a learning community in a naturalistic context of a heterogeneous classroom. As the study commenced one broad question guided the study: How does the teacher create a classroom environment in which students appear to engage in learning mathematics through inquiry and discourse? As the study developed throughout the year, two focused questions guided our data collection and informed our analysis. These included:

- What roles did the teacher play in facilitating students' inquiry and discourse about mathematics?
- What were the key aspects of the teacher's instruction that helped facilitate the development of a learning community?

Theoretical grounding: Community of Learning

The theoretical background of learning community has been influenced by the work of John Dewey (1902, 1916, 1963) as it relates to discovery learning and democratic classroom practices. More recently, these ideas have been elaborated in the work of Ann Brown (1992) and her colleagues (Brown & Campion 1990, Brown, Campione, Reeve, Ferrera, & Palinscar 1991), who have used the concept of community of inquiry in classroom contexts to study its promise for developing students' cognitive strategies. Another interpretation of the notion of a learning community has been proposed by Lipman and his colleagues (Lipman, 1991, 1993; Lipman, Sharp, Oscanyan, 1980; Sharp & Spliter 1995). In this notion of a learning community, a special emphasis is placed the development of students' critical thinking. Despite some theoretical and pedagogical differences between the two versions of the learning community, both versions emphasize the role of social interaction and reflective inquiry in mediating the development of students' learning.

The theoretical underpinnings of a learning community have also been strongly influenced by the sociocultural theories of learning. In the sociocultural perspective, language and other semiotic tools are seen to play an important role in the construction of proximal zones for learning, during which socially shared meaning making rouses new perspectives and possibilities yet to be discovered (Vygotsky, 1962, 1978; Wells 1999). From this perspective, learning is a social and cultural process that takes place through participation in socially shared activities with more knowledgeable members of culture (Rogoff & Toma 1997; Wertsch, Hagstrom & Kikas 1995).

The metaphors of the zone of proximal development, and expert Scaffolding (Wood, Bruner & Ross 1976) have influenced many researchers to devise instructional procedures or pedagogical modes for guiding classroom interactions and learning, with a special focus on calibrated assistance and the nature of the interactional support adults and peers can offer to learners. A well known pedagogical strategy grounded in the socio-cultural perspective is called Collective Argumentation, which was developed by Brown and Renshaw (2000) in order to create diverse communicative spaces in the elementary classroom. The notion of collective argumentation draws on five principles that are required for coordinating different perspectives in classroom interaction. These are the principles of generalizability, objectivity and consistency as well as consensus and recontextualizaton principles. In drawing on these principles, a key format of Collective Argumentation is introduced to the students; namely, represent, compare, explain, justify, agree and validate which the students can use in coordinating the phases of interaction in their small groups. The use of these strategies is realized in small group situations in which the teacher first guides the students in sharing their personal views or interpretations of the problem or task in question. This is followed by comparing, explaining and justifying various perspectives in small groups, by establishing a joint agreement, and then by presenting the group's joint representation to the whole class for validation. The teacher's participation in the interactions of the small groups includes allocating management of the problem solving process to the group, reminding the students about the norms of participation, supporting the development of conjectures and refutations, modeling ways of constructing arguments and the use of appropriate domain specific language, encouraging the class to engage in the evaluation of co-constructed arguments or perspectives, and providing strategies for dealing with interpersonal conflicts (Cobb, Yackle, & Wood 1996; Borwn & Renshaw 2000).

In the current work, we aimed to elaborate on the notion of facilitating learning in a classroom that worked as a learning community. By investigating the nature of social interactions in which the classroom community engages, the study aimed at highlighting the teacher's modes of participation when orchestrating and guiding classroom discussions following the notion of a learning community.

Methodology

The data was collected over the course of a year long teaching experiment in a sequence of two mathematics courses designed for prospective elementary teachers. The teacher whose work we traced espoused and enacted a philosophy that matched the goals of a learning community. In the research class, the instructional time was divided into three parts. First, the teacher posed a problem or task. Students broke into small group discussion of problems. A collective, whole group discussion followed the small group work during which the students shared their ideas with one another and negotiated their accuracy and validity using mathematics. Our plan was not to fit the unfolding of classroom interactions into strict coding schemes but rather attempted to convey a holistic and comprehensive view of the teacher's strategies, which supported the development of a learning community from the cognitive, social, and socio-emotional viewpoint during the teaching episodes.

Data sources consisted of interviews with the teacher and classroom observations. All class sessions over 32 weeks of instruction were videotaped. The data used in the analysis of this paper come from 40 hours of classroom videotapes and 35 hours of interviews with the teacher. The selected units of analysis for this report although representative of the type of events occurring throughout the year, contained critical incidents in which the teacher's decisions about dealing with students' mathematical work on the one hand, and her treatment of their work became explicit. Here, critical incidents refers to a series of interactions between the teacher and students eliciting actions that resemble the descriptions of a community of learning and teacher's role as appear in various reform documents, such as the Principles and Standards for School mathematics (2000), and in the literature.

All data sources were analyzed individually (in this case, the sequence of teaching episodes in each class or related to each critical incident) using a model of analytical induction (Bogdan & Biklen 1992; Miles & Huberman 1994) and then together in an effort to challenge developing assertions and conceptualizations. In this analytical approach, conceptions or working hypotheses to explain the phenomenon of interest are continually formed as data are analyzed. Emerging hypotheses and explanations are consistently tested against subsequent data (of either different type or source) and typically expanded to encompass new cases (in this situation, different teacher roles and strategies) that do not fit the previous formulation. The ultimate goal is to arrive at a relatively universal explanation (for the phenomenon under investigation) that has been derived from the consistent formulation and then reformulation of emerging explanations during data analysis. The numerous sources and types of data (e.g. interviews, observations, and instructional materials) allowed for triangulation of data and the construction of value profile of the teacher's practice.

Results

Characterizing the teacher's practice

Eight key characteristics emerged from triangulating the various data sources. These included: situating instruction in authentic tasks, sensitivity to mathematical detail and precision; importance of grappling with problems and tasks; development of a mathematical point of view; fostering collaborative problem solving and group decision making; the importance of sense making and personalization of knowledge; collaboration of teacher and learning; and assessing validity and soundness of arguments.

Teacher roles within the learning community

The teacher adopted and enacted eleven different roles within the learning community as she attempted to initiate and sustain a learning community:

1. The role of *motivator* involved the teacher encouraging students to take responsibility for their own learning.
2. The role of *diagnostician* involved the teacher giving students opportunity to express ideas in order to discern their understandings.

3. The role of *guide* involved the teacher directing students and helping g them develop strategies.
4. The role of *innovator* involved the teacher designing instruction by using new ideas.
5. The role of *experimenter* involved the teacher trying out new ways to teach and assess students.
6. The role of *researcher* involved the teacher evaluating his or her own teaching and engaging in solving problems.
7. The role of *modeler* involved the teacher showing the attitudes and attributes of the mathematician by example.
8. The role of *moderator* involved the teacher coordinating small and large group discussions, as well as individual growth on the part of each student.
9. The role of *curriculum decision maker* involved the teacher developing teaching materials to be used in class according to the unfolding of students' mathematical understanding.
10. The role of *learner* involved the teacher opening oneself to learning new mathematical concepts and tools in light of her realization of students' cognitive needs, or their discoveries.
11. The role of *instructor* involved that the teacher recognize the mathematical importance of the ideas presented in class and be able to manipulate her own knowledge to make that knowledge accessible to students.

GUIDING THE MATHEMATICS OF MATHEMATICAL DISCUSSIONS

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Despite calls for reform that support the enactment of collaborative learning environments in mathematics classrooms (NCTM, 1989, 2000), implementation of such environments is not widespread. There remains a pressing need for research that focuses closely on teachers' successful enactment of reform pedagogies (Boaler, 2003). An assumption of this study is that one reason for the lack of implementation of reform ideals is that insufficient attention has been given to the mathematics and how teachers can productively guide the development of mathematical ideas during collaborative activities. Findings from an in-depth longitudinal case study of a collaborative high school mathematics classroom are presented. A conceptualization of the teacher's role in guiding the mathematics is developed, and particular strategies that supported this student-centered collaborative environment are identified.

Introduction

In recent decades, there has been a multitude of reform efforts in mathematics education aimed at generating more collaborative and student-centered learning environments. In the United States, the vision of reform has been guided by the National Council of Teachers of Mathematics (NCTM) *Standards* documents (NCTM, 1989, 2000). In classrooms that exemplify this vision, students actively engaged in mathematical sense making and communicating with one another (NCTM, 2000). There is mounting evidence demonstrating the value of these environments for students' learning (Boaler, 1997; Stein et al., 2000; Wood & Sellers, 1997) and fostering a productive disposition towards mathematics (Boaler & Greeno, 2000). However, such classrooms remain relatively rare, particularly on the secondary level (Stigler & Hiebert, 1999).

One factor contributing to their lack of prevalence is our underdeveloped understanding of the complex pedagogies required to support these more interactive, discussion-based learning environments (RAND, 2002). In particular, insufficient attention has been given to the mathematical ideas and how they are developed through collaborative activity. Too often "surface features" associated with reform practices, such as groupwork or high levels of interaction among students, are taken as evidence of successful realization of the reform ideals. Yet, classrooms supporting these features may not support the quality of mathematics instruction espoused by the Standards. As the most recent version of the Standards (2000) states:

> As with any educational innovation … the ideas of the Standards have been interpreted in many different ways and have been implemented with varying degrees of fidelity. Sometimes the changes made in the name of standards have been superficial or incomplete. For example, some of the pedagogical ideas from the NCTM Standards—such as the emphases on discourse, worthwhile mathematical tasks, or learning through problem solving—have been enacted without sufficient attention to students' understanding of mathematics content. Efforts to move in the directions of the original NCTM Standards are by no means fully developed or firmly in place. (p. 5)

Thus there remains a pressing need for research that focuses closely on teachers' successful enactment of reform pedagogies (Boaler, 2003), and specifically, how teachers guide students' engagement with mathematical ideas and participation in mathematical practices.

The study presented here begins to address this need. Through an in-depth analysis of one class of a highly accomplished teacher, Ms. Nelson, I sought to better understand the organization and development of collaborative mathematics practices in high school mathematics classroom over time (see Staples, 2004). Ms. Nelson's students regularly worked collaboratively to forge new understandings of mathematical concepts. They attended to and extended each others' ideas thereby engaging in a highly generative kind of intellectual activity not often supported in mathematics classrooms. There was also constant attention to conceptual understanding and sense making, as well as justifying and supporting one's results. The results presented here focus on how the teacher successfully guided the mathematics as the students participated in whole-class discussions and other collaborative activities.

Conceptual Underpinnings

Research has demonstrated that effective implementation of collaborative, student-centered classrooms is difficult and demands more complex pedagogies than traditional instruction. One challenge germane to this innovative instructional format is developing the mathematical ideas in a manner that maintains a central focus on students' thinking while also addressing content learning goals. Jaworski (1994) referred to this as a "didactic tension," where teachers want to value, honor, and pursue students' thinking and yet also develop students' understanding of a particular body of mathematics. Effectively resolving this tension requires that the teacher be able to leverage a student's idea in a manner that is instructive for class's learning as a whole.

Understanding how teachers can effectively work with students' thinking has been the focus of several analyses of classrooms aligned with NCTM's vision of good practice (e.g., Ball, 1993; Lampert, 1990; Schifter, 2001). Teachers must be able to elicit a student's thinking as well as identify the core mathematical ideas the student is trying to make sense of. Ball (1993) refers to this as "hearing students' mathematical insights." The teacher must also have an understanding of the *mathematical terrain* of the idea (Lampert, 2001), which comprises related mathematics concepts and the connections among them. Without such an understanding, the teacher is limited in her ability to build on a student's idea to extend the class's mathematical exploration.

Other analyses have focused on the nature of the tasks set for students and how the teacher shaped task implementation to support student discussions and productive collaboration (Cohen & Lotan, 1997; Stein et al., 2000). The task has a set of constraints and affordances (Greeno & MMAP, 1997), but the specific implementation of the task directs the mathematics students' encounter and how ideas are developed during the course of a lesson. Stein and her colleagues (Stein et al., 1996; Stein et al., 2000) have demonstrated that tasks initially set at a high level of *cognitive demand* are likely to decline in demand during implementation. They explore factors that lead to this decline, such as the teacher taking over demanding aspects of the task or students' activity declining to unsystematic exploration. When this happens, the teacher's thinking and approach become primary and it closes down the collaborative process for students.

Although there are notable exceptions (e.g., Ball, 1993; Chazan, 1999; Lampert, 2001), the preponderance of the research on organizing discussions in mathematics classrooms has examined the shortcomings or downfalls of lessons where collaboration has not been sustained, student thinking not pursued, and mathematical concepts not developed (e.g., Chazan, 1999; Cohen, 1990; Ensor, 2000; Heaton, 2000; Jaworski, 1994). In general, we are well aware of the challenges, but relatively uninformed about how to productively conceptualize and understand the teaching that supports the development of mathematical ideas through collaboration. This study takes steps towards building that knowledge by analyzing a teacher and class that regularly worked collaboratively to extend their mathematical understandings.

Methods and Data Sources

Data for this study was collected during the 2000-2001 school year. The focal case was Ms. Nelson's ninth-grade pre-algebra class comprising twenty students. Ms. Nelson was a highly experienced (Nationally Board certified and a recipient of the Presidential Science and Math Award for Teaching) and consistently organized collaborative inquiry activities that attended to the development of students' conceptual understanding regardless of course of class composition. This was validated by a researcher team for the Stanford Mathematics Teaching and Learning Study (Principal Investigator, Dr. Jo Boaler; see also Boaler & Staples, 2003). The sampling strategy then was *purposive* (Yin, 1994) and the case was chosen to offer insight into teacher practices that support mathematically intensive collaborative learning environments.

The research was conducted from an interpretivist paradigm, drawing upon ethnographic methods described by Eisenhart (1988). Data collection included videotapes and observations of lessons (105 hours); interviews with students (19); teacher interviews (4); and video viewing sessions with the teacher (7). In addition Ms. Nelson and I had frequent conversations after lessons. Data analysis followed principles of grounded theory (Strauss & Corbin, 1990). From the process of coding field notes and video content logs patterns were identified. This process included codifying particular teaching strategies that seemed to support students' participation in mathematical collaborative activities. A refined set of codes was applied to ten transcribed videotapes of whole-class collaborative activities. In addition, videos of sequential lessons were analyzed to trace the development of mathematical ideas over time. Finally, analyses were refined and validated by reviewing videotapes and my analyses with Ms. Nelson. She confirmed that the results presented an accurate portrayal of her practice, both the description of the practice and how it functioned to support students' participation and learning in the classroom.

Results

Results indicated that Ms. Nelson guided the development of the mathematical ideas during whole-class discussion by simultaneously centralizing students' ideas and thinking, pursuing high-level task implementation, and attending to mathematics (here, algebra) as a body of knowledge and set of practices that students were learning. I discuss each of these three commitments and examine particular pedagogical strategies that Ms. Nelson used to guide development of the mathematics during collaborative discussions.

Centralizing students' thinking

At the heart of the class's collaborative discussions was students' thinking. Ms. Nelson regularly used students' ideas as the basis of the class's exploration of a problem or concept. She constantly elicited students' ideas, asked for explanations, and had them present their solutions at the board. These strategies are familiar ways to bring students' ideas to the fore.

Ms. Nelson went beyond this however. She recognized that for a student's idea to be used as the starting point for the class's explorations, other students needed to comprehend and consider the idea in order to have an opportunity to learn from the subsequent discussion. Towards this end, Ms. Nelson employed a variety of strategies. For example, she frequently asked students to repeat their ideas and record them on the board. She also restated them verbatim herself. Indeed, it was rare to have a student's idea only stated once. Ms. Nelson also asked questions that helped the class access the presenting student's idea. For example, she might ask, "Why did you decide to use *n* instead of *x* in your work?" or "Can you tell us more about how you set up your graph?" These questions were not asked for the purposes of evaluation, but to create an opportunity for the class to make sense of another's representation and understand their idea.

Ms. Nelson also directed conversations so that the students built upon one another's ideas. Guiding such conversations can be particularly challenging when a student's idea is incorrect mathematically. The teacher must manage social and intellectual aspects of the exchange in a manner productive for the class and supportive of the presenting student. One strategy Ms. Nelson used to maintain the class's focus on the original work was to insist that the next contributor ask a question of the presenter or identify what he thought might be problematic and why. This was particularly important to maintain coherence of the conversation as students were often quite eager to tell the "right" answer. The ensuing discussion often focused on analyzing the presenter's work. The students would probe the logic behind the solution or idea, and then attend to the limitations as well. This validated the presenter's work, drawing out its logic and mathematical soundness, and also raised important mathematical issues that were then discussed.

In many respects then, Ms. Nelson guided the mathematics by following. She attended carefully to students' ideas and pursued the mathematics offered in their thinking. When I asked Ms. Nelson about this approach, she commented that she trusted the student's idea would lead them to some productive issue or discussion. Within their ideas was important mathematics and her charge was to draw these out and help the class make sense of them.

Implementing tasks at a high-level of cognitive demand

As Ms. Nelson centralized students' thinking, she also guided the mathematical discussions by implementing tasks at a high level of cognitive demand (Stein et al., 2000). Indeed, these pursuits were complementary. Even with potentially routine tasks, Ms. Nelson engaged students in sophisticated mathematical thinking and reasoning.

One important strategy Ms. Nelson used to maintain a high level of cognitive demand and guide discussions was ongoing assessment of students' understanding using a *small inferential gap*. Teachers constantly make inferences about what students understand from the evidence available to them, which in turn inform how the teacher proceeds (e.g., deciding what question to ask; whether or not to move on). The inferential gap is the "distance" between the understanding that the teacher infers and what the student understands. Inaccurate inferences can jeopardize the maintenance of a high level of cognitive demand. Although mathematical activity may continue, the ideas develop on a weak foundation and the quality of the discussion is compromised.

Ms. Nelson consistently used a small inferential gap, tightly coupling the available evidence with her inferred level of students' understanding. This helped her effectively uncover student difficulties and guide the discussion to address these. It also maintained a level of rigor where there were high standards of evidence and reasoning. For example, in one lesson, they were trying to determine what the graph of the equation M=2F would look like, where M was the number of hired hands and F the number of families. The class had some experience with plotting points and with graphs of lines and rays. This problem proved quite challenging for them and there was a lot of discussion about valid points and the graph. At one point, the students had sorted out that the "odd numbers would be missing," as it was impossible to have an odd number of hired hands. Ms. Nelson inquired again what the graph would look like. Some responded that it would be a ray. Others were not sure. Notice that although students seemed to understand that there would be gaps, Ms. Nelson did not infer that they fully understood what the graph would look like. She continued to explore students' understanding of the term ray, and their understanding of what points would and would not be included in the graph. In this manner, she did not make assumptions about where their difficulty might be, but gathered further evidence to make an inference. Indeed, this led them to a discussion of discrete verses continuous.

Maintaining a high level cognitive demand during task implementation is particularly challenging at times when students are not generating new ideas or showing signs of frustration. Ms. Nelson did not solve this problem by taking over cognitively demanding aspects of the task. Rather, she kept students positioned as mathematical thinkers and provided them with "food for thought" to spawn their thinking. She might clarify an intellectual issue with which they were wrestling, or offer a new representation to help them think about the mathematics from another perspective. At other times, she recounted their intellectual journey, describing how they came to their current point in the problem solving process. These intellectual scaffolds support students' thinking about the problem without taking over demanding aspects of the task. In this manner, Ms. Nelson provided students with something to consider and use to generate ideas—food for thought—and not with a "thought" itself. Behind this approach was Ms. Nelson's stance that students *can* do the math. When students seemed to be having difficulty with a problem, her assumption was that they had not yet had enough experience thinking about the problem to have developed their ideas, and not that they needed her to explain the mathematics to them.

Guiding with a developmental map of algebra learning

Ms. Nelson had a well developed conceptualization of how students learn algebra which was brought to bear in her interactions with students around various mathematical ideas and concepts. Ms. Nelson saw today's task and ideas in relation to tomorrow's and next week's. During lessons, then, Ms. Nelson requested, pushed for, or pursued particular ways of representing and thinking based on how she understood students to develop proficiency over time. I describe this conceptualization of how students learn as her *developmental map of algebra learning*. This map comprised transition points and ways of knowing that were critical steps for students as they developed proficiency with various topics.

To illustrate this aspect of Ms. Nelson's teaching, I consider the topic of functions and report on the activities of the first six weeks of school. During time the emphasis was on recognizing a pattern in a table of values and representing that relationship symbolically. Over the six weeks of instruction, Ms. Nelson guided the students towards increased proficiency by following a trajectory that attended to these "transition points."

• identify a pattern relating the values in the table, and fill in missing values
• verbally explain a process for finding missing values in one's own words
• verbalize a single relationship between the input and output values (e.g., they add to 18)
• write a rule using the words IN and OUT (e.g., the IN and OUT together make 18)
• write the rule as an equation, using only symbols (e.g., IN + OUT = 18)
• write the rule in the form OUT = some expression (e.g., OUT = 18 – IN) As Ms. Nelson worked with students' ideas about various problems, she pushed the students to formulate the pattern and think in these particular ways, even when students could complete the problem without doing so. For example, a student Kurt explained in response to a particular table of values (oriented horizontally) in the following way: "Multiply the bottom by 7 if the top is missing, and if the bottom's missing, divide the top by 7." Kurt thus saw a relationship, but stated it as two rules with a general reference to the "top" and "bottom." Ms. Nelson pressed on his thinking, asking him to come up with *one rule* that worked to find all the missing values, and to use the words IN and OUT as he explained. In this manner, she was pushing him to a new way of thinking that (she believed) would support the development of his proficiency.

Similarly, later in the six-week sequence, Ms. Nelson explained to students that a rule such as IN + OUT = 18 is good and correct, but that the *best way* to write rules is in the form "OUT =." Thus Ms. Nelson honored their present approach, but pushed them to recast the relationship they saw in a manner that more closely aligned with how explicit functions are conceptualized. Indeed, the difference between conceptualizing a pattern as IN + OUT = 18 and OUT = 18 – IN, while trivial for those well versed with mathematics, was significant for these students. Ms. Nelson recognized the different demands placed on the students and carefully attended to these differences as she worked to develop new ways of thinking and proficiencies.

Discussion and Conclusion

The study provides a detailed analysis of how one teacher guided the mathematics during lessons aligned with current calls for mathematics reform by centralizing students' thinking, implementing cognitively demanding tasks, and attending to a developmental map of algebra learning. The results reported here echo some earlier findings, such as the importance of eliciting students' ideas. They also point to other aspects of teaching practice that may be critical components of organizing productive, collaborative learning environments, such as using a small inferential gap to guide task implementation. Further research is needed to examine the potential value of these constructs in other settings.

The role of teacher knowledge also comes to the fore in this study. Ms. Nelson's developmental map of students' algebra learning is a particularly interesting component of her knowledge. This knowledge can be classified as *pedagogical content knowledge* (PCK) (Shulman, 1986), and yet it seems to extend beyond this. The knowledge represented in this developmental map in some respects is similar to what Ma (1999) has called *profound understanding of fundamental mathematics*. Her focus was Chinese and American elementary teachers' knowledge of basic operations. Some of the Chinese teachers had an intricate and well-connected conceptualization of the connections between mathematical ideas called *knowledge packets.* The organization of the knowledge packet was based in part of mathematics and relationship among ideas, but to large extent was informed by students' thinking and how students develop proficiency by progressing through a certain sequence of activities (e.g., addition of digits, addition within 20, etc.). Ms. Nelson's map is similar in that respect. It relies on a deep understanding of algebra, but is organized based on how students might learn, including critical transitions in students' ways of thinking. More research is needed to identify the kinds of teacher knowledge needed to support classrooms aligned with NCTM's vision of good practice.

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A WORK IN PROGRESS: THE CHALLENGES OF CHANGE IN SECONDARY MATHEMATICS TEACHING

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Listening to students' ideas and responding in ways that further the mathematical development are not easy tasks for the teacher. Furthermore, we know little about how experienced teachers can learn such listening and responding within the context of their existing practices. In this paper, we report on how an experienced secondary teacher worked to develop a practice that would be characterized by less telling and more listening. We describe how the teacher listened and responded to his students, while simultaneously developing alternative pedagogical strategies to direct telling. The teacher was able to drive the lesson forward by assessing students' partial understandings, responding with analogies, and supporting multiple students in expressing their ideas about the equivalence of two exponential function transformations.

Large gaps continue to exist between the vision of reform espoused in the national standards documents and the practice found in many secondary classrooms. Many arguments have been given for the continued difficulties in achieving substantial change in teaching practice. Some researchers have pointed to the challenges inherent in changing the larger systems within which the activity of teaching is embedded (Grant, Peterson & Shojgreen-Downer, 1996). Others have argued that the lack of a professional knowledge base for teaching and the limitations of teachers' mathematical knowledge for teaching (Cooney, 1999; Hiebert, Gallimore, & Stigler, 2002) are impediments to substantial change in practice. In this paper, we argue that research on the development of teachers' knowledge in practice is an essential step in understanding how substantial change can occur. We present the results of a study of an experienced secondary mathematics teacher as he implemented innovative curricular materials on exponential growth and decay. The results illuminate the challenges found in "not telling" and in finding ways to use student thinking to drive the lesson.

Theoretical Background

Despite the rather large body of research on students' thinking about functions (Dubinsky & Harel, 1992; Leinhardt, Zaslavsky & Stein, 1990; Sfard, 1991; Vinner & Dreyfus, 1989) and important related work on teachers' understandings of function (Even, 1990; Even & Tirosh, 1995; Zazkis, Liljedahl & Gadowsky, 2003), there is relatively little research that focuses on how teachers learn to use an understanding of students' thinking about functions in their practice at the secondary level. Current reform-based rhetoric that would exhort teachers to listen and to "not tell" does little to provide insight into what teachers could do and when and why (Chazan & Ball, 1999), while simultaneously eroding teachers' sense of efficacy when left with little sense of a new role (Smith, 1996). Furthermore, such exhortations tend to leave the notions of listening and responding as unproblematic. As Hammer and Schifter (2001) pointed out, if teachers are to invite their students to articulate their thinking, then the teachers must "hear those ideas, diagnose their virtues and weaknesses, and incorporate them into the substance of instruction" (p. 442). Listening to and responding to the multiplicity of conceptual developments that may be taking place in the classroom is not an easy task for the teacher. The teacher needs to choose various strategies to further the students' mathematical development. Such strategies could include the use of appropriate representations and the connections among those representations, a repertoire of probing questions, or insightful ways of using computational technologies. The ways in which a teacher might respond to students' mathematical activity is, of course, dependent on what it is that the teacher hears, sees and interprets in that activity in the first place.

In this paper, we draw on a theoretical perspective of models and modeling (Doerr & Lesh, 2003) in order to examine the development of practitioner knowledge about students' mathematical learning. Our emphasis in this research is less about what teachers *do* in their classrooms and more about how they *see*, *interpret*, and *respond* to the events in their classroom. In other words, our goal is to examine teachers' thinking as it is revealed in the context of their interactions with the developing mathematical reasoning of their students. We seek to understand *how* and *why* the teacher was thinking in a given situation, that is, interpreting the salient features of the event, integrating them with past experiences, and anticipating actions, consequences, and subsequent interpretations. Researchers have argued that teachers need to be more attentive to their students' mathematical reasoning, to interpret and analyze their students' mathematical ideas, and to challenge them to revise or extend those ideas in order to become more powerful mathematical thinkers. Yet teachers do not automatically hear their students' reasoning or deeper understanding, rather, they tend to listen to their students' responses for the purpose of evaluating the correctness of their answers (Davis, 1997; Evan & Tirosh, 1995; Heid, Blume, Zbiek, & Edwards, 1999). However, listening to another person's reasoning is not an unproblematic activity, and the task of responding in appropriate ways is equally challenging. Moreover, images of teachers who are able to listen and respond do not necessarily provide us with the foundation for understanding how it is that teachers can learn to listen and respond to the multiplicity of ideas that are present in a classroom of learners.

Our goal in this study is to understand two aspects of a teachers' learning through his interactions with his students as they engage in tasks involving transformations of exponential functions. First, we describe how the teacher listened to and interpreted the salient features of his students' problem-solving activities. Second, we examine how the teacher responded to and supported his students' developing mathematical reasoning about the task. In so doing, we reveal the challenges that the teacher encountered as he worked on shifting his teaching practice to find ways to use students' thinking more centrally in his instruction.

Methodology

This study is part of a two-year research project using modeling tasks to investigate the teaching and learning of the mathematics of exponential growth and decay. Using the design of the multi-tiered teaching experiment (Lesh & Kelly, 1999), teachers examined the development of students' learning and researchers examined the concurrent development of teachers' learning. The students were engaged in making sense of a sequence of model development tasks (Lesh, Cramer, Doerr, Post & Zawojewski, 2003) for exponential growth and decay. The curricular materials were designed to engage students in the development of significant mathematical ideas about exponential functions and to shift the central tasks of teaching from direct instruction to one of understanding and using students' thinking. The particular lessons reported in this paper focused on an exploration of transformations that occurred near the end of the sequence. The essential task was to understand how a function such as $y = 2^{x+2}$ could be been as both a horizontal translation and as a vertical stretch ($y = 4 \cdot 2^{x}$).

The participant in this study was a secondary teacher with 18 years experience and a class of 28 pre-calculus students. The teacher participated in summer workshops and school-year meetings with eight other teachers in the project. Collectively, these teachers had investigated a sequence of modeling tasks designed to elicit the development, exploration, and application of exponential functions. Throughout the workshops, the teachers were encouraged to examine the ways that students' ideas about exponential functions might develop and to engage their students in expressing their developing ideas throughout the unit. The particular lesson that is the focus of this study provides students with an opportunity to explore the relationship between the graphical and algebraic representations of the transformations of an exponential function.

These two lessons on transformations occurred during the second year of the study. The data sources included researchers' field notes from classroom observations of two consecutive lessons, the videotapes of the lessons, the transcriptions of the videotaped lessons and informal conversations with the teacher that occurred after the lessons, and semi-structured interviews before and after the unit. The data analysis was done in three stages. The first stage of the analysis involved open-ended coding (Strauss & Corbin, 1998) of the field notes and the transcripts of each lesson. Each author did this coding independently for the first lesson. We then met to compare our codes; differences in coding were resolved by finding references to early codes and making comparisons and revisions to the codes. In the second stage of the analysis, we viewed the videotapes for each lesson, adding annotations and clarifications to the transcript that were visible from the videotape. Codes were clustered so as to identify the critical features of the lesson. In the third stage of the analysis, we re-examined conversations with the teacher that took place during and after the lesson and we coded the interview data. This enabled us to gain further insight into the teacher's perspectives on the lesson.

Results

Our findings are organized around the challenges the teacher encountered as he began to use students' ways of thinking as a basis for the lesson. First, we report how the teacher interacted with the students as they began their investigation of the transformations and how this led to an unanswered question in the first lesson. Second, we describe how that question was subsequently used by the teacher to further student learning through student explanations.

Assessing Student Thinking and Using Partial Understandings

As one small group of students was investigating the graph of the equation $y = 2^{x+2}$, they were unclear as to whether they would call it a stretch (dilation) or a horizontal translation. They knew that adding two to the independent variable would transform the graph in some way, but they were unclear as to how to describe the change in the graphical representation of the function. After they attempted to describe the associated transformation, they asked the teacher:

S1: How can you tell if it's a dilation or a translation?
S2: What's the difference between them?
T: Maybe nothing.
S1: How can you tell the difference between the two?

Rather than directly tell the students the difference (as had been his practice in the past), the teacher assessed their current thinking and posed a seemingly analogous question:

T: Well let me ask you a question. Is there a difference between multiplication and division?
S1: Yes.
T: What?
S1: One you're increasing and the other you are decreasing?
T: What if I had 2 divided by one half?

S1: Well, it's still different though because multiplying and dividing are two different things. You can't say they are the same.
T: Well, how do you divide by one half?
S2: You multiply it by 2, but you're still dividing.
T: But you have the same net result, right?
S2: Yeah.
T: Can every multiplication be written as a…
S1: You have to divide and multiply by different numbers, so they can't be the same.

The teacher quickly encountered and responded to the students' confusion about multiplication and division; this suggests that he had recognized this common confusion, likely from his many years of teaching experience. Despite this as a source of confusion, it seemed that the teacher wanted the students to use the arithmetic example as a way to sort out what the teacher interpreted were their emerging ideas about stretches and translations. He attempted to move the students' thinking toward understanding that some transformations can be thought of and represented algebraically in more than one way. For the teacher, this was analogous to seeing that every multiplication can be written as a division.

Supporting Revision and Refinement of Developing Ideas

The teacher continued to work with the students, as he saw them as being close to seeing two equivalent ways of describing the transformation:

T: Okay. So maybe your translation and dilation would be different values?
S1: Okay, then how do we distinguish the two? That was what I'm saying.
T: Maybe they don't need to be other than by perception.
S1: So what you're saying is if we wrote translation or dilation, those are both right?
T: Yeah.
S1: So it could be translation or dilation, those are the exact same things?

The student attempted to refine her thinking about the equivalence of the translation and dilation (stretch). The teacher posed another analog hoping to support the students' transfer of their understanding of the rules of exponents to see that $2^{x+2} = 4 \cdot 2^{x}$:

T: Perhaps let me ask the question a little differently. Could I write a square root as an exponent?
S1: Yes.
T: Yes, so they are just different ways of writing the same value.
S2: So you're not right or wrong. You're not wrong if you wrote a translation and somebody else got a dilation.
T: That would be my perception. However you need to get a little more defined. There are obviously some values you need to get. But it was a good question because it seems to be doing either one, right?

As this exchange ended, the teacher confirmed that either the dilation (stretch) or translation could be correct and encouraged the students to be more precise in their description. He used the connection between multiplication and division and laws of rational exponents in an attempt to help the students think about the equivalence of the stretch and the horizontal translation. The teacher continued to listen to other groups of students as they worked on this question of how to see such an equivalence. The question was not resolved within many of the small groups as the class ended and it became the central idea of a discussion during the lesson that followed the next day.

Engaging Students in Explanation

The teacher began the next day's class by asking the students questions about the homework from the previous day. The whole class discussion began with a focus on the third problem, which read, "Compare each of the following functions to $y = x^2$ and tell what transformations have taken place." The first function, $y = 2^{x+2}$, had been a source of some confusion within a small group conversation the previous day. The teacher wanted to clarify the connection between a stretch and a translation, continuously questioning multiple students until there was a clear explanation and algebraic justification for the equivalence:

T: Mark, describe the transformations in 3a.

Mark: 3a?

T: Yeah.

Mark: I said it was stretched by 4.

T: Okay, so it is a stretch by a factor of 4. All right. Interesting question. Why if the graph was Y equals 2, and a stretch implies what operation?

Mark: Multiply.

T: Multiplication. How do you get multiplication out of Y equals 2 to the X plus 2?

Mark: Well, 2 plus 2 is 4 and 2 times 2 is 4.

This student was unable to give a clear explanation for how he thought about this transformation; he merely recited correct arithmetical calculations that seemed disconnected from the question. In the face of this partial student understanding, the teacher pressed on with more questions. This constituted a change from his usual practice of offering the correct reasoning. The teacherappeared to be exploring ways to bring this student and others into a mathematical conversation about how to see that a horizontal translation can also been seen as a vertical stretch.

T: Do you see a 4 anywhere on the board?

Mark: No.

T: Oh, so we just make one up?

Mark: Sort of.

T: Sort of. Think about it. If we are going to have multiplication, if we have said that stretch, the dilation is a multiplicative factor, where could it come from? [pause] Well, as a hint, let's see. This should end up as Y equals A times 2 to the X. Right? If it is going to be a dilation, so it should fit in the form of Y equals A times 2 to the X? Because the dilation or that stretch means something is multiplying each Y value. And Mark, what did you say A should be? You said it was a stretch by a factor of 4. So it seems that Mark is saying that this is the same thing as Y equals 4 times 2 to the X. My question is, how can that be or why is that or how could we show it is true algebraically?

S3: You could plug it into a graph.

T: You could plug it into your graphing calculators and see if they what, were the exact same graph?

S3: Yes.

T: Ah.

At this point the teacher was listening for an algebraic explanation and he gave a hint (as was usual in his practice) to move the discussion in that direction. Although he acknowledged the graphical approach to the question, he moved on quickly. However, the videotape of the lesson shows that numerous students quickly got out their graphing calculators and followed the suggestion that was offered by the student. But in this instance, the teacher did not follow the students' actions. It is possible that the teacher did not see the students verifying the graphical equivalence of the two forms of the equation. It is also possible that the teacher did not see clear ways to a graphical approach at that moment. At this point, a student re-offered the suggestion about solving it algebraically. The teacher pursued this line of reasoning and began to draw other students into the discussion.

S4: Solve it algebraically. Just a suggestion.

T: Well, this is really more like an identity or a proof. What it is about this ($y = 2^{x+2}$) that says it is equivalent to this ($y = 4 \cdot 2^x$)? Tom do you have an idea?

Tom: Not really.

T: Isaac?

Isaac: If we plug in zero for X, then we will get for the first one, we would get 4 because 0 plus 2 is 2. And 2 times 2 is 4. Then the second one, if we have 0, so 2 to the 0 is going to be 1 and 1 times 4 is going to equal 4. So….

T: So obviously the points correspond, right?

Isaac: Yeah.

T: Jack?

Jack: To do the first one could you like try to…(inaudible)…2 to the X times 2 to the second in order to get the second one? Because 2 to the second is 4, so…(inaudible)…the same.

An alternative of value substitution was offered by a student to explain the equivalence of the stretch and the translation. Again the teacher acknowledged the idea but pressed on, seeing that another student had his hand raised. This time, the teacher heard what he was expecting in the explanation based on rules of exponents. The tension involved in "not telling" was eased for him, as he saw that the formal mathematical explanation had been presented. He furthered the dialog by calling on multiple students who had not yet been verbal in the conversation.

T: Don, do you understand what he just said?

Don: Surprisingly, yes, I did.

T: Surprisingly yes you did. Could you restate it?

Don: What he is saying is like the X plus 2, he is saying, I think he is saying that if you square 2, that will give you 4 and you just bring 4 down and multiply them by 2 and you get X.

T: All right. Jenny, make sense to you?

Jenny: Sort of.

T: Could you explain it in your words?

Jenny: No.

T: Well, try!

Jenny: I don’t know.

T: Kevin, how about you?

Kevin: Well, Don’s words kind of sufficed.

T: Oh, Don’s words kind of sufficed. But he is an individual and you are very much an individual, so we will go for your individual interpretation.

Kevin: He was saying was that you multiply the 2 times the 2 to give you 4 and you multiply that by 2 to the X leaving…. That is all I got.

Perhaps the strategy of calling on multiple students was merely intended to keep students listening. However, it also seemed to be used by the teacher to create an opportunity to assess what gains students were making toward understanding the relationship between the two different transformations. It became clear that not all students could yet verbalize an argument about equivalent algebraic expressions to explain why the stretch and translation resulted in the same graph. One last time, the teacher pushed for the precise algebraic explanation:

T: Okay, Isaac?
Isaac: When you multiply two bases that are alike, you add up the exponential….
T: All right, good, keep going.
Isaac: So for that one you got 4 times 2 to the X. If you make that for the same base you can say 2 to the 2 times 2 to the X.
T: 2 to the 2 times 2 to the X.
Isaac: Which will give you 2 to the X plus 2.
T: Okay.
Isaac: And that is the same thing as...

The discussion about this one homework problem was, at the very least, a change in the teacher's usual practice of offering answers to homework problems that were partially or not at all understood. This discussion was centrally driven by the teacher's goal of expressing the algebraic equivalence of the two functions. While some student ideas did not fully play out (e.g., graphical or numerical substitution approaches), the teacher did work to engage multiple student voices in expressing the symbolic argument.

At the end of the class, the teacher commented on his efforts to engage students in giving explanations:

T: Left over from yesterday was that [question], well is it a transformation or dilation, or does it even matter? … I really wanted to have them explain it and I worked hard not to say anything. I worked hard not to say anything!

The teacher had begun to work toward using student thinking to encourage the generalization that the horizontal translation of an exponential function can be viewed equivalently as a stretch of the function and that this can be seen algebraically by applying the rules of exponents. The teacher began to shift toward helping students understand each other's explanations. Thus he was able to meet the challenge of "not telling".

Discussion and Conclusions

We claim that the development of this teacher's knowledge in practice evolved through meeting the challenges in "not telling" that occurred when listening and responding to student inquiry. He first had to comprehend students' emerging ideas and then work at responding in a way that allowed them to reason their way toward the formal mathematics of the activity. In his interactions with a small group, his changing practice included the strategy of quickly thinking of and using related questions, examples and analogs based on what he perceived and expected was students' prior knowledge. Through these interactions, he attempted to bridge emerging student thinking with the mathematics that was his intended lesson. In working at "not telling," the teacher encountered the challenge of finding ways to engage students in articulating their understandings. In so doing, his powerful understandings of symbolic approaches drove the discussion. However, the teacher did work to engage students in the expression and re-expression of the symbolic equivalence. This was a substantial and conscious shift in the teacher's practice as he acquired new ways of listening and responding to student thinking.

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COMPARATIVE ANALYSIS OF CONCEPTUAL UNDERSTANDING OF CALCULUS IDEAS AMONG TEACHERS WITH DIFFERENT CALCULUS EXPERIENCE

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The purpose of this study is to examine the differences in conceptual understanding of calculus ideas among in-service teachers with various calculus backgrounds. One of the main components of the research was to explore how effectively could calculus ideas be introduced to students with different mathematical experiences. The research sample included 24 participants –in-service teachers that had various backgrounds, ranging from middle school teachers to community college level instructors in different disciplines (mathematics, science, language arts, music, etc.). The mathematical level of proficiency varied: 9 participants had never taken calculus classes, 8 participants had taken Calculus-1 and/or 2, and 7 participants had taken up to Calculus-3. The participants were pre-assessed on current level of content knowledge by testing their understanding of calculus concepts of graphing, rate of change, and integration. Teaching intervention consisted of a set of conceptual activities to support students' understanding of calculus ideas. Finally, a concept post-assessment was given to observe if the activities helped to improve the participants' conceptual comprehension of calculus.

Theoretical Framework

During the last two decades Calculus is at the forefront of research and curriculum reforms in mathematics education. The majority of research in Calculus learning has been done at the level of undergraduate education and some at the high school level. Researchers observed that students enter calculus courses with a primitive understanding of concepts of function, change, continuity, etc. (Tall, D., 1996, Ferrini-Mundy, J., & Lauten, D., 1993). They also noted that students have cognitive difficulties in coordinating function concept in algebraic and graphical representations, which is critical in constructing a foundation for fundamental calculus ideas (Schnepp, M., & Nemirovsky, R., 2001). Other researchers concentrate on different approaches to teaching calculus principles: comparison study on technique-oriented approach vs. conceptual and infinitesimal approaches of learning calculus shows that different approaches have different impact on students' language use and sources of conviction (Frid, S., 1994).

Researches have also determined that cognitive obstacles to the learning of calculus arise in at least two different ways – one related to linguistic/representational aspects and the other related to intuitions. Given that so many of our algebra and calculus courses are immersed in symbolic manipulation, often at the expense of understanding, it is not surprising that linguistic/ representational factors give rise to cognitive obstacles. Since learners basically want to understand and make sense of what they are being asked to learn, the intuitions that students bring to bear on the concept of calculus often play a crucial role in the appropriate construction of those concepts. Researches "propose that a potentially useful framework in which to embed considerations of cognitive obstacles lies in the framework of Krutetskian cognitive processes of reversibility, flexibility, and generalization" (Norman, A., & Prichard, M., 1994, p. 76).

There is an emerging importance of making connections between different representations (concrete, visio-spatial, numeric, graphical, algebraic, etc.) in helping students' to learn calculus concepts. Visualization may be a major tool to develop this understanding. One of the guiding principles of Harvard Consortium Calculus text is the multiple representations, "which says that wherever possible topics should be taught graphically and numerically, as well as analytically. The aim is to produce a course where the three points of view are balanced and where students see each major idea from several angles" (Hughes-Hallett, D., 1990, p. 121).

One of the most significant points that come from the analyses of research in Calculus learning is that there should be more emphasis on conceptual learning using multiple representations and connections before students immerse into symbolic manipulations. In order to build a rich conceptual foundation for successful learning of Calculus at the high school and college level there should be a lot of preparatory work done at the early years of schooling. "Calculus needs to be studied across many years of school, from early grades onward, much as a subject like geometry should be studied" (Kaput, J., 1994, p. 132). While this approach is common among upper class and private schools, it is lacking in high minority lower socioeconomic school programs.

The project is based on the following key assumptions about learning and teaching:

- Conceptual learning leads development of cognitive acquisition of formal procedural operations. Lev Vygotsky claims that development of formal operations depends on the creation of a successful learning environment, which includes the use of conceptual tools. The development of students' procedural calculus skills is a derivative of students' conceptual understanding of big calculus ideas.
- An application of the Davydov's accelerated pedagogy of teaching and learning mathematics using method of ascending from big, general ideas to specific procedures is a key methodological tool for designing a rigorous conceptual mathematics curriculum. Thus, development of students' conceptual understanding of calculus principles should be achieved by ascending from multivariable calculus concepts to single-variable principles.
- Cognitive-visual conceptualization (CVC) through the use of modeling and technology plays a critical role in learning of calculus principles.

In contrast to previous remarkable attempts in early introduction of advanced Calculus concepts (e.g., SimCalc project, CoVis project), which basically considered development of single-variable Calculus concepts, this study starts teaching Calculus principles from general multi-variable to single-variable concepts: from generic 3-D surface to arbitrary 2-D curves and then to specific elementary curves (linear, quadratic, exponential, etc.), from tangent plane to tangent line (including concept of gradient), from general infinitesimal methods to procedural calculations of derivative and integral, etc. In teaching multi-variable Calculus concepts we used one of the advantages of local Greater El Paso landscape – mountains (a natural model of generic arbitrary 3-D surface). In parallel with this students were introduced to basic 3-D Geometry concepts (3-D coordinate system, projections of 3-D objects, sections of arbitrary 3-D surface, etc.). 3-D Geometry is a mathematically natural way to introduce multi-variable Calculus concepts.

Piaget's idea that development of geometric concepts in children follows an anti-historical order is probably familiar to most readers. The idea is that, whereas historically the earliest geometrical operations were developed to deal with specific problems of terrestrial mensuration and hence had a Euclidean character, the child only arrives at the specific concepts of similarity, congruence, and proportion after a long process of developing these refined concepts from more global, or general, ideas about spatial relations. "Historically, the development has been from the particular, measurement-bound, practical "real-world" geometry to the more general, abstract, and non-metrical relationship found in projective geometry and ultimately in topology. For the child, according to Piaget, the earliest and easiest spatial relations to grasp (in a very intuitive way) are those concerned with general futures such as contiguity, neighborhood, closed contour, and so on – that is, topological futures" (Dodwell, 1971, p. 179). These developments are held to occur through the agency of the child's own active exploration of, and interaction with, its environment (Piaget, J., & Inhelder, B., 1956). Current research on designing learning environment for developing students' understanding of geometry and space seems to support the Piagetarian idea (Lehrer, R., & Chazan, D., 1998, Balomenos, R., Ferrini-Mundy, J., Dick, T., 1987, Yakimanskaya, I., 1991).

Research Design

The 24 participants were given a concept pre-test consisting of four open-ended questions. The first question assessed the participants' ability to read a graph that showed the distance traveled by a car and to analyze it. The second question evaluated the participant's competence to create a distance vs. time graph and a speed vs. time graph for a given context. The third question was related to finding the distance a car travels based on two given graphs: a distance vs. time graph and a speed vs. time graph. The last question was linked to understanding the concept of slope by using contour diagrams.

Teaching intervention consisted of a set of conceptual activities. These activities mainly dealt with calculus concepts of derivative and slope, integration, gradient, and optimization. The activities are explained in more detail below.

Activity One (Contour Diagram of Produce): This activity gave the participants a chance to both create and read a contour diagram. They were given a fruit or vegetable and were then instructed to make a contour diagram of half that fruit or vegetable and to keep the other half in a place where no other participants could see it. During the activity, they were able to decide on a reasonable spacing of contour lines depending on their fruit or vegetable and to cut the fruit or vegetable in layers of same thickness. They then traced each slice on a piece of paper and put them together to make a single contour diagram. The participants then swapped their contour diagrams and attempted to guess which fruit or vegetable was being represented and what interesting features it had. They then reconstructed the fruit or vegetable by making layers (out of cardboard or modeling clay) that corresponded to each of the contours. At the end of the activity the participants compared their reconstructed fruits to the ones that the other people saved. They also analyzed the differences and similarities between the two halves. This activity helped the participants to understand different concepts as contour diagrams, contour lines, elevation, layers, steepness and smoothness, and slope.

Activity Two (Volume of Produce): In this activity the participants estimated the volume of fruits and vegetables by chopping them up into small pieces. The participants were given either a fruit or vegetable and asked to give an estimate on the volume of the produce. Once the students had their estimate, they needed to see how close they came to the correct volume by using a measuring cup with water. The students needed to put the produce into the measuring cup and see how much water was displaced. This helped the participants to obtain the right volume. Then, the participants were asked to find an organized way of finding the volume of the produce by cutting the produce into small pieces, and adding up the volume of the smaller pieces. The participants were told not to cut the pieces to small because they would be to hard to handle, likewise, cut the pieces to big or they would make the cutting null and void. Otherwise, why bother cutting? Once the participants had sliced up the produce into small pieces, they needed to measure the volume of the small pieces and put this information into an organized table. The students also needed to put an upper and lower bound of the small pieces into the table. The participants used the table to get the estimate of the produce, the upper bound and the lower bound. They then were asked to compare this estimate to the original estimate and the volume from the measuring cup. Finally the students were asked to try and think of anything that they might be able to do differently to get a more precise answer. This activity helped the participants on solving for volume of regular and irregular objects. The activity also introduced a variety of mathematical concepts to the participants such as lower and upper bounds, length, height, estimation, and diameter.

Activity three (Zooming into a Mountain/Slope of a Mountain): In this activity, the participants investigated different slopes described at a particular point on a clay mountain and what each slope had to do with one another. The participants were given clay to make a small mountain model. Once the students had their model mountain, they placed it on a large enough paper to hold the mountain. They were asked not to move the model any more throughout the rest of the activity. The participants then marked the paper North, South, East, and West and were asked to give a story of a lost hiker on a mountain whose scale model was the clay mountain. The hiker was supposed to have a cell phone and a compass and a way to measure accurately the slope or grade of the mountain where he is. This in turn allowed the participant to tell the rescuers where to look. The participants repeated this activity until they had a grasp of different slopes at a particular spot on a model of a mountain. This activity allowed the participants to understand how to use slopes/ grades on three dimensional objects.

Activity Four (Trips to the Dump): In this activity the participants tried to optimize a function with two variables. The only items that were needed for this activity were couple sheets of paper and a calculator. The participants were given a problem that dealt with the dimensions of a box and the volume the needed to come up with the least amount of cost. The problem was that the cost of the box varied. The cost of the bottom of the box as well as the sides was \$10/ m^2, and the cost for the ends of the box was \$20/ m^2. The participants were also told that they were allowed to make more than one trip to get the amount of volume they needed, however the cost for each trip was \$2. The participants needed to come up with the minimal total cost to make a specific amount of volume. In this problem the students had two variables to deal with. They were told that there were many ways to solve or approach this problem. A couple of these methods are: Make a diagram of the box and label the box with variables; trial and error; make a formula; and they could even use a table. Once the students had applied their methods and come up with the correct answer they explained their method to the rest of the class. After the students finished this activity they had a better understanding on solving or approaching problems with more than one variable.

Activity five (Area of a Scab): In this activity the participants were able to find the area of a real-life two-dimensional object. The participants not only focused on their estimate but also on how accurate the estimate was. The students will be told to analyze a scab and think of a way to dissect the scab in an organized fashion by giving a lower bound as well as an upper bound of the area of the scab. They were then told that they should have some type of system to their dissection. They could not eyeball the area for measurement. Their estimated lower bound really had to be less than the actual area, and similarly the upper bound. The method the students came up with had not to be that complicated. The upper and lower bound of the pieces had to be very close as possible to each other and equal to each other. Once the students were done with the dissection of the pieces they organized the data into a table. Then they used the table to get the upper and lower bounds of the entire scab area. After they finished their activity the instructor asked them if there could be a different or better way to get the upper and lower bounds closer together. When the students had completed this activity they were able to estimate the area of an irregular shape object. They were also able to give an upper bound to the area as well as a lower bound too. This activity gave the participants a glimpse of calculus without them realizing it.

Activity Six (Match it, graph it): On this activity the participants were able to match their motion to a given graph by moving back and forth in front of a motion detector. The activity started by setting up a motion detector on a table or a desk. The detector was aimed to be at the waist or chest of an average person. Meter sticks were then used to measure ½ meter intervals from the detector. Each interval was marked on the floor with a piece of masking tape. The distance from the detector was then written on the tape. The program MATCHIT on a TI-83 calculator helps to continue with the activity following the directions on the calculator. A distance vs. time graph was generated and the participants moved back and forth of the motion detector to match the graph. They then described their motion by writing a piecewise-defined equation for the motion. This activity helped the participants to develop a better understanding of different calculus concepts as: positive and negative slope, y-intercept and its physical representation, constant and changing speed, equation, and reading and understanding a distance vs. time graph.

Activity Seven (Walk This Way): In this activity the participants were able to create a distance versus time plot that is always increasing by moving away from a motion detector. From this plot, they computed the total distance they traveled by finding the difference between the starting and ending points. They also realized that if they changed directions while walking in front of the motion detector, this method would not work. The rectangular heights were computed at the midpoint of each width. Although the trend was for the distance approximation to get better as more rectangles were used, it was possible for a smaller number of rectangles to give a closer approximation to the total distance traveled. This activity helped the participants to understand how distance can be determined by finding the area under a speed versus time curve.

Activity Eight (Verifying Velocity): The last activity the participants took part in was to learn more about how distance, time and speed or velocity are related to each other. This activity was done with the help of the DISTANCE program on the TI-83 calculator. The participants used some data and plot to determine the time intervals where the velocity was positive. They also identify intervals where the velocity was negative and give their reasoning. The participants were also able to describe what portions of a given graph on the calculator represented a velocity of zero by estimating the slope. This activity helped the participants to understand better the concepts of positive and negative velocity, average velocity, slope, tangent lines, distance versus time graph, and velocity versus time graph.

Following the activities, a concept post-test was administered to the participants to evaluate if there was a significant change in calculus reasoning after the activities took place. A concept post-test consisted of four open-ended questions and was similar to the pre-test in terms of the main Calculus ideas taught through the intervention.

Results and Recommendations

The research showed that the participants that had taken no calculus classes gained the most from the activities (28.20%). Where as the group that had taken Calculus-1 and/or 2 gained the least (3%). The highest post-test result came from the group of people that had taken up to Calculus-3 (77.00%). The following table (Table 1) consists of the research results.

Table 1. Results of the study

Groups	Pre-test (%)	Post-test (%)	Gain (%)	Pre-Test (max-25)	Post-Test (max-25)	Gain
"Non-Takers"	32.80%	61.00%	28.20%	8.2	15.25	7.05
Calculus 1-2	53.00%	56.00%	3.00%	13.25	14.00	0.75
Calculus 3	52.00%	77.00%	25.00%	13	19.25	6.25
Total	45.90%	64.70%	18.80%	11.48	16.17	4.69

Taking into account the limitation of this study (e.g., small sample size, short intervention time), we feel safe to conclude:

1. In this study in-service teachers with traditional Calculus 1-2 coursework were reluctant to re-learn Calculus concepts. Once student has formalized a procedure, it is difficult to re-visit the underlying concept for deeper understanding (Hiebert & Carpenter, 1992, Lee & Wheeler, 1989, Skemp, 1978, Wilson & Goldenberg, 1998).
2. Study also showed that in-service teachers with no Calculus experience are more open to learn Calculus concepts than teachers with traditional Calculus 1-2 coursework. It sounds like "no knowledge is better than a little knowledge".
3. Analysis of teachers' understanding from Calculus-3 group revealed that they were able to overcome "the zone of conceptual resistance" and capable to generalize calculus procedures.
4. Classroom observations and interviews with teachers showed that the method of ascending from general to specific helps teachers to focus on conceptual generalization of calculus ideas. It also contributes to vertical alignment and flexibility of teachers' content knowledge in order to make transitions from multi-variable to single-variable Calculus concepts and reverse.
5. Study showed that connections between different representations (concrete, visual-spatial, numeric, graphical, algebraic, etc.) are important tools in improving teachers' understanding of calculus concepts.

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GRAPH THEORY: A TOPIC FOR HELPING SECONDARY TEACHERS DEVELOP INNOVATIVE TEACHING STRATEGIES

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Discrete mathematics has curricular importance and provides opportunities for teachers to develop innovative teaching strategies. Arguments from the National Council of Teachers of Mathematics, recent secondary curriculum materials, and from other prominent mathematicians and mathematics educators support this position. An activity for a mathematics course for secondary teachers is described. The activity illustrates some of what is claimed about curricular importance and innovative teaching. Finally, results of a recent study of secondary teachers' conceptions provide support for the arguments made.

Purposes

Discrete mathematics not only has curricular importance, it also provides opportunities for teachers to develop innovative teaching strategies. This paper presents arguments from the National Council of Teachers of Mathematics (NCTM, 1989, 2000) *Standards*, secondary curriculum materials developed to support them, and from other prominent mathematicians and mathematics educators. It also describes an activity illustrating some of what is claimed about curricular importance and innovative teaching. Results of a recent study of secondary teachers' conceptions provide additional support this claim.

The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989, p. 176) stated that in grades 9-12, the mathematics curriculum should include topics from discrete mathematics so that all students can--

- represent problem situations using discrete structures such as finite graphs, matrices, sequences, and recurrence relations;
- represent and analyze finite graphs using matrices;
- develop and analyze algorithms;
- solve enumeration and finite probability problems

The Discrete Mathematics standard recommends the inclusion of discrete mathematics in the 9-12 curriculum. The more recent NCTM document, *Principles and Standards for School Mathematics* (NCTM, 2000) also emphasizes how discrete mathematics can be implemented and integrated throughout the secondary curriculum. For example, it stated:

> Because students' interests and aspirations may change during and after high school, their mathematics education should guarantee access to a broad spectrum of career and educational options. They should experience the interplay of algebra, geometry, statistics, probability, and discrete mathematics. NCTM, 2000, p. 287.

In the United States, secondary curriculum development projects, such as the *Core-Plus Mathematics Project (CPMP)* have developed high school curriculum materials designed to implement these calls. For example textbooks for both grades nine and ten of the *CPMP* series (Coxford, et al., 2003) contain units dealing specifically with graph theory.

Many prominent mathematicians and mathematics educators have argued for the inclusion of discrete mathematics in the secondary curriculum (e.g., Rosenstein, 1997) partly because it is so useful in the real world. Others have stated that discrete mathematics affords students opportunity to participate actively in the mathematics process and to experience success and enjoyment in mathematics (Holliday, 1991, Kenny & Bezuszka, 1993). The literature and our experience also provide evidence that it provides teachers with opportunities to adopt positive dispositions about innovative teaching strategies (Wilson & Spielman, 2003).

Theoretical Perspective

We believe that innovative teaching involves teachers helping their students to explore important ideas, solve real problems, make meaningful connections, and work in cooperation with other students (NCTM, 1989, 2000). Mathematical ideas should be correct because they make sense and work, not just because a teacher or textbook say so. We believe that student-centered instruction often requires teachers to share *mathematical authority* with their students (Wilson & Lloyd, 2000). Such sharing sometimes involves the use by teachers and their students of computers and calculators. Rather than simply being told or shown important relationships and mathematical concepts, with technology students more easily explore and solve meaningful problems themselves.

Graph theory is a topic that we believe lends itself to such sharing. For example, real-world applications are extremely prominent in graph theory and the *Mathematical Modeling* software (Graubart, C.B., 1997) upon which the example we discuss later in this paper is based, encourages students to explore the meanings of problems and to represent the problem situations in multiple ways. The software is dynamic in many of the same ways as *Geometer's Sketchpa*d (Jackiw., 1997). For example, when changes in the vertex graph are made, corresponding changes are displayed in the adjacency/connectivity matrix representation, and vice versa. Partly because the software enables students to experiment and make sense of problem situations without having to concentrate on computational aspects, they are more easily able to explore problems without the teacher's direct input. Multiple representation and cooperative exploration in the solving of real-world problems are important components of sharing by teachers of mathematical authority. Engaging in these processes help students develop sense of mathematical ideas based on internal voices, not solely on the basis of what their teachers tell them about conventions and important relationships (King & Kitchener, 1994). To a greater extent then when simply told by their teachers, students gain an internal voice concerning the correctness of the concepts considered.

Methods

Many arguments have been presented for including discrete mathematics in the secondary curriculum. For example, Rosenstein (1997) expressed that discrete mathematics is applicable (provides different ways to represent real-world problems), accessible (basic mathematics such as arithmetic is sufficient in order to understand the application of discrete mathematics), attractive (discrete mathematics problems catch the attention of students and lend themselves to discovery and exploration) and appropriate (discrete math is for students that are accustomed to success and for those who are not). Kenny and Bezuszka (1993) stated that discrete mathematics is appropriate for illustrating and emphasizing NCTM's process standards. The reader is encouraged to keep these points in mind as he or she considers the example task described later in this paper.

A recent study of 15 secondary teachers enrolled in a mathematics course for teachers (Wilson and Spielman, 2003) suggests how and why graph theory also has positive prospects for mathematics teacher education. For example, one student said:

> Discrete math enlightened me. I like how you can take a concept in math and make it practical for a student. I didn't really know what discrete math was in the past or how it

was applicable. In the future as a teacher, I would try to make math more applicable and include more discrete math.

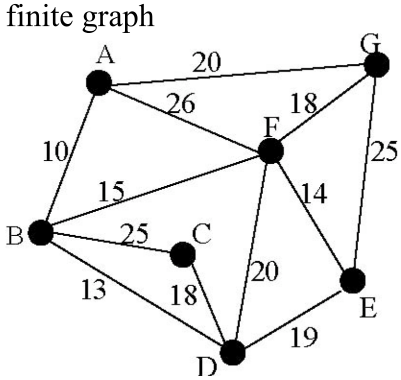
Based on these and similar student comments, and the claim that technology is essential in teaching and learning mathematics (NCTM, 2000), we suggest an instructional activity assisted with mathematical computer software to provide a way to incorporate discrete mathematics, in particular graph theory, into teacher education and mathematics curriculums. We point out that although it is our firm belief that the development of mathematical (and pedagogical) authority by teachers is best accomplished by allowing them to create and lead lessons, there is also a need for structure and specific examples to be provided by teacher educators The activity that follows exemplifies such structure. In the introduction section of the activity, students become familiar with *Mathematical Modeling* software (1997) and with basic graph theory concepts. The students (secondary teachers) learn:

a) how to create finite graphs (and rearrange them),
b) the interrelation between graphs and adjacency or connectivity matrices,
c) to find the minimal length of a path along all vertices,
d) the differences between a connectivity matrix and a shortest path matrix (distance matrix), and
e) the properties of isolated and central vertices.

In the investigation section, students explore the concept of shortest path in three different contexts:

1) cheapest airfare,
2) optimum manufacturing process, and
3) shortest route between towns.

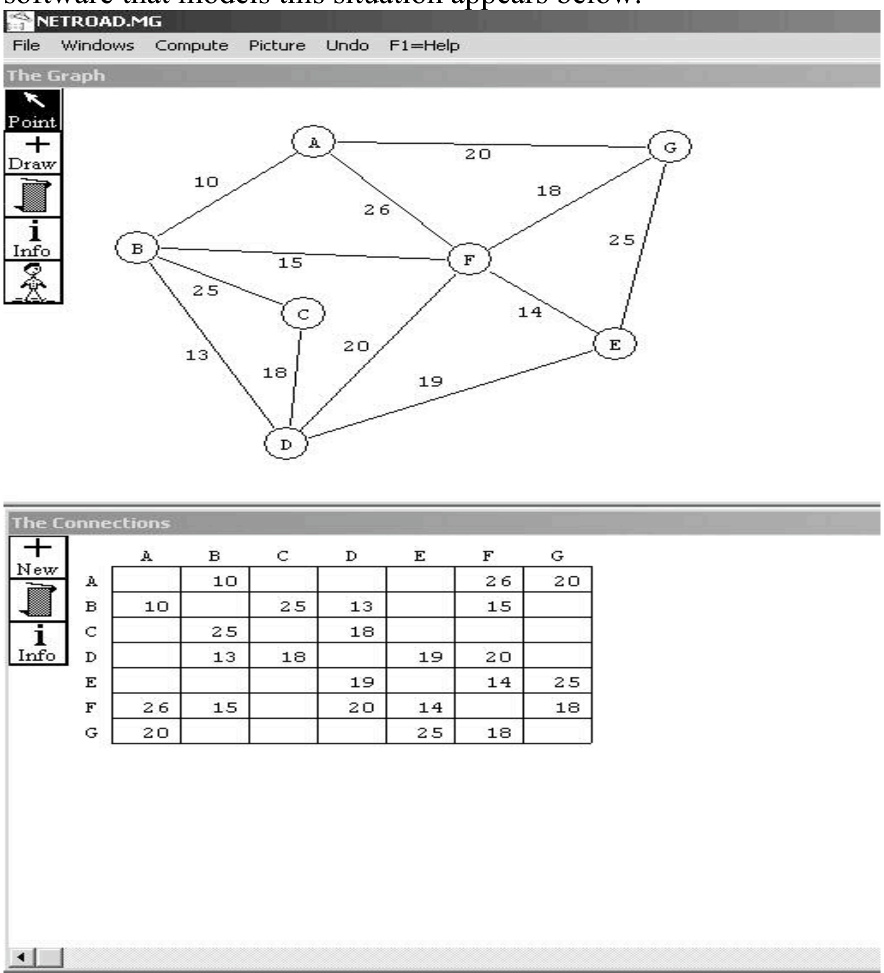
The activity emphasizes all five NCTM process standards (problem solving, communication, connection, representation, and reasoning & proof) (NCTM, 2000). A variety of problems from the real world are solved using a computer program that emphasizes connections between the two most common graph theory representations: finite graphs and adjacency matrices. With the software, as the graph or adjacency matrix is adjusted, the other representation adjusts automatically. Users are also able to view the shortest path matrix, which the program creates upon request from the adjacency matrix. The activity requests that students enter the information and explain the meanings of the things represented by the software. For example, one activity of the investigation section titled "road network" explores the concept of shortest path in the context of distance between towns. The students (secondary teachers) are given the following finite graph



The vertices represent the towns and the numbers along the edges (the "weights") represent the distances in miles between the corresponding towns. For example, town B is 13 miles from town

D and town D is 19 miles from town E.

The students are asked to create the adjacency matrix corresponding to this finite graph. They can copy or draw the given finite graph into the software (without line weights), or they can enter the adjacency matrix and see the graph (perhaps with different vertex positions). One might assume that all students will just copy the vertex graph into the software (line weights must be changed in the matrix), but it is easier to enter a matrix than a graph. Also, entering the matrix allows students to check their interpretations. If after entering the matrix, the graph generated by the software is equivalent to the one given, students know they have interpreted the graph correctly. The most likely process will be to enter the graph directly into the program and then change the line weights (distances between towns) in the adjacency matrix. A screen from the software that models this situation appears below.



	A	B	C	D	E	F	G
A		10				26	20
B	10		25	13		15	
C		25		18			
D		13	18		19	20	
E				19		14	25
F	26	15		20	14		18
G	20				25	18	

The task gives numerous opportunities for students to talk about the problem and decide how to solve it. For instance, if the problem is initially assigned to small groups of students, they talk about the meaning and solution of the tasks. After the initial task, the students respond to several questions related to the graph. For example, the task asks students to find different paths from A to E and to find the shortest path between these towns. They are also asked to describe the relationship between those numbers and the matrix representations that are displayed by the software. They can either answer the questions by looking at the finite graph or other representations of the situation displayed by the software (both the adjacency and shortest path matrices), or by using matrix computations such as row sum, column sum, matrix multiplication, which can also computed be by the software. Another specific task in this particular activity asks:

> Suppose a new county hospital is to be built in one of the towns to serve all seven towns. In which town should it be built? Why?

This task also demands that verbal and written skills be used in the process of solving the problem and communicating about it. Students are asked to explain the meanings of their responses. The use of critical thinking and reasoning are also promoted (the task is a problem, not just an exercise involving the use of the software). The task promotes connections among multiple mathematical representations (between matrices and graphs). Moreover, it encourages students to see how linear algebra (matrices) is connected to graph theory. Throughout the activities students are asked to create mathematical representations of different situations and explain in writing, the meanings of those models.

Data Sources

We have illustrated in several ways how graph theory provides opportunities for teachers to develop innovative teaching strategies. Many individuals have provided theoretical arguments supporting our case (some are cited in the Purposes section of this paper). Second, we have described and analyzed an example (an instructional activity that uses specific software) illustrating how teachers can be helped to share mathematical authority and implement technology in meaningful ways (Methods section of this paper). Third, results of a recent study suggest that teachers are more willing and able to adopt innovative teaching strategies when considering graph theory than when considering other important secondary mathematics topics (Wilson and Spielman, 2003). Students both felt and acted as if graph theory provided a more suitable context for teaching about the usefulness of mathematics in the real world than other topics (e.g., functions and graphs). In the course, the students (secondary teachers) not only claimed that graph theory was a better context for innovative teaching, they presented lessons about graph theory that were more student centered and involved more sharing of mathematical authority than lessons about other course topics. For example, to a greater extent, lessons contained problems involving real-world applications. One of the students in the class said:

> I have enjoyed working with discrete math. It is something that I have not had the opportunity to do much of…. These topics are terrific ways to interest students in math by giving real-world applications.

A study in the summer of 2004 of two prospective teachers' understandings of graph theory will pursue our belief more vigorously about innovative teaching and the sharing of mathematical authority by teachers. In a mathematics course for secondary teachers, we will intensively interview two students and analyze lessons they prepare and lead about graph theory in an attempt to identify how their conceptions of graph theory and technology are related to their more general conceptions about mathematics teaching and learning. Results of this study will be shared at the conference.

When graphing technology first became popular in the early 1990s, many individuals (Wilson, 1994, Demana, et, al, 1993) and professional organizations (e.g., NCTM, 1989) predicted that secondary classrooms would become "laboratories" as students learned about functions. It was believed that graphing technology would help teachers share mathematical authority with their students as they learned about functions and graphs. In our experience, this has not occurred as extensively as predicted. Perhaps teachers see functions in traditional ways since for the most part that is how they learned about them themselves. Graph theory is not as prominent in the secondary curriculum, but teachers' understandings of how to teach graph theory seem more open to innovative ideas. Perhaps this is because they did not learn the concepts in traditional ways. As the quote above suggests, many have not learned about graph theory at all and perceive that they do not understand it very well themselves. Perhaps their understandings of graph theory concepts are not as formal and traditional as their views about functions. It might also be the case that teachers are more willing to "experiment" with content they perceive to be not a major part of the curriculum, but regardless of why they seem to be more open to innovative teaching ideas in the context of this topic, they seem to be. The statement above (and others like it) leads us to wonder about graph theory being a good context for helping teachers to develop orientations that support innovative teaching.

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PROSPECTIVE TEACHER EDUCATORS' DEVELOPING PERSPECTIVES ON TEACHER EDUCATION THROUGH THE CREATION OF MULTIMEDIA CASE STUDIES

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Research on the use of multimedia case studies has found that these can support (a) the professional development of pre-service teachers, and (b) the goals of teacher educators using them with pre-service teachers. Using multimedia case studies with their pre-service teachers may even serve as professional development for teacher educators. This is an area that needs further research since the professional development of teacher educators is largely unexamined. In this research study, I examined the knowledge development of prospective teacher educators as they created multimedia case studies of practice for use with pre-service teachers using the methodology of the teacher development experiment. I found that creating multimedia case studies of practice allowed the prospective teacher educators to develop a greater appreciation of the complexity of teaching and learning, and to think in new ways about their own learning.

Purpose of the Study

Research on the use of multimedia case studies with pre-service teachers has shown that these case studies can be used to support the professional development of pre-service teachers by providing them with a wide variety of resources to study classrooms and begin to understand the complexity of teaching (Doerr & Masingila, 2001; Masingila & Doerr, 2002). Additionally, multimedia case studies support the goals of teacher educators using these cases with pre-service teachers. Bowers and Doerr (2001) found that the teacher educators who used a multimedia case study with their pre-service teachers report that using the case study helped them think more deeply about issues involved in preparing teachers. In many respects, using the case studies served as professional development for these teacher educators.

To date, learning about and with multimedia case studies and the related professional development of teacher educators is a largely unexamined portion of the research base on teacher education. Indeed, there is very little research on the nature of the knowledge base of teacher educators and the development of that knowledge base. In this study, I examined the knowledge development of prospective teacher educators as they created multimedia case studies of practice for use with pre-service teachers.

Perspectives and Guiding Frameworks

This work is embedded in the view that teaching is an ill-structured domain characterized by complexity, ambiguity, incomplete information and partial understandings, and within this situation teachers have to make reasoned judgments and informed decisions for action. Researchers have found the use of case studies to be powerful and effective with pre-service and in-service teachers to (a) promote theoretical understanding and bring theoretical principles to practice, and (b) promote understanding of the complexities of practice and help teachers become more analytical about the data of classroom practice (Ball & Cohen, 1999; Barnett, 1998; Lin, 2002; Shulman, 1992).

In this study, I investigated the knowledge development of those who are preparing to become teacher educators. Understanding the nature of teacher educators' knowledge and its development has broad implications for the preparation, support and professional development of teacher educators. Central to this investigation was an examination of how teacher educators understand what it is that teachers need to know and how that knowledge is acquired.

Specifically, I investigated the following aspects of prospective teacher educators' knowledge and its development:

1. How do prospective teacher educators understand the tasks of teaching and learning? How do prospective teacher educators define, select, and present elements of practice? How do their understandings of these elements develop as they engage in the process of creating a case study of practice?
2. What questions do they pose about the various elements of practice? What strategies do they envision for teachers to grapple with the issues that emerge from these elements of practice?
3. How do prospective teacher educators organize larger frameworks for teachers to use in understanding the inter-relatedness of teaching and learning? In what ways do they create contexts for teachers to examine that inter-relatedness? How do these frameworks develop through the process of creating a multimedia case study?

The knowledge that teacher educators need includes both the knowledge of teaching (that is, the knowledge that teachers need) and the knowledge of preparing others to teach. These research questions and the methodology, therefore, are focused on both these aspects of teacher educator knowledge. The first question focuses on what the prospective teacher educators themselves know about the tasks of teaching and learning. The second and third questions focus on how it is that they (as teacher educators) can support the development of this knowledge among those who are preparing to teach or are already teaching. My approach to understanding these issues was through an investigation of how prospective teacher educators create their own multimedia case studies of practice. The creation of the multimedia case studies simultaneously revealed the thinking of the prospective teacher educators about the tasks of teaching and learning and revealed their thinking about how these tasks, as instantiated in practice, can be used as sites for learning by pre-service teachers.

Methods and Data Sources

To investigate these issues, I used the teacher development experiment methodology (Simon et al., 2000) to study the development of prospective teacher educators as they engaged in the creation of multimedia case studies using Case Creator software (http://www.sci.sdsu.edu/mathvideo/cc/download/html). The prospective teacher educators' work was carried out during a two-semester course for doctoral students in several teacher education programs at a university in the northeastern part of the United States. There were 18 prospective teacher educators who participated in this research study and each worked on a team to create a multimedia case study. The 18 participants were spread across five teams.

The work with the prospective teacher educators involved in this project consisted of two phases: in the first phase, the prospective teacher educators engaged in making sense of the tasks of teaching and learning through discussion of readings on issues in teacher education, including the use of case studies, and constructed a multimedia case study of practice; in the second phase they reconstructed the records of practice in such a way that these records could be used by others to make sense of the tasks of teaching and learning.

In the first phase of the research study, the data collection and analysis focused on how the prospective teacher educators created and interpreted a case study of practice and its salient features and issues. In this phase, participants worked in teams to develop a case study of classroom teaching and learning by (a) identifying significant content being taught, (b) deciding what video and other artifacts would comprise the case, (c) collecting and editing video, documents, and artifacts for the case, and (d) integrating the materials into a coherent case study of practice using Case Creator software. Data collected during this phase consisted of team planning documents, individual critique of an existing multimedia case study, two semi-structured interviews of teams, the instructor's journal, and the preliminary version of the multimedia case studies created by the participants.

In the second phase of the research study, which corresponded to the second semester of the course, the participants engaged in the task of developing potential ways in which their case study could be used to support the professional development of pre-service or in-service teachers. They continued to work in their teams to develop facilitator guides and issues matrices. Data collected during this phase consisted of the records of the development of the facilitator guides and issues matrices, one semi-structured interview of teams, individual reflective essays on the created multimedia case studies, the instructor's journal, and the final version of the multimedia case studies with their supporting materials.

Following the methodology of the teacher development experiment, I used both ongoing and retrospective analyses of the data. The ongoing analysis, which occurred during the teacher development experiment, was the basis for continued interventions with the participants, the testing of emerging hypotheses, and the strategies for promoting further development of the prospective teacher educators' thinking. In the second phase of the research study, the ongoing analysis shifted from an analysis of the participants' understanding of the tasks of teaching and learning to an analysis of their understanding of how it is that one prepares someone to engage in the task of teaching and learning. During the retrospective analysis of the data, which is continuing, I am examining the larger corpus of data through a carefully structured review of all relevant data of the teacher development experiment.

Features of the Course and the Multimedia Case Studies

The course that the participants were students in was designed for graduate students interested in exploring issues in teacher education. The students examined current issues in teacher education, and looked carefully at the use of case studies as a vehicle for preparing teachers. The major assignment in the class was for these prospective teacher educators to create multimedia case studies to be used in professional development.

There were 19 students in the class and 18 of them decided to participate in the research study. Each of the 18 participants in the class had teaching experience at either the elementary, middle or high school level. The majority of them had also taught at the post-secondary level as teaching assistants. Additionally, some of them had experience working with pre-service teachers through working with a professor to teach a methods course and/or experience working with inservice teachers through internships or professional development workshops.

When working in their case study creation teams, the prospective teacher educators (a) decided on the content and grade level of their case study, (b) set goals and objectives for their work, (c) decided how to capture the lesson through video and audio equipment placement, (d) made editing decisions, (e) prepared questions to ask the case study teacher before and after the lesson, (f) edited the video tapes from the three cameras into one coherent video, (g) developed an issues matrix, (h) developed discussion topics and questions for the facilitator guide, and (i) imported text and video into the Case Creator software.

The Case Creator software has these built-in features for the five video slots: (a) QuickTime movies with frame control, (b) a bookmarking feature, (c) scrolling and searchable transcripts, and (d) an interactive timeline. The software also has an issues matrix to allow the creators to structure access to episodes in the case by creating categories of issues and linking them to specific episodes on video. Case Creator allows for links to websites, as well as the ability to have text documents.

The prospective teacher educators created facilitator guides to accompany their multimedia case studies. Each guide contained information about case studies, suggestions on how to use this case, and questions that could be used to promote discussion about the case.

Results

This paper will focus on answering the first research question. While the retrospective analysis continues, a dominant theme that emerged from the data is that the prospective teacher educators developed greater appreciation of the complexity of teaching and learning through the process of creating multimedia case studies. Some of the appreciation focused on the complexity of what is involved in preparing someone to be a teacher. During an interview one participant noted, "Teaching is very complicated; teaching someone else to teach is even more complicated." Another commented, "It is very challenging to think of what it means to have someone learn to teach well." A third participant reflected on the match between teacher education rhetoric and practice: "We tell our pre-service teachers they need to plan worthwhile activities for their students. We need to plan worthwhile activities for our pre-service teachers and think about what that means."

The participants found that thinking about, planning and creating a case that would prompt pre-service teachers to think in new ways and deeper ways about teaching is quite complex. They struggled to articulate what they hoped pre-service teachers would gain from using their multimedia case studies, and how to create cases so that the likelihood of these gains happening would occur.

Participants also focused on new appreciation of the complexity of the actual task of teaching. In the reflective essays written at the completion of the project, a participant wrote, "The technical challenges we faced brought to light one aspect of the difficulty of teaching—it's not easy to film a mobile object (the teacher)." This team's case study teacher moved around the classroom continually during the lesson to prompt and assist students. This participant gained insight into the decisions the teacher made through trying to follow her with a camera.

Creating multimedia case studies also afforded participants opportunities to think in new ways about their own learning. In his reflective essay, one participant wrote, "I learned the most [through this project] by having the opportunity to question how I learn. Throughout my teacher education program I have learned a great deal about the way others learn, how to assess students' learning, and how to create lessons for diverse learners. I had never framed these issues around my own learning. [Creating a multimedia case study] has enabled me to study my own learning in ways that I never had before."

Another participant found that the process of creating a multimedia case study supported her in gaining insight into bringing theoretical and practical ideas together. As she wrote in her reflective essay, "Making this case was an eye-opener for how we can bring together theory and practice for our students (pre-service teachers)."

There is much more analysis to do on this rich set of data. I anticipate coming to better understand what and how the participants learned about the preparation of teachers through the creation of multimedia case studies.

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MATHEMATICS FOR TEACHING: FACILITATING KNOWLEDGE CONSTRUCTION IN PROSPECTIVE HIGH SCHOOL MATHEMATICS TEACHERS

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This paper reports results of an ongoing design-based research project that seeks to both promote and characterize the kind of deep, well-connected and flexible conceptual understanding of mathematics that is advocated for teachers (e.g. Ball & McDiarmid, 1990; CBMS, 2000; Fennema & Franke, 1992; Shulman, 1986). In particular, this paper investigates how students access relevant mathematical information in the context of mathematical problem solving.

Background

A subject's mathematical background is an important component of problem solving abilities (Schoenfeld, 1985); yet adequate mathematical preparation does not guarantee success. Studies show that undergraduates (Schoenfeld, 1985; 1992), graduate mathematics students (Carlson, 1999), and even some professional mathematicians (DeFranco, 1996) struggle to access the appropriate mathematics needed to solve a particular problem. In their work with mathematicians, Carlson and Bloom (2004) suggest that a well-developed and well-connected conceptual understanding of mathematics facilitated the mathematicians' problem solving success. Additionally, the mathematicians in their study reported that content knowledge was more useful when solving difficult problems than other factors such as general heuristics.

Research into the mathematical understandings of teachers indicate that even when teachers (both preservice and in-service) appear to be adequately prepared, their mathematical knowledge base is shallow and compartmentalized (Ball, 1990; Bloom, 2001; Bryan, 1999; Post, Harel, Behr, & Lesh, 1991; Tirosh & Graeber, 1990). A rich, well-connected knowledge base allows teachers to conduct the kind of inquiry and discourse recommended by NCTM (Ball, 1991; Ball & McDiarmid, 1990; Koency & Swanson, 2000; Ma, 1999; McDiarmid, Ball, & Anderson, 1989; NCTM, 2000). It has been reported that inadequate mathematical understandings tend to inhibit efforts to implement "reform" curricula (Behr, Khoury, Harel, Post, & Leah, 1997; Koency & Swanson, 2000; Mathematical Sciences Education Board, 2001; Post et al., 1991).

Theoretical Framework

Carlson and Bloom studied working mathematicians to gain insight into the problem solving process (Carlson & Bloom, 2004). The resultant framework makes transparent the complex interactions between the aspects and processes of mathematical problem solving. While analyzing the transcripts of the problem solving sessions, the authors noticed a regular pattern of cognitive and metacognitive actions. These actions led to the identification of *orienting, planning, executing, verifying* and *monitoring* phases during problem solving. The actual solution path was found to be cyclic in nature as the solver would run through the cycles until the problem was solved or abandoned, with accessible mathematical knowledge, heuristics, emotions and metacognition influencing each step along the path. The framework also revealed that transformational reasoning was a powerful tool employed by these expert problem solvers as they selected or rejected strategies.

The multiple dimensions and diverse components of the *Multi-Dimensional Problem Solving Framework (MPS)* developed by Carlson and Bloom (2004) suggest that learning to become an effective problem solver requires the development and coordination of a vast reservoir of reasoning patterns, knowledge, and behaviors, and the effective management of both resources and emotional responses that surface during the problem solving process, as well as a great deal of practice and experience.

Thus, the *MPS Framework* provides the means to scrutinize the resources accessed or overlooked in the process of solving mathematical problems. By attending to both the repetitive cycles of *orienting, planning, executing* and *verifying,* and the aspects of *resources, affect monitoring* and *heuristics* that influence each of the phases, one can tease out the nature of the subject's knowledge base and its influence during the solution process.

The Study

This study used *design-based* research methods —the iterative refinement of instructional innovations employing thought-revealing activities and artifacts in conjunction with inquiry into theoretical considerations regarding the nature of the development of mathematical knowledge (Kelly & Lesh, 2002; Lesh, 2002). The subjects were 17 preservice high school mathematics teachers who had completed most or all of their required mathematics courses and intended to student teach within the next year. The study was set in a three-credit upper division mathematics education course called *Mathematics in the Secondary School*. Data sources for the study included clinical and task-based interviews, video recordings of classroom sessions, homework, class work, reflective journals, pre/posttests consisting of NAEP (Grade 12) items and a *Views About Mathematics* (VAMS) survey (Carlson, 1999) administered at the beginning and end of the semester.

The curriculum for this course has been developed over several iterations of the experiment and provides students with opportunities to revisit the major concepts of school mathematics while concomitantly enriching their knowledge base and improving their problem solving abilities. Challenging problems were selected for their capacity to reveal student thinking and understanding and to stimulate mathematical discourse. For instance, the problem "*In rhombus EFGH the coordinates of E and G are (-6,-3) and (2,5) respectively. Find the area of the rhombus if the slope of segment EF is 2*" is very effective in revealing student thinking and understanding of geometric concepts such as various properties of rhombi, as well those of linear functions such as slope and distance. A set of homework problems might look like that seen in Table 1. Problems selected from a variety of mathematical topics force students to think more carefully about the concepts involved and tend to generate multiple solution paths.

Students also took typical high school mathematics problems and were asked to generalize them and analyze them for mathematical structure. Students shared their work and their reasoning, orally and in written form, with their peers on a regular basis. For example, the problem*, "Person A sets out in a car going at 50 mph. Starting 3 hours later, person B tries to catch up. If person B goes at 75 mph, how long does it take to catch up?"* is a fairly common type of problem. Prospective teachers can quickly determine that the answer is *6 hours.* By simply fixing the speed of Car A at 50 mph and varying the speed of Car B, they can explore how a change in the speed of Car B impacts the time it takes to catch Car A. This type of activity, called *Extended Analyses*, pushes the students beyond thinking about the answer to thinking about the relationships among the various components. While early Extended Analysis tasks are carefully scaffolded, students eventually complete the tasks independently.

Table 1
Sample Homework Set

1	How many 5-card poker hands can be dealt from a deck of 52 cards?
2	What is the probability of being dealt a royal flush? (i.e., ten, jack, queen, king and ace of the same suit)
3	For $x > 0$, what is the smallest value of $x + \frac{5}{x}$? Solve this problem two different ways.
4	When an integer is divided by 15, the remainder is 7. Find the sum of the remainders when the same integer is divided by 3 and by 5.
5	Apple weighed her 4 dogs in pairs. Together, Hilbert and Euclid weighed 110 pounds, Euclid and Gauss weighed 103 pounds, and Gauss and LaGrange weighed 108 pounds. How many pounds would Hilbert and LaGrange weigh together?
6	Answer the following: a Prove the following: If two numbers are divisible by 4, then their average is divisible by 2. b Write a more generalized version of the statement in part (a). c Prove the statement you wrote in part (b).

Results

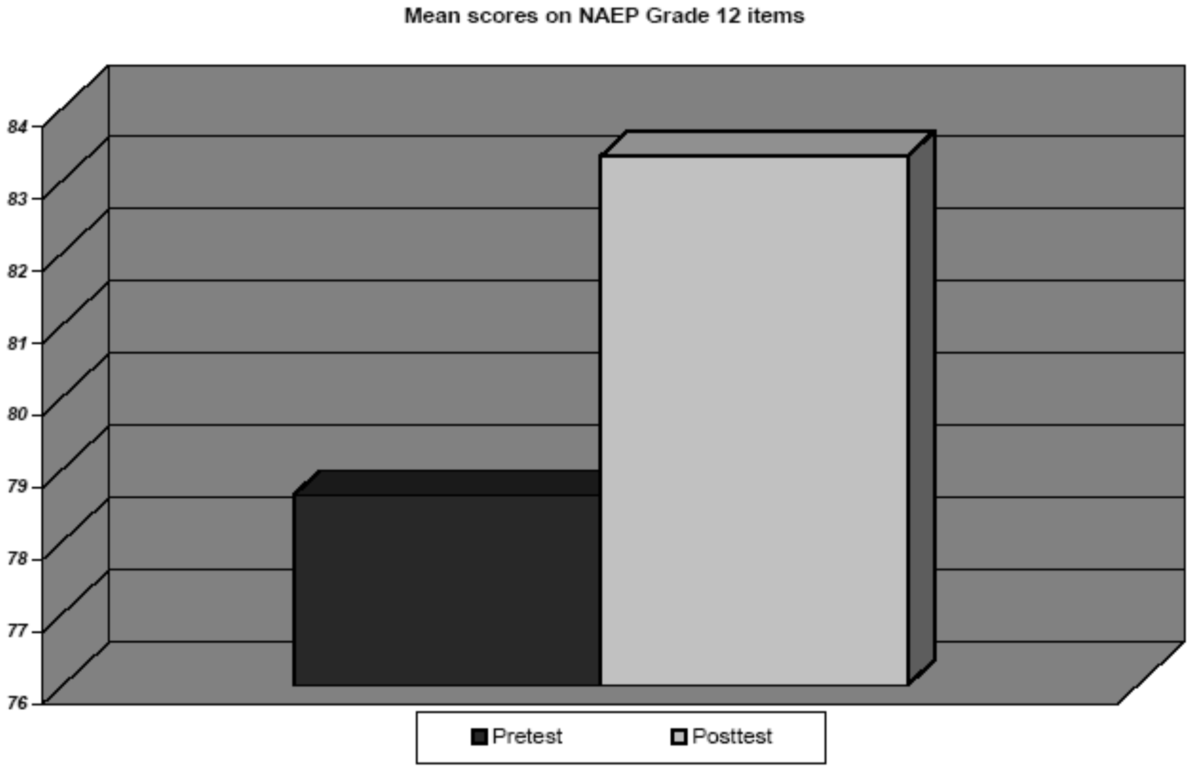
Although the pretest results (see Figure 1) verified that all participants were proficient in high school mathematics, analysis of classroom sessions, problem solving interviews and written work showed that their knowledge base was compartmentalized and their problem solving approaches were proceduralized. In addition, while a majority of subjects self-reported a high level of persistence, examination of their work early in the semester revealed that they could be easily frustrated and gave up quickly. Mathematical problems that would be routine if found at the end of a section of text became quite challenging when stripped of familiar cues. As a result, students sometimes failed to access the appropriate mathematical concepts required for the situation. For example, when asked to find an instantaneous rate of change, many found the average rate of change over an arbitrary period of time, rather than calculating the derivative at a point.

Students also demonstrated difficulty expressing themselves. Although students quickly gained a kind of fluency, in the beginning many students were unable to explain their reasoning or to articulate what mathematical concepts or problem solving heuristics they used, even when pressed by the instructor. For example, when Donald was probed as to how he determined the number to be placed in the center of a Magic Square, he could only respond with “I don’t know, I just *knew* it was 5, that’s all.” Students also evidenced difficulty following the work of their peers and pinpointing the errors and misconceptions of others. By introducing a classroom culture where explaining one’s reasoning in large and small group discussions, and where critiquing the work of others was routine, fluency in mathematical discussion progressed fairly rapidly.

By mid-semester, data showed that students were more persistent in their problem solving efforts, more confident when presenting mathematical arguments, and displayed metacognitive behaviors in small group interactions. End of semester measures showed positive shifts in content knowledge, problem solving behavior and beliefs.

Mean scores on the pre and post tests (see Figure 1) revealed significant gains ($p<0.001$), with non-traditional students exhibiting the greatest gains.

Figure 1

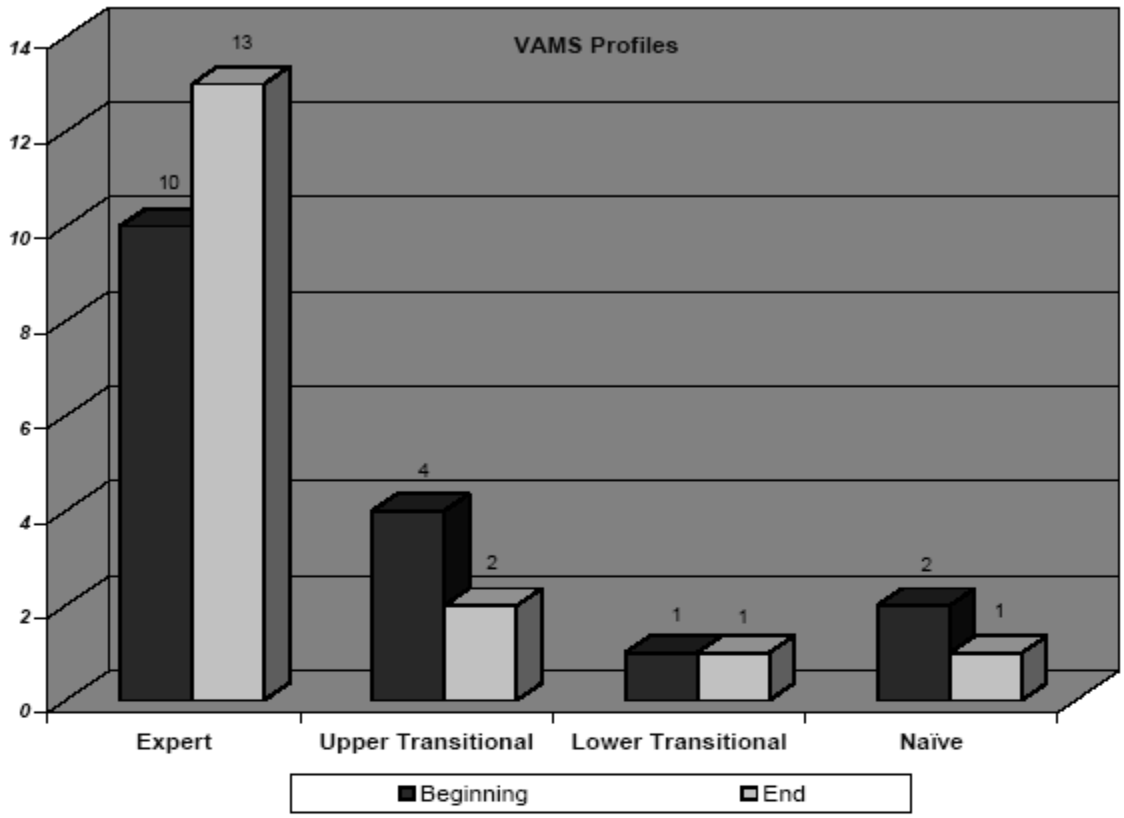


At the end of the semester, problem solving sessions were markedly different. During task-based interviews, subjects were able to articulate the mathematics needed and verbalize a solution path. Self monitoring behaviors and sense-making behaviors were detected more frequently and more consistently in end of semester interviews. During preliminary interviews, the subjects were observed frequently looking to the interviewer for assistance or feedback, but analyses of end of the semester sessions found few instances of this as students were more likely to depend on their own skills and judgment during problem solving.

In both interview transcripts and reflective journals, these students professed a better understanding of how topics fit together and relate. They consistently claimed greater confidence in problem solving in general, and the content of high school mathematics in particular. Improved scores on the posttest support this evidence. All students involved in peer tutoring and teaching internships reported feeling more effective and confident in these settings because they don't have to "see how the book did it" first, and are able to explain concepts in multiple ways. Most of the students also remarked on a renewed enjoyment of mathematics and mathematical problem solving as well as a desire to introduce future students to that pleasure.

Analysis of pre-and post-VAMS (see Figure 2) revealed shifts from *naïve* views about mathematics toward expert views. Even the one student who remained in the *naïve* category made some shifts toward a more expert point of view. The most dramatic shifts were found in two items pertaining to mathematical instruction. For the item asking *The role of a mathematics teacher is,* a significant number shifted away from the naïve view *to show me how to work specific problems* and toward the expert view *to guide me in learning to solve problems.* The item stating *A major goal of mathematics instruction is* exhibited a shift away from the naïve view of *to impart information* toward the expert view of *to equip students to solve problems independently.*

Figure 2



Students noted three aspects of the curriculum as being particularly important and influential in enriching their understanding of mathematics. One was the interesting and varied set of problems they were presented with. As Terry remarked, "You really had to use all the math you knew at some point." Second were the Extended Analysis tasks such as the Catching Up problem mentioned above. Karen said, "I really liked the ability to follow my curiosity and figure things out on my own." Other students reported that the tasks helped them to see connections between topics and to appreciate the value of multiple representations of problem situations. Finally, most students valued seeing problems solved in a variety of ways. They appreciated being responsible for presenting their work and explaining their reasoning, and being exposed to other points of view. Colleen remarked, "You know, I always thought there was really just one best way to solve a problem, and I had to find that way. I see now that's just not always true."

This study demonstrated that a course for prospective high school mathematics teachers which focuses entirely on school mathematics from a content (as opposed to a pedagogical) point of view, and which promotes problem solving can positively impact preservice teachers in a variety of ways. As a result of the experience, the subjects enhanced their mathematical knowledge base, improved their problem solving skills, achieved greater verbal fluency in mathematics and demonstrated important shifts in their attitudes regarding the nature of mathematics and mathematics teaching. This ongoing project has provided some insights into a process by which teacher educators can successfully enrich the content knowledge base of prospective mathematics teachers.

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ASSESSING TEACHER KNOWLEDGE IN ACTION

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In this paper we explore possibilities for assessing teachers' mathematics for teaching. Data are drawn from a teacher in-service program in which the principles of complexity science are used to inform the teacher educators (in-service providers). From the research we have learned that although one can assess an individual's knowledge through paper and pencil tests, such tests are limited in that they occasion the teacher's immediate response and that response is not necessarily reflective of their mathematics for teaching. Further, our work leads us to believe that teachers' mathematics for teaching arises in collective activity rather than individual activity. We are left questioning the value of traditional paper and pencil assessments of individual knowledge.

"Twenty-three times thirty-five equals eight hundred and five," the woman (a teacher educator) at the front of the room said as she wrote 23 x 35 = 805 on the board. Facing her were some 20 teachers sitting in groups of 4 or 5 around tables. These were all teachers who identified themselves as nonspecialists in the area of mathematics and who teach children and youth in grades K-12. "I want you to show how you know this is true," she said as she put the chalk down.

The teachers picked up their pencils and quietly began to jot things down on the leaves of papers and in scribblers in front of them. A few of the teachers were seen talking to one another and pointing to their papers. No more than a minute or two went by before the faces were directed back on the teacher educator who had been wondering among the tables. She asked two people to put what they wrote down in their books on the board for the others to see.

$$\begin{array}{r} 23 \\ \text{x } 35 \\ \hline 115 \\ 690 \\ \hline 805 \end{array} \qquad 23 + 23 + 23 + 23 + 23 + \text{É} \; = 805$$

(thirty-five times)

"Does anyone have anything different?" The teacher educator asked.

"I have 23 groups of 35," one woman said. "It's just the opposite."

"Come put that up on the board," the teacher educator suggested.

The teacher at the board drew twenty-three circles on the board and then put a ring around them. "I can't draw all of them but if there were 22 more groups then I would have 805," the teacher explained.

"So, any other ways to think about this multiplication?"

"You could do it with base-10 blocks. That's not what I did. I just did it the normal way, but you could do with the blocks," another teacher suggested.

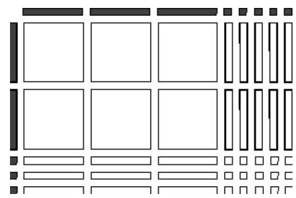
"What do you mean by the normal way?"

"Like the first one up there. Long multiplication but I don't put a zero in the second row. I put an X so as not to forget to move over."

"Oh, I understand, and now the base-10 blocks can you show the others how you would use them."

"Sure, well you take 2 longs for the 20, and then 3 units, and repeat that 35 times. Then you would group all the longs into tens to make hundreds and the units to make tens and so on." The teacher explained.

"Ah, when you said use the hundreds blocks, I thought you might take them and arrange them like this. Then place the product inside" the teacher educator drew on the board as she spoke.



"Area is equal to length times width," another teacher blurted out. Many heads nodded with this suggestion.

Assessing mathematics knowledge of teachers and for teaching

Recent work in the area of teacher education has explored the nature of teachers' mathematics or mathematics for teaching (Ball and Bass, 2002; Davis and Simmt, 2004; Simmt, Davis, Gordon and Towers, 2003). What might we say about these teachers' mathematics knowledge? What is it they know and of that what is the role that knowledge plays in their teaching? In other words, how might we assess the mathematics and the mathematical understanding of these teachers?

Calls to address perceived deficiencies in teachers' knowledge of mathematics have often been answered by suggesting that teachers need more university level mathematics courses. However, there is evidence to suggest that student achievement in mathematics is not correlated with greater number of undergraduate mathematics courses taken by teachers (Begle, 1979; Monk, 1994). So then, how might we educate teachers for the teaching of mathematics?

In our work we have been studying mathematics cognition through the frame of complexity science. We have theorized that teacher knowledge must be developed both around established mathematics (that is the existing body of mathematics) and the establishment (or generation) of mathematics. Teachers' knowledge of established mathematics includes both conceptual depth and curricular breadth. Teachers need to understand concepts deeply but they must also know how these concepts are sequenced and elaborated across the curriculum. The second main area of teacher knowledge, the knowledge of how mathematics is established, includes the knowledge of how mathematics is constructed both at the personal level and at the collective (or community) level. (See Davis and Simmt, 2004, for an elaboration of these points.)

Our work is oriented by the conviction that generic mathematics courses which focus only on content, as well as traditional methods courses that address how mathematics should be taught, fall short of attending to the complexity of human learning systems and collective projects like mathematics. As we reconceptualize teacher knowledge we find ourselves with a need to rethink the assessment of teacher knowledge. In our view, standard forms of assessment (paper and pencil tests written by individuals) used in most university mathematics courses familiar to us provide little insight into teachers' understanding of mathematics for teaching.

Indeed, we believe there is a need for forms of assessment more in line with the kinds of assessment mathematics educational researchers are promoting for school children. To this end we are interested in creating assessment items and strategies that will help teacher educators, teacher certification agencies and employers better understand the quality and content of teachers mathematical understanding. In this paper we begin to explore ways of interpreting and assessing teachers' mathematical knowledge with some tasks that we have been using in a teacher in-service project.

Our Research Site

The data used to explore assessing teacher knowledge are taken from a teacher in-service research project in which 20 teachers who teach grades K-12 are engaging in mathematics, not to find classroom resources and strategies, but to strengthen their own mathematical understandings.

As researchers, we have approached this in-service project from the perspective that by engaging in mathematical inquiry and processes teachers themselves will develop more robust and extensive mathematics for teaching. This happens in the process of presenting and re-presenting their mathematical understandings. Following principles developed in detail elsewhere (Davis & Simmt, 2003; 2004), our work with these teachers is guided by the principles observed in complex systems. In particular, we understand the possibilities for learning and the generation of mathematical knowledge to be enhanced by having learners work together on joint tasks that include "liberating constraints". The lessons we set up take advantage of the diversity of understandings, interpretations and talents of the participants at the same time as ensuring sufficient redundancy of knowledge and abilities among participants is present. As well, we structure the lessons such that ideas and interpretations will bump up against one another. It is only in this way that the possibility for the generation of new knowledge is possible. Finally, the teacher in-service gives up a common assumption of mathematics instruction at the university level—namely the "transfer" of already established concepts, algorithms, and strategies for solving particular mathematics questions and problems. In these in-service sessions tasks are posed with the intention that the members of the learning community will fold back to image-making (Pirie and Kieren, 1994) in order to notice properties and formalize structures developed in action. Thus, it is in action that mathematics knowledge is created among members of the collective rather than asserted by the professor.

Observing/Assessing Teacher Knowledge in the Collective

In this paper we pose the question, "How might we assess teacher knowledge in the context of the collective project of doing mathematics?" Consider the limited number of responses to the multiplication task posed to the teachers. In our view, the teachers' immediate (first) responses must suggest how they most commonly approach the task of multiplying multi-digit numbers. Notice how they were asked not to find the product but to show how they knew that the product was the amount given.

What can we say about the mathematical knowledge of those teachers who responded to the task with an algorithm? According to Pirie and Kieren (1994) the person might be acting formally. Reid (2002) might interpret the action as a form of mechanical deduction. That is, the teacher deduces the product through a mechanical process and possibly without conceptual understanding. He or she knows when and how to use an algorithm for multiplying and from it is able to compute a correct product. Is this adequate knowledge for teaching mathematics? Clearly we cannot yet answer this question because our understanding of the person's understanding is simply inadequate. We might guess that the person who offered the first algorithm has some understanding of place value given the zero found in the second partial product in contrast to the person who puts an *x* in the position to hold its spot. Which of these two people understand place value? What do our observations allow us to assume? At this point very little; with just this response we have no more information than what we might have gathered from a paper and pencil test. Already, it is clear such tests are inadequate in terms of assessing mathematics knowledge for teaching.

However, with the simple act of having teachers share their ways of knowing with others in the group the teacher educator creates the possibility to interrogate and to observe teachers' mathematics more carefully. When the teachers talked about the methods used for multiplication later in the session it became evident that the algorithm was little more than a procedure for some of them. Indeed there was at least one teacher who did not realize the relationship between the '*x*' placeholder and the fact that in the second stage of this algorithm one is multiplying not by three but by thirty, hence producing products that are multiples of ten not one.

Sharing in the group becomes an opportunity to assess teachers' knowledge of mathematics for teaching; implicit in the request to share is the obligation to explain in terms the 'other' can understand. As the members of the group express their understanding in terms the others can understand, we, as teacher educators and researchers have an opportunity to assess the their conceptions and explanations. We begin to see mathematics for teaching revealed. But the sharing leads to a phenomenon that could be said to confound the assessment.

In our sessions with teachers, we have repeatedly observed that a collective project of understanding emerges and soon we find mathematics arises that cannot be said to be the understanding of a single person but of the collective itself. When one person offers an explanation—take, for example, when the researcher used base-10 blocks to illustrate what she thought the teacher had said—a new understanding of multiplication began to emerge. In another instance, when the participants compared their strategies for multiplying, space began to open up for the participants to offer multiple interpretations— something that is impossible with paper and pencil tasks completed by an individual in isolation from others and their ideas. Emerging from the discussion of multiplication as groups of objects and as area, the discussion turned to what makes those two contexts different. With further conversation the collective brought into its understanding (in action) a distinction between discrete and continuous phenomena. So we return to our question, what does it mean to assess teacher's mathematics for teaching?

Yes, individual teachers did have an understanding of "multiplication as area-producing" but it took interactions among the teachers for this understanding to be articulated in such a way that the teacher educator could observe it. Further to that point, and even more significant, it took a collective project around the nature of multiplication in order for distinctions between discrete and continuous phenomena to emerge. We speculate that these distinctions were part of the teachers' knowledge of mathematics for teaching (i.e., something they knew and enacted in their classrooms), but, for the most part, not part of their explicit mathematical knowledge (i.e., something that they knew that they knew).

Conclusions

In terms of assessing teachers' mathematics, we find it significant that first responses to a question reveal little more than one possible response and likely the response afforded by little thought. When we ask teachers to share their responses (or explain for others) we find they begin to use their mathematics for teaching. As they do a collective begins to form among members of the group and the nature of the mathematical understanding begins to transform. No longer do we observe strictly the mathematics of individuals but we observe the emergence of mathematical understanding within the collective itself. At that point it becomes difficult to attribute particular understandings to individuals. However, we do begin to

observe transformations in individual understandings (this is often reported by teachers themselves, "I never thought about it that way before!"). Generally speaking the collective understandings that emerge are observed to be connections between concepts, strategies, and explanations. Recall the move from multiplication as groups of objects to multiplication as area to distinctions between continuous and discontinuous phenomena. We believe most teachers in our group would have struggled with trying to make distinctions between continuous or discrete examples of multiplication prior to the session. Further, those understandings were not "transferred" from teacher educator to teacher in the session; those understandings emerged in the collective that formed as teachers explored multiplication together.

In this paper we have explored ways in which we might begin to assess teachers' knowledge of mathematics and of knowledge of mathematics for teaching. From that exploration we have come to see how there is a need to develop ways of assessing teacher knowledge that can be used in the context of both pre-service and in-service education. Those means must take advantage of the dynamic nature of mathematical understanding and the collective nature of knowing that arises when people work together on joint tasks. In our paper we have illustrated formative assessment strategies, which include taking common tasks teachers give students and asking the teacher to "show how you know"; then following that with group discussion which bump people's ideas up against one another thus providing an opportunity to occasion the generation of new connections and mathematics at the collective level. Our next challenge is to create summative assessment strategies for working in the space of the collective understanding.

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STUDENTS' PERSPECTIVE ON ARGUMENTATION IN A MATHEMATICS CLASSROOM

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This paper reports research into whole class argumentation from the perspective of second grade students who experienced mathematics in a problem centered learning setting. We will report what second grade students themselves say concerning their experiences in arguing about various solutions presented and in defending and justifying their thinking. Somewhat surprisingly, they all indicated that arguing about their mathematical solutions and strategies was important to their learning of mathematics. Several students who had been in this second grade class the two years previously were also interviewed to probe their thoughts on arguing. They also shared positive memories of their experiences in second grade. A number of them contrasted these positive experiences with experiences in other classrooms where discussing their ideas and arguing about solutions was not as valued.

In recent years there has been increased focus on the importance of learning settings in which students discuss their mathematics and explain and justify their solutions (Yackel, 2001). Mevarech (1999) indicates children learn through opportunities to explain, justify, and listen to one another's ideas. Explanations are the best means for elaborating meaning and making connections. Krummheuer (1995) notes that participation in negotiations during argumentation has a strong positive influence on students' conceptual development in mathematics. Sfard (2000) actually proposes that this discourse is equivalent to thinking itself.

The many positive effects for students' explaining, justifying, and arguing about their solution strategies have been found as researchers have examined classrooms where the culture is one that encourages students to act with intellectual autonomy (Kamii, 1989), devise their own meaningful solutions to mathematics tasks, share these strategies and solutions with the whole class, and challenge one another's ideas. However, much of the research has focused on the researcher observing, analyzing, and interpreting these classroom events rather than students themselves deliberately sharing their ideas about the importance or otherwise of mathematical discourse.

Research Setting

This research was conducted in a second-grade classroom in a low to middle income suburban public school with students from various backgrounds. Using a problem-centered learning approach (Wheatley, 1991), each mathematics period Kathleen chose tasks based on her evaluation of her students' mathematical needs and sophistication in their thinking; tasks that she thought would be challenging but doable and that she anticipated would help them grow mathematically. Students spent about half the mathematics lesson working with a partner to make sense of the assigned tasks and develop solution strategies. The remainder of the time was devoted to whole class sharing of the different solution strategies, with Kathleen acting as facilitator. She encouraged students to make sense of the different ideas presented and ask questions about or challenge ideas that were being presented. The research reported here is part of a larger study of approximately 60 students spanning three years of investigating the argumentation process in which students openly participated in solution strategies, particularly during whole class discussion. The data analyzed is taken from the second year of this study (Cassel, 2003).

Methodology

We used Cobb and Steffe's (1983) teaching experiment methodology where the researcher is a facilitator, participant, observer working with the teacher and students, and listening to their interactions and ideas (Steffe & Thompson, 2000). We observed the second-grade mathematics lesson at least one day each week throughout the school year, monitoring and questioning students where appropriate to clarify their thinking. Extensive field notes were taken, supplemented by video recordings of each lesson observed. The structure of this research also gave the team, including the teacher, opportunities to meet for thirty minutes after each session. During this time we shared our observations about how students solved the various tasks and the ideas they shared, particularly focusing on whole class discussion events that indicated differences among students' mathematical thinking. We used this discussion to help plan the tasks to be used over the next few days with these students. We also interviewed several third and fourth grade students who had previously been in this same second grade classroom to solicit their thoughts about these experiences.

Data Sources

One theme that emerged in our observations of the class was the importance Kathleen, the teacher, placed on argumentation. She took a proactive stance in challenging her students to explain, justify, and question the various ideas presented. During one debriefing session Kathleen commented: "I always feel like we had a great day if they are lining up to go to specials, music, P.E., whatever, and they are arguing about more math." We observed numerous cases where students engaged in such argumentation. During one particular mathematics period in the second semester students actually discussed what "arguing" meant to them. This discussion was triggered by one student commenting to Kathleen: "You like it when we argue, don't you?" We will analyze the subsequent discussion, incorporating relevant comments from Kathleen's former students who were now in third and fourth grade.

Kathleen: Last time Betty you brought up the fact about arguing. Why is it arguing? How does it help you?

Betty: umm…It gives you questions to try to solve them.

Kathleen: Okay, you try to solve it. How does arguing help, or what do you think about the arguing we do?

Cort: Ummmmm…How does it help you?

Kathleen: Yes

Cort: Some people have different problems and different answers…

Kathleen: Do you know what Cort is saying?

Class: No

Kathleen: Okay Cort, say it louder.

Cort: Some people's answers might be wrong and some people might say these wrong, so they say something else like, one time we're doing shapes on the paper. You have to argue over everything because some people said some shapes count or don't count. Sometimes people know the answer and other people don't, and they don't want to get theirs wrong.

Here, Betty has introduced the idea that arguing raises questions about the ideas being discussed – something we will return to shortly. Cort has emphasized the idea that arguing about different answers helps students decide if their answer is correct. When we interviewed

Kathleen's former students several also commented on this idea. Karen, a third grader, remarked that having right and wrong answers in her second grade class was "good." Arguing about these answers helped her to learn. She said she was not afraid or embarrassed to give solutions because the class would talk about them and they would figure out the answers. However, she did not argue in her other classes, only in second grade. Another third grade student, Jon, commented that while he was in first grade he "usually got his math wrong." That is why he never went to the board in other classrooms. While in Kathleen's room he participated in arguing and "going to the board did not bother me; when people argue I would think harder."

Research shows that students who feel embarrassed or threatened by being wrong will not perform well academically. Once a situation is perceived as a threat, even being embarrassed in front of a peer, for example, our bodies throw up defense mechanisms (Jenson, 1998). Students in both traditional and nontraditional classrooms share answers on the chalkboard, and/or orally, both of which are in the public domain. Students want to succeed and feel good about themselves and what they have done. If students are called on for answers in an environment that is not risk-free, they feel embarrassed and will not perform their full potential. Students in Kathleen's classroom learn that it is okay to have conflicting answers; actually it is more than okay, it is important in helping them construct their mathematics.

The conversation about arguing continued:

Kathleen:	What about when somebody has a wrong answer or a different one than you are thinking about? Does it help you to listen to their ideas?
Class:	Yes!
Kathleen:	Why does it help you? Tina, why does it help?
Tina:	Because arguing helps you know more things like, shapes don't have to have corners and sides to actually be a shape. (She is talking about a previous class setting where they were talking about shapes.)
Kathleen:	Yeah! We had a big argument about that didn't we?
Tina:	I know corner, and I know, actually, because everyone was arguing, everyone knows it. It doesn't have to have a corner so arguing lets people tell you.
Kathleen:	When we were arguing about the shapes, Bill did you hear that?
Bill:	Yes.
Kathleen:	What did you think?
Bill:	More…What I think it does, you have more people arguing the easier it is to find out because you have this amount of people saying this is right and this amount of people saying this is wrong. You really go over it for a long time.

Tina and Bill have highlighted the notion that arguing helps them think about the different ideas. Bill actually introduces the idea that the time taken in talking about the different ideas helps the learning: "You really go over it for a long time." Again a student from last year said, "You get more problems because you have to think in your head how to explain so all can agree on a solution." She further stated that this way of arguing about tasks gets you "thinking a lot."

In this second-grade classroom the students discuss the different solutions and try to make sense of the mathematics. As students present their solutions and discuss them, they end up spending time making sense of these different ideas and reflecting on the viability of the various solutions. Through this argumentation process the students' dialogue and activities become compatible (Yackel, 2000); or as Bill said above: "(Y)ou have this amount of people saying this is right and this amount of people saying this is wrong. You really have to go over it for a long time." These students are talking here about what Kamii (1989) refers to as "intellectual autonomy." Also, Wheatley (1992) indicates that reflection, which gets little attention in more conventional settings, is an integral part of mathematics learning and that teachers should allow ample time for whole class discussion of the various ideas so that students can reflect on their thinking, an idea so well put by Bill: "You really have to go over it for a long time."

As the discussion continued it became evident that students are convinced that it is their responsibility to take time to reflect on the different ideas presented and judge their viability. From **their** perspective, taking this stance helps them learn.

Kathleen: I want to ask you something… you notice I don't go over here and go, okay this is class, this is what a shape is, and this is a shape, and there is no arguing…

Class: Yeah! Because arguing it helps us work it out ourselves…kids can figure it out…teachers are not supposed to tell us the answers.

Kathleen: Do you think it's better when you figure it out yourself?

Class: Yeah!

Kathleen: Why do you think it is better?

Class: It's better because you get smarter and you won't have to ask or…

Kathleen: Maggie, what do you guys think about all this?

Maggie: I think you shouldn't tell us. You have to let us figure what it is, that helps.

Kathleen: Why does that help Maggie?

Maggie: We learn how to do things and we remember when you let us figure it out. That is what helps us learn.

Kathleen: Do you remember better if I go, okay this is the way it is? This is how you do it?

Class: No, we have to learn we have to do it ourselves. We wouldn't have to go to school if you told us things our parents could tell us. Then we wouldn't be learning.

Kathleen: Why do you think you learn better if you figure it out yourself?

Mack: You figure it out yourself you remember it better than if you tell us what it is.

The comment from one student: "Teachers are not supposed to tell us the answers," is contrary to how students traditionally expect teachers to act, yet for these students, if the teacher takes this traditional role then they are not given the opportunity to think for themselves. They have confidence in their own ability to discuss their ideas and reason about their mathematics: "Kids can figure it out." This "figuring it out" for themselves helps them to remember the ideas better.

Five fourth grade students were asked what they remembered about Kathleen's room. Each indicated they remembered arguing during math class. They said that math was easier then because they could "figure it out." These students said that their teachers now let them discuss sometimes, but for the most part they are told what to do. Math for them is harder without the arguing. They do not see this year's teacher taking part in their limited discussion whereas they saw Kathleen taking part in all their discussions and encouraging them to argue about their ideas: "She (Kathleen) was interested in how we thought about math" (Gay Arla). These five students were not necessarily the "top" math students and yet they remembered positive mathematical learning experiences in this second-grade environment where arguing was a negotiated and accepted norm. Thus, the children themselves have identified aspects of argumentation that are identical to findings presented in research literature on argumentation. For example, Krummheuer (1995) states that the relationship between engaging in argumentation and learning is reflexive. Students' active involvement in the argumentation process influences their individual learning by enhancing their cognitive capabilities that influence both the course and the outcome of the argumentation process. Further the argumentation process has a strong positive influence on students' conceptual development in mathematics.

The discussion then began to focus on "how" this arguing should be done:

Kathleen: Cort, your jacket is green! (He is wearing a red and black jacket) I like your green jacket.

Cort: It's not green!

Kathleen: What if a teacher tells you something that doesn't make sense? Is it okay to argue? Respectfully?

Class: Respectfully, yes. Don't be disrespectful.

Mary: It's not green!

Kathleen: Is it important for kids to figure things out themselves?

Mary: YES!! My mama always told me if you want something done you gotta do it yourself.

Kathleen: You want something done.. you gotta do it yourself. But how does…how does the arguing help when we all get together and share ideas? You know how we are doing that?

Class: Take turns. Let people share.

Kathleen: What if you disagree?

Class: Do it nice. Yeah, nice.

Kathleen: That is a good idea to say it respectfully…because important issue is respect. Do you think if someone gave an idea and another person went duh! That's dumb! Is that respectful?

Class: No!! You don't say that's wrong!

The class has, under Kathleen's guidance, folded back to their earlier discussion about this arguing only being possible if they feel "safe" to express their ideas in the classroom, without fear of being embarrassed or threatened. While it is apparent that Kathleen is proactively orchestrating this part of the discussion (she introduces the word "respectfully" into the conversation here) there have been a number of occasions over the year where these students have shared their feelings about how they need to act towards one another in this classroom. For example it has been the topic of conversation in several "morning meetings" where students take time to discuss disagreements that may have occurred in the classroom or on the playground that they are upset about. During these meetings Kathleen takes the stance that students themselves must discuss and decide how these issues should be resolved (Cassel, 2002). Kathleen's prior students also alluded to this idea of "respect" when they talked about feeling "safe" in Kathleen's room to present their solutions, even if they turned out to be "wrong," something they did not necessarily feel comfortable about in other settings: "Going to the board (in second grade) did not bother me; when people argue I would think harder" (Jon).

Kathleen brought this discussion to a close by asking the following question:

Kathleen: Have you ever thought you had an idea this is the way it was, and then someone showed something else and it made you think about it?

Cort: When Mary said that thing about shape it made me change my answer.

Kathleen: Did you hear that Mary? Say it again Cort.

Cort: When you got that thing about the circle it made me change my answer.

Kathleen: You saw something a different way; almost like a bird's eye view. So you think arguing is an important thing to do in school?

Class: YES!! In a respectful way!

Cort changed his answer because he had made sense of the arguments. Through interactive discussions and arguments students do make sense of the mathematics and, as they reflect on the ideas, will change their answers. The students in Kathleen's classroom feel comfortable enough in sharing mathematics and know it is acceptable to change answers once they have made sense of the ideas. Kathleen provides opportunities for the students to engage in reflective thinking (Wheatley, 1992). Dewey (1910/1991) defines reflective thinking in the following way: "Active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it, and the further conclusions to which it tends, constitutes reflective thinking" (p 6).

Conclusions

There is a distinct kind of discussion occurring in the episode from this second grade classroom. The discussion does not focus on mathematical concepts or strategies, but focuses on ways of working together. This has been referred to as talking about talking about mathematics, as opposed to just talking about mathematics (Yackel, 2000). When the discussion is talking about talking about mathematics, the subject of discussion is primarily social in nature; how to act and contribute. It is not specifically about how to make mathematical constructions. This primarily social aspect illustrates how social norms (rules for behavior, but not generally written down) are peculiar to each classroom.

In the above discussion Kathleen and the students have taken time-out from doing mathematics for the specific purpose of letting the students openly express their ideas about arguing within a mathematical classroom context. As is evident in the student responses, they identified the type of open interactive arguing that is acceptable, and they expressed its use in the classroom as very beneficial to learning. Third and fourth grade students who had been in this second grade classroom previously also highlighted these ideas about "arguing" as being important to their mathematics learning. In Kathleen's mathematics classroom arguing is routine, but essential, and very important in its contribution to student learning. In these examples from Kathleen's current and former students it is apparent that they **themselves** learned to value argumentation as an important part of learning mathematics.

These students have expressed in their own meaningful ways what previous research has reported about the importance of argumentation. When students explain their solutions they actively communicate with each other and the teacher. In order for that communication to be successful, they must negotiate meanings, not just recite facts. As they negotiate they adjust their interactions by presenting rationales for their strategies. This argumentation process gives rise to learning opportunities as students question and express their thoughts while attempting to make sense of each other's ideas. From our research we identified several aspects resulting from students' talking about what they considered important in the argumentation process:

- It helps them construct meaning.
- It enables them to reflect on their own thinking as well as that of other students.

- They ultimately find the "right" answers and develop strategies without the teacher telling them.
- They remember their mathematics better.
- They experience greater learning.

According to Caine and Caine (1994) and Sprenger (1999), brain research indicates that arguing is helpful to students in their development of mathematical intelligence and in becoming autonomous learners. From the students' perspectives in this study, "arguing" is an important component in **their** making sense of mathematics, becoming confident in **their** own ability to do mathematics, and ultimately **their** learning of mathematics.

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MAKING SENSE OF LEARNING MATHEMATICS

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A phenomenon is what is interpreted as common to a collection of incidents which seem to someone to share some similarities. Seven classroom incidents are described, and proposed as examples of a phenomenon. These are then analysed from the perspective of learning as an effect caused by teaching, and from the perspective of learning as a maturation process involving changes of the structure of attention analogous to changes of state in physics. The latter is shown to make better sense of the phenomena, and is conjectured to better inform teaching.

Theoretical Base

Human beings make sense of the world through cognising and re-cognising similarities and differences. Stressing similarities leads to categorising and assimilating (which includes dismissing) in order to feel comfortable (Piaget 1971, Lakoff 1987). Stressing differences leads to disturbances of varying intensity, and so either to accommodation or to undertaking enquiries and explorations in order to resolve surprise or broken expectation (Festinger 1957, Fischbein 1987, Moshovits-Hadar 1988). Resolution leads to feeling comfortable with what had previously disturbed, and hence to pleasure.

Paying attention to similarities and differences is a powerful pedagogic strategy (Brown & Walter 1983, Brown & Coles 2000) corresponding as it does with a pervasive theme throughout mathematics: invariance in the midst of change. As Gattegno (1987) pointed out, stressing some features (whether attending to similarities or attending to differences) inevitably involves ignoring others, and this is the basis for generalisation and abstraction. At its best it leads to constructs and to labels for these constructs; at its worst it leads to stereotypes and to overlooking of relevant details.

The incidents which comprise a phenomenon may all be experienced by the same person, or may be described and negotiated within a group. Awareness of an incident as being an example of a phenomenon is recognisable when there is a sense of 'familiarity, of 'oh, that again', with perhaps even a categorising label coming to mind. Where the incidents happen to different people, negotiation of what it is that is similar between incidents is necessary in order to reach useful agreement; where they come from one person, task-exercises inviting others to see if they recognise incidents as a phenomenon can be used to engage colleagues and check out whether it is sensible to suggest a phenomenon (Mason 2002). That is the approach taken in this paper.

Note that what one person reports noticing tells others as much about the individual's sensitivities to notice and re-mark, as about the situation or incident itself (Mason 2002). This is most especially the case when an incident is labelled by some collective term, which may even play a apart in the rising sense of 'sameness'.

Method

Incidents involving learners, teachers, or both together are the life-blood of educational research which aims to inform teaching and improve learning. Incidents which are recognised not as isolated but as having something in common with other incidents from your own past are particularly pertinent, because they indicate a potentially fruitful domain for analysis. As a variety of incidents coagulate around a common label, they form a rich bed of potential for recognising and noticing similar incidents in the future, even to the extent of alerting you before or just as the incident itself develops. This makes it possible to choose whether to follow the habitual route, or to respond freshly to the trigger.

In considering incidents of your own or from others, it is helpful to cast about for descriptions of other incidents which seem to be similar. The act of ‘recognition’ which makes an incident vibrant is an essential part of the process of moving from incident to phenomenon. Actively seeking incidents with a similar flavour, and negotiating the similarity with others is part of the refinement of the phenomenon and of the analysis which can lead to informing teaching. Different theoretical approaches or perspectives can then be used to try to make sense of and to account for the phenomenon. Sometimes the incidents themselves will suggest useful pedagogic strategies which could be used in other contexts when something similar seems to be happening. Although it is usual to treat the initial incidents as ‘data’, the methods employed here treat these as triggers; the real data are the incidents which come to mind in the reader. The Discipline of Noticing (Mason 2002) elaborates and develops this approach to researching and informing teaching.

Seven Incidents

Three incidents connected with algebra classrooms are described not as singular and peculiar, but because they are likely to resonate with most teachers of algebra. Three further incidents are then included because although they concern much younger children, they came to mind when considering the first three.

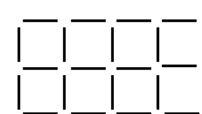
Incident 1

A class at an early stage with algebra are undertaking a task in which they repeat the same operations on different data. In response to the teacher’s question: “What are you doing each time?”, a pupil says "Every time, I am, well, for example with 5 you multiply it by 2 and ...". [Rob Shadbolt, 2004 private communication]

Incident 2

Practicing teachers doing a distance learning course (Floyd *et al* 1981) faced the following question in the examination.

The picture shows a rectangle made up of two rows of four columns and of squares outlined by sticks. How many sticks would be needed to make a rectangle with R rows and C columns?



Teachers on the course had been exposed to the notion of expressing generality in simple stick and other patterns, and to various techniques, such as specialising, specialising systematically, and attending to how you draw chosen examples, but only as a small part of the whole course of 400 hours study.

Many of the mathematically less experienced candidates found great difficulty with this question. They all specialised in one direction (either rows *or* columns but not both), and many made progress in specialising in the other direction. But only a few reached the double generality, finding a formulae involving both R and C. Many made comments along the lines of “I have a sense of what is wanted, but I can’t quite see it” [Mason 1996 p84].

Incident 3

Binns (1994) studied the awarenesses of parameters and variables displayed in contexts such as $y = mx + c$ as the equation of a straight line with slope m and y-intercept c, by 17-18 year old students doing university entrance mathematics examinations and expecting an A grade. She found a wide variety of appreciation of the roles of x, y, m, and c. For example,

> Asked for the equation of a straight line with slope *m* and passing through the points *p* and *q*, Frank identified the form $y = mx + c$ but did not get any further until asked what information would be needed to draw the line, which led him into needing particular values for *M*, *p* and *q*. [Bills 1994 p304-305]

Incident 4

A young child said 'goed' instead of 'gone' on several occasions, despite adults offering the correct word. Some time later, the past tense was used correctly and fluently.

Incident 5

A child of 3 pointed to a cow and said "horse"; pointing to a goat she said "animal". A child of 4 pointed to a crow and said "crow", and to a sparrow and said "bird". When asked if the crow is a bird or if a horse is an animal, both denied the relationship. Later crows suddenly became birds, and horses and cows were animals.

Incident 6

> Some 6-7 year old children were playing with a green plastic 'field' and some toy farm animals. One child had 7 animals, the other 4. Each could describe the relation between their farmyards in terms of "7 is 3 more", "4 is 3 less", "7 is more than 4", "4 is less than 7". Neither reflected back to the teacher the full sentence "7 is 3 more than 4", even when the teacher rehearsed it with them. A few weeks later, both children spontaneously included all three numbers in their statements. (Floyd *et al* 1981, Mason 1996)

Notice that the description does not say "neither child *could* say …" because that would mean subtly including in the account *of* the incident, an explanation accounting *for* what was observed: attributing inability to non-performance is at best inappropriate and at worst dangerous. All we know is that neither child did something at that time in those conditions.

Generalised Incident 7

Learners who performed adequately if not well on tasks one day, perform much less well subsequently, and when asked in a science lesson if they have met the topic in mathematics, claim not to have seen it before.

Accounting For Phenomena

In educational research it is vital to distinguish data and analysis. With regard to incidents, this means giving an account of an incident which is as free from value judgement, explanation, and justification as possible (Mason 2002). That is why the incidents have been separated from some analysis. These incidents are typical of situations in which learners are invited to generalise: they resort to the particular rather than struggling to express the general. It may even be that the learner is looking at the particular, but not holding in mind the similarity across several examples. In incident one, the learner appears to be fully absorbed in the particular, and adopts a behaviour to which they have been enculturated by many adults and all teachers; giving of particular examples rather than stating a generality. The fourth and fifth incidents have been noted and studied in some detail as a general phenomenon. For example, the way in which young children deal with the past tenses of irregular verbs and with class inclusion is widely reported (Brown 1973). Incident 6 may be to do with attention fully absorbed by particular details, blocking out articulation, or even awareness, of the full relationship, and this may also account for incidents 1 to 6. The seventh speaks more to the effects of expecting that trained behaviour is sufficient to produce robust and stable learning, which is taken up in later sections.

Each of these incidents suggests a particular phenomenon to the extent that people who have taught algebra or who have been with young children find that they recognise them, or find themselves noticing similar incidents in the future. For example, in relation to incident 2 especially, but also incident 1, a teacher reported recently observing that 'learners who can make generalisations to one variable, but then find extreme difficulty in seeing generalities across their various 1-variable generalisations'. [Rob Shadbrook 2004 private communication] which by the inherent generality in the language suggests recognition of an underlying phenomenon. Regarding incident 3, Bills found it useful to distinguish between using a form such as $y = 3x + 1$ to name a particular straight line, and summarising or denoting a whole family of straight lines by the form $y = mx + c$ [Bills 1996 p206]. This dual use of forms requires a shift in how you attend, It becomes second nature for those who succeed in mathematics.

From Phenomena to Phenomenon

The fact that these seven incidents have been juxtaposed suggests that I detect something similar about them. A short contemplation brings to the surface the observation that people sometimes show that they have learned something only later, sometimes even much later, despite attempts to 'teach them now'. Each of the incidents can be seen as an example of what the teacher thinks they are teaching not having an immediate effect. In other words, learning does not always happen in discrete observable steps.

Recognition

Do other people recognise this phenomenon? One example comes from Denvir and Brown (1986, 1986a) who found that having organised mathematical topics according to logical relationships of the underlying ideas, learners showed that on tests their performance improved on topics which were sometimes completely unrelated (in the sense of logic relationships) to what was being taught. Sometimes these were previously encountered ideas which were supposed to have been mastered but had not; sometimes these were topics not even recognised by the teacher as relevant or as part of the teaching.

An important component of Guy Brousseau's comprehensive analysis of the *situation didactique* (Brousseau 1997) is the implicit *contrat didactique*: if learners accomplish the tasks they are set by teachers, then they will necessarily learn what is required. In trying to meet their side of this implicit contract, teachers encounter an endemic tension: the more precisely the teacher specifies the behaviour sought from learners, the easier and more likely it is that learners will display that behaviour without actually generating it or internalising it for themselves. The incidents described suggest that accomplishing tasks may take time, but that not yet accomplishing a task does not mean that no learning is taking place. In other words, if you tell learners exactly what they are to do, they can and may do it without learning much of anything from doing it. Put another way, *doing ≠ construing:* learners busily engaged in doing things does not necessarily imply that they are learning anything.

Cause & Effect as an Explanation

If a cause and effect mechanism underlies the relation between teaching and learning, then teaching causes learning, and good teaching will be followed by evident learning. Lesson objectives stated at the beginning will be met by learners (all of them?) displaying the intended behaviour by the end of the lesson.

This is not an unfairly extreme description of a position: in the U.K. inspectors are sent into schools to, among other things, visit (parts of) lessons during a single week. They then write reports which can have a significant impact on the functioning and the finances of the school. They are charged with looking for signs of learning in the lessons they visit. Consequently teachers experience a strong force to teach in a manner which displays learners apparently actually learning, with lesson objectives at the beginning stated in behavioural terms, and a closing plenary intended to 'ensure' that the stated objectives have been met.

A cause and effect explanation of the phenomenon has to be that in each reported incident the teaching is either badly conceived and or badly conducted, because learners are expected to display learning in each lesson according to and corresponding to the stated objectives. Clearly this did not happen.

Maturation and Change of State as an Explanation

When energy is applied continuously to a substance such as water, the temperature rises, for a time, and then stops rising, until suddenly it starts rising again. This is the point at which water turns into steam. The same applies to ice melting, and in reverse, when energy is taken out of the substance rather than being put in. The term *latent heat* is used to refer to the energy which is required in order to effect a change of state.

There is a possible analogy with teaching and learning. Sometimes a teacher's explanations appear to fall on deaf ears. There is no visible effect of explaining or correcting. Sometime later the learners display apparently spontaneous evidence of having sorted things out, of having now learned. Unlike the physical situation in which the experiments can be repeated over and over, in the teaching situation the amount of explanation given and the time delay until new behaviour is observed are highly dependent on the people, the topic, and the situation.

Whereas in physical state change it is possible to account for the disappearance of energy in terms of reorganisation of molecular structure, for human beings it makes more sense to think in terms of maturation, such as the baking of bread, making of cheese, or the fermenting of beer and wine. There are complex actions going on, which are highly sensitive to changing conditions. Furthermore the result (bread, cheese, beer, wine) looks and tastes nothing like the ingredients that went in. So too the sense that people make of their encounters with the material, mental and symbolic worlds (cf. Bruner's enactive, iconic and symbolic forms of representation: Bruner 1966 p44-45) can involve significant alterations to what they are subsequently sensitised to notice, and how they respond and react to certain situations.

This perspective suggests that what we call learning can usefully be seen as a change of state, a change in what learners attend to and how they use that attention. The incidents described capture moments when learners are in transition.

Once sensitised to the fact of irregular verbs, the language producing parts of our brains can be left to get on with doing it correctly; once there is sufficient attention to encompass three numbers in relationship, children will articulate relationships using all three numbers; once sufficient attention is freed from keeping control of symbol manipulation, multiple meanings and interpretations are easier to countenance.

Implications for teaching

Having attention drawn to commonalities amongst various incidents suggesting a phenomenon, and having encountered different metaphors for interpreting the phenomenon, the next step is to consider implications and ramifications of the two explanations.

Cause and Effect Approach

If learners are not changing their behaviour according to what the teacher is teaching, then a cause and effect mechanism for teaching and learning points to a strong need to alter the teaching, since it is proving to be ineffective. Lessons therefore need to be constructed so that learners are attending to what they are supposed to be attending to, and immersed in relevant practices (mathematical techniques, ways of seeing and speaking, …). Where learners do not pick up practices immediately, those practices need to be broken down into simpler steps and components, which can then be trained sequentially. This principle is readily seen to be at work in materials designed for the 'slower' learner, the 'low attainers'. Typically their textbooks spread topics out over more pages, breaking them down into smaller steps, with more practice exercises(textbook references).

I was once visiting a class described as ‘low attainers’ and found some children working rather slowly and laboriously through pages of exercises, so I asked them what they were doing. They took me through the question they were currently working on. They seemed to have little difficulty or challenge. I then asked them a sequence of questions which they answered perfectly competently, verbally. When I pointed out to them that they had now covered all the exercises on the next two pages, they told me that they were expected to work slowly, so they did. In working slowly they were unchallenged and consequently often did not integrate what they were doing through subordinating the need for attending to details (Hewitt 1994, 1996). On the contrary, they were using minimal attention to do the work, attending to maintaining a just-fast-enough pace so as not to attract attention from the teacher, and otherwise proceeding unreflectively.

A similar account comes from the work of Jenny Houssart (2001, 2004) who over a number of years sat in lessons as a teaching assistant and listened to what the 6 and 7 year old ‘children at the back’ were saying and doing. She came to call them the whisperers, because they would whisper to her and to each other comments about what the teacher was doing, answering his questions and even correcting his mistakes. But when it came to tests, they never performed very well. Explanations might be that they were bored by the lack of challenge on the tests; that they had short attention spans for such activity; that they found writing difficult to the point of pain; that they were slow learners. By contrast, various projects (LAMP: Ahmed 1987; IAMP: Watson *et al* 2004) have shown over the years that ‘low attainers’ are just as capable of making use of their natural powers of sense making in order to think mathematically about mathematical phenomena as anyone else, if given the opportunity and encouragement.

Maturation and Change of State Approach

Seeing learners as engaged in a change of state, a process of maturation, releases teachers from having to produce observable effects in over short time intervals. Teaching can be seen as taking place *in* time, while learning takes place *over* time (Griffin 1989).

One immediate implication is not to be dissatisfied if learners appear not to be taking in what they are being taught. Of course this is too bald and general a statement. It is not license to go on expounding despite evidence that learners are lost or mystified. It does however suggest that if learners can be attracted into thinking about what they are doing, into reflecting overtly as well as implicitly on the structure that underpins the tasks they are being set, then their state transition may be eased and may be more robust over time.

For example, where learners appear to be concentrating on particulars rather than experiencing a generality, it may be that their attention is fully absorbed by discerning details and recognising relationships within that particular. There are specific teaching actions that can be taken, not so as to produce an immediate change, but to provide more specific experience of a different way of looking at mathematical objects, of seeing the general through the particular, rather than simply attending to the particular. Getting learners to ‘chant’ several particular cases in sequence with the invitation to listen for what is the same in what they are saying, done in short bursts over time rather than in one long heavy interaction can assist learners in making the ‘phase transition’. Getting learners to construct a similar example for themselves, then another, then another, often releases awareness of a general class of such examples (Watson & Mason 2002, in press).

A current educational fashion is to extol the use of peer discussion as well as learner-teacher discussion, since trying to articulate thinking to others is a well known way of deepening learning (the best way to learn is to teach). But discussion alone is no panacea, since discussion can serve to reinforce inappropriate ideas just as well as it can serve to support appropriate reconstruction. Any strategy will have to engage learners in active mentation, in using their natural powers of sense making if it is to contribute to learning. But in the end, learning is not something which people do, but rather something which happens to them over time as a maturation process.

A maturation perspective puts a premium on trust: teachers trust learners to become active, to construct and reconstruct, to be assertive about rather than merely assenting to what happens in lessons (Mason & Johnston-Wilder 2004). They construct tasks which encourage choice making and initiative, rather than trying to control everything that learners do.

Discussions between learners can serve a useful purpose if learners are put in situations which call upon them to attend in certain ways and to use certain language patterns until they become fluent (Floyd *et al* 1981). It is the fact of an audience which impels someone to try to express themself, that contributes so powerfully to learning. Getting thoughts outside of you so they can be more carefully tested and modified supports transformation of thinking and attention It is not discussion *per se* that aids learning (else there would be convincing research results to this effect), but rather the inner transformatory actions in the learner which discussion encourages.

Summary And Comparison

Learning seen as the effect caused by a teacher teaching suggests that well designed teaching leads to observable learning. Certainly that is the underlying perspective of school inspections in the UK and perhaps elsewhere. The analysis of the phenomenon at the core of this paper suggests that such a mechanism does not fit well with observations of how learning actually takes place. Cause and effect as a mechanism makes sense with machines which continue in one state until altered by fatigue or adjustment; it does not make sense when applied uncritically to organisms, and especially to human beings. Even medical doctors are beginning to realise that drugs do not have the same effect on all patients. Furthermore, the ramifications of a cause and effect mechanism for teaching and learning have been in place for many generations, and have proved not to have succeeded.

An alternative ‘mechanism’ of maturation and change of state has significant implications for the conduct of teaching and learning, and for the education system as a whole. By engaging the individual in a process of personal construction and social adjustment through enculturation, by encouraging and trusting learners to make sense using their own natural powers and to become aware of their use of those powers, significant learning can be fostered and sustained.

Further analysis could account for the related phenomenon that learners who appear to have learned, who have picked up and reproduced desired behaviour patterns (performing techniques, articulating concepts) often appear some weeks or months later to have no memory at all of the topic or technique, and appear to have lost all previous fluency.

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TEACHING AN APPLICATION-ORIENTED HIGH SCHOOL MATHEMATICS CURRICULUM: THE TEACHER'S PERSPECTIVE

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Applications are central to mathematics education as a basis of demonstrating and validating the utility aspect of mathematics. An application-oriented, project-based curriculum has the potential of enhancing school mathematics. However, the curriculum by itself is unlikely to make a significant difference in the classroom if teachers deliver it in a traditional way. This paper reports on a study that investigated mathematics teachers' perspective of what is necessary to teach this curriculum in a non-traditional way.

The study is framed in the theoretical perspective of teacher thinking in which teachers are viewed as creating their own meaning to make sense of their teaching. Studies have focused on teachers' content knowledge, beliefs/conceptions, classroom practices, learning, and change (e.g., Chapman, 1997; Leder et al., 2002). These studies have provided us with insights on, e.g., the relationship between beliefs and teaching, deficiencies in teachers' content knowledge, and the challenges of teacher education and change. They provide justification for ongoing research to understand the high school mathematics classroom from the teacher's perspective.

The application-oriented curriculum is the new Alberta [Canada] Applied Mathematics Program for students of grades 10 to 12 who are unlikely to pursue an academic area at university that requires mathematics. The curriculum was implemented in 2000 to 2002, one grade per year. The curriculum is officially described in its introduction by the designers as follows:

> [It] focuses on the application of mathematics in problem solving. Through challenging and interesting activities and projects, students further develop their skills in mathematical operations and in understanding concepts. …The curriculum, in general, emphasizes the application and relevance of math in daily life.

The textbooks for this program consist predominantly of projects. An example of a grade 10 project topic is: "In this project, you will work in a group of three to design your own line of jewelry. The members of your group will include two jewelry designers and one marketing manager. You will work with area, volume, scale factors, and metric units of measurement." An example of a grade 11 project topic is: "In this project you will explore some of the mathematics of population growth. You will study different mathematical models that describe the growth of the world's population, write a report on the subject, and create a poster display."

A case study was conducted with two experienced high school math teachers. They were selected because they had been teaching the new curriculum from its initiation [about 3 years], they seemed to use a non-traditional teaching approach based on why they were recommended as participants by the school system and they were willing to participate. Data collection consisted of semi-structured interviews of mainly open-ended questions, and written accounts of project-based lessons. The interviews focused on the teachers' thinking and teaching experiences with the new curriculum from initial implementation to the date of interview. The interviews were audio taped and transcribed. The teachers wrote detailed descriptions of six lessons they taught, documenting the projects involved, their behavior, the students' behavior, the treatment of the mathematics content and the classroom environment. Analysis involved scrutinizing the interview transcripts for significant statements that conveyed the participants' perspective of what was necessary to teach the applied curriculum and organizing them into themes.

The findings focus on what was common to the two teachers in terms of their perspective of teaching the new curriculum. In implementing the curriculum, the teachers found the "stand and deliver" approach to be counter-productive. For example, explaining the concepts and using the ideas in the project to show how they relate to the real world made the projects seem unnecessary and the curriculum no different from a traditional one. After a few semesters of trying alternative ways of working with the curriculum, their thinking and behavior started to articulate a shift in perspective that conveyed what they came to understand as being necessary to teach the curriculum in a non-traditional way. The three categories of the dominant, common factors of their perspective are summarized here. Understanding of Mathematics: Mathematics should be viewed and treated as follows: *(i) A problem solving activity:* This involves "a social activity shared between students and the teacher". *(ii) Integrated and connected:* As one teacher explained, "I came to realize that mathematics should not be seen as independent units taught independently of each other." *(iii) Relevant and useful:* For example, "It is important to present applied math so that it is relevant to the experiences of students…. It is important for students to see the connection between the mathematics they were learning in the classroom and its use in the real world." Understanding of the Teacher's Role: The teacher should: *(i) Establish relevance:* That is, "attempt to make applied math relevant to the real-world and particularly to the students in the class." One teacher invited guest speakers for several topics, e.g., a car sales person, a financial planner, someone from the logistics industry. *(ii) Know students:* In particular, anticipate areas where students may encounter diffficulties either interpreting or completing the task presented. *(iii) Support students:* For example, provide scaffolding to support students. Provide students with enough and appropriate information to begin working on the problems. Encourage students to persevere and enjoy the challenge of doing mathematics. *(iv) Encourage perturbations:* For example, allow students to "bump up" against things that do not seem quite right. One teacher explained, "I believe in letting students experience the success and the failures and the frustrations and then the exhilaration of getting beyond it … I really try to get them to think through possibilities." *(v) Use questioning:* This involves "ask questions that move students along in the process of solving problems", and "ask questions to understand the students' thinking". *(vi) Use technology:* This involves "understanding how to use technology in a responsible way with the goal of enriching students learning of mathematics". Understanding of Students' Role: The students' role should be to: *(i) Participate actively:* This involves "work hard during class time", "raise questions", "solve problems", "apply reason to a problem or task", and "analyze, synthesize or evaluate what they or the teacher is doing". *(ii) Interact with peers:* This involves working collaboratively and to "listen and follow each other reasoning and problem solving activities". *(iii) Be problem solvers:* This involves developing a sense of ownership for the problem/tasks, and pursuing multiple ways of solving problems.

The curriculum allowed the participants to construct a different instructional approach. The findings can provide a basis for teachers to reflect on what it means to teach such a curriculum.

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SECONDARY TEACHERS' UNDERSTANDING OF RATE OF CHANGE

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This paper reports an analysis of 13 CPMP teachers' understanding of rate of change. Interview tasks probed different aspects of rate of change knowledge. Teachers demonstrated a similar understanding on tasks dealing with constant rate of change and varied understanding on tasks involving changing rates of change. Teachers who had more experience teaching the CPMP materials showed a strength in connecting average rate of change and instantaneous rate of change using various representations, in particular, graphical representations. Some misconceptions were held by less experienced teachers as well as more experienced teachers. The findings of this study suggest that curriculum materials may support teachers as they learn ideas involving rate of change, but that a more thorough understanding of the concept may require additional resources.

Background

Change/rate of change is an important concept (Nemirovsky, 1993; Noble et al, 2001). It has its basis in everyday experiences like motion and growth. Change/rate of change is a fundamental organizing idea for relationships between variables. The rate of change concept is fundamental to the study of much of theoretical and applied mathematics. In the Core-Plus Mathematics Project (CPMP) curriculum[1], ideas of change/rate of change ideas are addressed in each grade level course while continually growing sophistication in ways of "making sense" and articulating mathematical ideas more than a set of symbolic rules. Also, the CPMP materials engage learners in meaningful and challenging discussions. They launch into discussions about why, for example, the graphs are changing, how they are changing, and how their actions and inputs affect the resulting graphs. These dialogues are valuable, for they encourage learners to rethink their own understanding, to consider multiple representations, and to look at a problem from various points of view.

A Guiding Framework for Analyzing Rate of Change Knowledge

A framework was designed to provide a comprehensive guide for analyzing different aspects of rate of change knowledge, incorporating existing frameworks developed for the domains relative to this study (e.g., Stump, 1997; Wilson, 1994) and NCTM's recommendations (NCTM, 1989; 2000). The components of the framework include: a) concept image and concept definition, b) Multiple representations and connections, c) linear connection, and d) mathematical modeling.

Results and Discussion

All teachers included a description of "slope" of a straight line in their image of rate of change. Only slightly more than half of the teachers (7 of 13) used functional language to illustrate their image. More experienced teachers with the CPMP curriculum were more likely to mention functional representations of rate of change. It was a surprising finding that the functional representation of rate of change was missing from the descriptions of many teachers' concept images although a functional nature is inherent in the term rate of change. However, all but one teacher preferred a functional representation of rate of change as their formal definition of rate of change. All teachers demonstrated the well-developed understanding and flexible use of different representations in tasks dealing with linear functions that exhibit constant rate of change. Teachers who had more experience with the CPMP curriculum were generally more successful in dealing with changing rates of change and recognizing similar and contrasting characteristics of different types of representations across contexts. Teachers generally had more difficulty using graphical representations, compared to numerical representations. Teachers who had less experience with the CPMP curriculum were less flexible with graphical representations

than their peers. Context played an important role with regard to teachers' ability to explore rate of change. More teachers were able to interpret situations involving changing rates of change when they were embedded in a context-rich setting. Teachers who had taught all CPMP courses showed a strength in their thinking and ability to approach problems involving instantaneous rate of change and, in particular, the concept of derivative in a sense-making way using their understanding of rate of change in linear relationships, which reflected a strong conceptual understanding of rate of change ideas and the ability to distinguish between constant rate of change and non-constant rate of change. However, understanding the connections between finding the slope of a line and methods for estimating the rate of change for nonlinear functions proved to be challenging for most teachers. In particular, teachers' understanding of derivative was often held as separate bit of knowledge. Some teachers even after having a great deal of experience with the CPMP materials had difficulty interpreting a graph as a relationship between two variables. These same weakness have been noted in other research studies (Monk & Nemirovsky, 1992; Porzio, 1997), indicating that these ideas my require more time to develop, regardless of the particular curriculum used to develop the concepts.

The findings of this study suggest that curriculum materials may support teachers as they learn ideas involving rate of change, but that a more thorough understanding of the concept may require additional support.

Endnote

1. The CPMP curriculum is a Standards-based high school mathematics curriculum materials, funded by NSF. The term "Standards-based" is used to indicate the curriculum materials reflects the recommendations in the NCTM Standards documents.

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RECONCEPTUALIZING THE MATHEMATICAL KNOWLEDGE AND UNDERSTANDINGS OF ELEMENTARY-SCHOOL TEACHERS

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Our analysis on the knowledge of elementary-school teachers attempts to answer three questions:

- *What knowledge do elementary teachers need to effectively teach mathematics to children?*
- *How is this knowledge subtly different from previous descriptions of knowledge?*
- *How can we help teachers, more specifically preservice teachers, construct this knowledge?*

To answer these questions we had to rethink the types and the description of the knowledge we believed preservice and inservice teachers need to be more effective teachers of mathematics. This paper draws upon the context and the theoretical foundation of the Connecting Mathematics for Elementary Teachers Project, CMET. Our goal is to help preservice teachers connect the mathematics they are learning in content courses with how children learn mathematics--to tie research on children's learning of mathematics with practice.

The Problem

Often, the mathematics prospective elementary teachers learn in content courses is disconnected from what they will be teaching. Typically they have only their own, often negative experiences learning mathematics to relate to the mathematics they are learning. As a consequence, mathematics is learned as disassociated facts and procedures without meaning; mathematics is not learned as a sense-making activity. A final dilemma in our analysis was that the concept of pedagogical content knowledge (Shulman, 1986) encompasses such a wide range of knowledge and skills that it was not a viable tool for studying the mathematics education of teachers. Especially with preservice teachers, pedagogical content knowledge is often prescriptive without taking into account children's viewpoints. Rather than combining subject matter and pedagogical knowledge into pedagogical content knowledge, we are suggesting that they be considered complementary interpretative orientations. Each is a lens that can provide insights into the learning of teachers.

Reconceptualizing the Knowledge Necessary for Teaching Mathematics

To address the aforementioned problems, we had to rethink the types and the description of the knowledge that we believed that preservice teachers would need in order to be more effective teachers of mathematics. We felt it important to think about how children acquire knowledge, not only in a given context, but in their long term development. Consequently, we discerned more subtle differences and descriptions in the knowledge that teachers need to teach mathematics to children.

Basic Subject Matter Knowledge

Basic subject matter knowledge is the mathematical knowledge one would need to complete all the mathematics in the elementary-school textbook or other sources used in the teaching of mathematics to children.

Conceptual Subject Matter Knowledge

Conceptual subject matter knowledge is a conceptual or relational understanding of mathematics—understanding the how and why behind the basic subject matter knowledge. It is more than the understanding of mathematical procedures.

Knowledge of Connections to Higher Later Developments in Mathematics

This is knowledge of how the mathematics one is teaching is related and connected to higher level mathematics.

General Knowledge of How Children Learn and Understand Mathematics

We believe that if preservice teachers have appropriate knowledge of how children understand mathematics that this will help them, not only in their teaching, but in their own learning experiences (Bransford et al. 1999).

Knowledge of Individual Children's Mathematical Thinking

Children often have their own personal ways of making sense of mathematics. This is knowledge that is derived from teachers making sense of individual children's thinking.

Knowledge of the Fundamental Underlying Concepts of Mathematics

This is knowledge of what are the key fundamental concepts that underlie the mathematics that one is teaching. One can understand mathematics procedurally and conceptually, know how children think about mathematics generally and idiosyncratically, and know how the mathematics is connected to higher mathematics; but one also has to know what is at the heart of the mathematics—what concepts are important in understanding the mathematics. This knowledge includes an understanding of how these underlying concepts fit into the long term mathematical development of children.

Conclusion

Our brief mention of problems with the mathematics education of pre-service elementary teachers certainly suggests these teachers need more and richer subject matter mathematical knowledge. We contend there are subtle and salient differences between prior descriptions of content knowledge and our reconceptualization of this knowledge. Likewise, the knowledge we are suggesting is not the same as pedagogical content knowledge – knowledge of best methods for content instruction (Shulman, 1986). We are suggesting a more detailed description and categorization of the knowledge teachers need in order to more effectively teach children mathematics. We parallel Ball and Bass's (2000) integration of content knowledge and pedagogy. From this perspective we agree that promoting subject matter knowledge in isolation is not the most productive means of helping teachers develop the mathematical knowledge they need to teach children mathematics. However, the content knowledge we are attempting to help pre-service teachers develop is not based on how to teach this knowledge to children directly, but on how children learn. In turn, understanding how children learn and think mathematically is an essential component of effective teaching.

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THE DEVELOPMENT OF A KNOWLEDGE BASE FOR TEACHING AMONG UNDERGRADUATE MATHEMATICS FACULTY TEACHING CALCULUS

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Changing undergraduate populations and low student achievement / retention in mathematics have resulted in mathematics education reform at the university level (Robert & Speer, 2001). University-level teaching is in critical need of systematic research and little is known about the knowledge held by undergraduate mathematics faculty [UMF] related to mathematics pedagogy (Bass, 1998; Selden & Selden, 2001). K-12 research (Fennema & Franke, 1992; Grossman, 1990; Kinach, 2002; Shulman, 1987) on the knowledge base for teaching [KBT] has provided a solid foundation for exploring related issues in post-secondary education. Since Shulman (1987) first articulated a framework of the KBT, considerable research has been conducted in this area; yet, existing models still need to be refined and expanded (Rahilly & Saroyan, 1997).

This study was designed to examine the knowledge base of UMF in order to uncover the components of knowledge that constituted a framework of their KBT and to identify themes that existed in their KBT calculus. Based on earlier research, a framework of the KBT mathematics was developed (six components) prior to data collection. Briefly, *context specific knowledge* includes knowledge of the institutional culture/norms, the mathematics department, the classroom culture, and undergraduate students. *The nature of disciplinary mathematical knowledge* is composed of subject matter knowledge and pedagogical content knowledge [PCK]. *Personal practical knowledge* is the experiential knowledge of teachers. *General pedagogical knowledg*e includes knowledge of learners/learning, general principles of instruction, classroom management, and the aims and purposes of education. *Knowledge of research*, added to the framework following pilot work, reflects the direct/indirect relationship between mathematical research activities and teaching practices. *Beliefs* are a subset of constructs that name, define, and describe the mental states that are thought to drive a person's actions and can influence teaching practices. The subjects were 7 Ph.D. mathematicians teaching calculus at colleges/universities in the Northeast. Data were collected using audio-recorded interviews, with questions developed around the pre-existing categories in the KBT framework. Interview data were coded line-by-line by the categories in the KBT framework and sorted to extract themes within and across cases. Data unable to be coded by these categories were reexamined to revise the framework.

Results of this study supported the inclusion of the existing categories in the framework of the KBT, though several subcategories were refined. For example, *classroom management* was removed as a subcategory of general pedagogical knowledge since participants did not identify with the image of being a manager of a classroom. Most equated classroom management with behavioral issues, which were of minimal concern. Of interest was the addition of two subcategories of the nature of disciplinary mathematical knowledge - knowledge of the *history of mathematics* and the *aesthetics of mathematics*. Participants' knowledge of the contributions of various mathematicians and the timeline of events/discoveries in the history of mathematics is a rich component of knowledge that has the potential to enhance students' understanding of mathematical concepts and interest in the discipline and should be emphasized in the pedagogical development training of UMF. Participants' knowledge of the aesthetics, or beauty, of mathematics emerged as an important component of the framework. By engaging in research activities and other experiences with mathematics participants had developed a deep appreciation for the aesthetics of mathematics and were continuously able to connect with the discipline on a level not fully understood by education researchers. More research needs to be conducted to examine how this powerful component of knowledge can be drawn upon in instructional practice.

Themes and patterns emerged across cases within each category of the framework for teacher knowledge base. For example, with respect to knowledge of *learners and learning* participants were unfamiliar with literature on learning theory. Although content experts, participants had essentially no formal training in general pedagogy and thus, several participants reported drawing upon their own methods of learning mathematics or inventing their own theories about how novices learn mathematics. This result has important implications for the development of PCK, which blends one's knowledge of content and pedagogy (Shulman, 1987). For the participants in this study, their PCK was derived from their knowledge of content and their self-created understanding of "learning theory", and may or may not be aligned with the learning needs of undergraduate students enrolled in their calculus courses.

An examination of the KBT among UMF is critical to reform initiatives in the teaching and learning of undergraduate mathematics. Results of this study, which will be expanded upon in the short oral discussion, will impact the ways in which mathematicians/mathematics educators begin to reflect upon their own understanding of the components of knowledge required for teaching mathematics at the undergraduate level.

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LYN'S REPRESENTATION OF SLOPE AS AN ITERATIVE PROCESS

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Introduction

We explore an invented slope representation developed by a practicing elementary teacher participating in the first of five mathematics courses in a three-year professional development program. The program was designed to prepare twenty-five teachers to work as mathematics mentors in their schools. Teachers worked on rich, open-ended tasks in collaborative groups. They frequently presented work in progress to the entire class, and engaged in investigative ways of learning mathematics to impact not only their perspective of what it means to do mathematics, but to envision and implement improved classroom practices and conditions for their students.

Theoretical Perspective

Our lens on the social collective as a learning group, positions the teacher within the community of learners where mathematical thinking and justification actively build deep understanding through evolutionary adaptation of ideas (Simmt, Davis, Gordon, & Towers, 2003). Responding to dynamical activities of the group, we carefully designed rich tasks to provide beneficial focusing phenomena (Lobato, Ellis, Burns, & Munoz, 2003) to assist teachers in confronting misconceptions and building understanding through intuitive explorations.

Method

Grounded theory informed our method of data collection and analysis. Data were collected from videotapes of two three-hour class sessions and field notes recorded by members of the research team. Specific videotaped episodes, in which teachers articulated, inscribed, or kinesthetically presented slope, were identified and analyzed. Story lines were constructed and narratives composed using methodology described in previous studies with video data (Speiser, Walter, & Maher, 2003).

Data and Analysis

Prior to comparing rational-number slopes, teachers collaboratively investigated numerical, graphical, and algebraic representations of linear functions, with positive integer rates of change and used Cuisenaire Rods to explore fractional relationships.

Teachers were asked to determine which is steeper, a slope of 1/2 or a slope of 2/3, and to provide two convincing representations to support their conclusions. Cuisenaire rods remained available on the tables where teachers were working. Lyn, a teacher without prior knowledge of slope, began to use the Cuisenaire rods to create representations. Lyn represented 2/3 slope by stacking pairs of light green Cuisenaire Rods which she called "threes". In succession, she stacked four light green rods and six light green rods on the right. Similarly, Lyn built a representation for 1/2 slope using the red rods, which she called "twos"

Other teachers questioned Lyn's choice to represent 1/3 with the light green rods and 1/2 with the red rods. Earlier, teachers focused on light green as 1/3. However, Lyn also saw the light green rod as representing 3. While presenting to the class, Lyn transitioned to "thirds" and "halves" instead of her earlier use of "threes" and "twos" but did not change her representation. She chose a 2-unit rod to represent 1/2. We believe Lyn generalized that the 1/3 rod was 3 units, so the 1/2 rod must be 2 units. Lyn demonstrated the relative steepness of the slopes by placing a ruler against the left corners of each collection of rods. The demonstration showed that her 2/3 slope was steeper.

Lyn spoke in terms of a potential graphical model and then drew on the white board her second representation, a table of recursively defined numerical data (Figure 1).

x	y
1	2/3
2	4/3
3	6/3

Figure 1. Lyn's table of data.

Lyn: I guess if you were on a graph you went over one and up two-thirds. And then I'm over two and then I'm up two-thirds, another two thirds, does that? I don't know?

Linda R: How come you only went over 1?

Brenda: That's two-thirds one time and two-thirds two times so you're adding two-thirds to it each time.

Lyn: I've never done slope in my life. Well would it be that, would you be saying then that if you were graphing it the x is one? And the y is two-thirds? Would it be doing that? I'm not sure. And if x was two, y would be four-thirds. I don't know would that work? And if x was three, y would be six-thirds? I don't know, I'm just trying to make it relate to that. All I know is I would rather climb a mountain that was one-half than one that was 2/3.

Discussion

Lyn built her Cuisenaire rod representations through iterative processes of repeated addition. We argue that Cuisenaire rods, from prior investigations of fractions, became focusing phenomena for Lyn's representations of slope, prior to experience with slope as rise-over-run. Although Lyn's 2/3 and 1/2 slopes actually represented slopes of 6 and 2, respectively, her collaborations with the other elementary teachers revealed an evolution of interrelated ideas, consistently grounded in repeated addition, as they worked to resolve misconceptions and inconsistencies between limited understandings of fractions, Cuisenaire rod models, recursively defined numerical data, and graphical intuitions for rational-valued rates of change. We suggest that evolutions of mathematical ideas toward deeper understandings may be supported by the persistence of focusing phenomena across multiple representations when learners are engaged over time in carefully designed, open-ended tasks.

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WAYS OF SEEING: UNEXPECTED STUDENT SOLUTIONS AND SUBSEQUENT TEACHER RESPONSES

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Preparing teachers for the often-unpredictable classroom questions related to reform is a challenge. Yet the reactions of teachers to students' ideas is crucially important. This discussion examines teacher's responses to the unexpected, and makes suggestions for relevant teacher education experiences.

Framework

Recent research been done on the types of mathematical understandings needed by teachers (Ball, 2002; Ma, 1999). Ma cites a need for deep conceptual understanding of the subject matter, while Ball defines teacher mathematics knowledge as specialized knowledge about mathematics, needed for teaching. Clearly such knowledge helps teachers to teach more effectively. But the reform agenda also seeks to promote problem solving and questioning, and supports student generated solutions. What preparation can we give teachers to respond appropriately to these?

Classroom Examples

Only one of several classroom examples will be described here due to space limitations. Mrs. B. was a grade five teacher who was already teaching when the strand of patterning and algebra was introduced into the elementary curriculum in Ontario. She admitted she sometimes needed to resort to the answers in the back of the book to correct pattern rules. Daniel was a student working in her classroom. For example, he worked hard on the following pattern:

Input	Output
4	7
6	13
11	28
13	34

Daniel's thinking was not what the teacher expected. Daniel "saw" the output numbers as being one greater than multiples of three. But they were the 'wrong' multiples – they were the result of smaller input numbers. Daniel put this idea into words and wrote: *You take the input number and subtract 2. Then you multiply that number by 3. Then you add 1.*When Daniel received his worked back it was marked wrong with an "X", with no explanation. After the incident, the teacher explained: *It's wrong because it's not the simplest answer. The correct answer is 3 times the input number minus 5. Why make it more complicated?* In this example, the way that Daniel wrote the rule was how he thought about it; to him his rule wasn't 'more complicated', it was the way he understood it.

While arguably the teacher may have been lacking in algebraic understanding, something in the value system of the teacher may also be in question. Other classroom examples show that even teachers who encourage multiple solutions often unintentionally illustrate a bias towards the 'simplest' solution by emphasizing it.

Davis (1996) defines a good mathematical problem as variable entry, open-routed, and open-ended. To follow is an example of the type of problem which fits this criteria and might encourage pre-service (and in-service) teachers to re-examine their values about multiple solutions in their classrooms.

Vicky was enrolled in a mathematics course for elementary pre-service teachers as part of a four year education degree. The students were given tiles and asked to find a general rule for the number of tiles needed to build a staircase of n steps. Students were encouraged to move the tiles around to come up with arrangements whose areas are easier to calculate, with several

types of rectangles being popular choices. Vicky's solution was quite unusual; she started with a square, subtracted the diagonal, and divided the rest in two. This gave her a formula for the amount to *remove* from an n-square to leave the staircase. Although initially Vicky thought her solution must be wrong because it differed so much from the others, she was thrilled when she was able to show it was equivalent. This experience was a watershed for Vicky and changed her opinion of herself mathematically, as well as showing her the value of student-generated solutions. For further discussion about such problems and their potential in teacher education see Dione, J. and Kajander, A. (in progress).

Discussion

Starting with the algebraic "answer" from the back of the textbook rather than having experience investigating patterns in a concrete way herself, may have reinforced the perception of Daniels's teacher of "the" correct answer. She did not appear to have any deep appreciation of how a solution might alternatively have been constructed. It is likely that she would have benefited from more experiences herself in which she had an opportunity to construct alternative solutions to problems, and also see the benefit of algebra as a tool to move from one type of solution format to another.

An example of such an environment might be the experience of Vicky, who had surprised herself by constructing an elegant solution based on a deep understanding of the problem. Vicky then saw the need for algebra as a tool to reconcile her solution with those of her classmates. As a practicing teacher, it is hoped that she would be open to varying types of problem solutions from her students, as be willing to investigate them fully to determine their correctness.

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INVESTIGATING TEACHING AND LEARNING OF SUBTRACTION THAT INVOLVES RENAMING USING BASE COMPLEMENT ADDITIONS

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This article reports on how "base complement additions" was used to help research participants, in grades 4 and 5, do compound subtraction (subtraction that involves renaming). The participants' understanding the use of "numeration cards" helped them to do addition, subtraction and to convert compound subtraction to simple subtraction. Findings revealed that participants prefer using base complement additions to the decomposition (borrowing) strategies.

In the history of mathematics education, teaching children to subtract has long been considered a problem area. Winch (1920), noted that "no methods give more trouble and are less successful than those of teaching subtraction" (p. 20). Thorndike (1921) believed that the controversy regarding how children should be taught to subtract centered on the argument of whether to use the "subtractive" or "additive" method. Today, learning subtraction that involves renaming continues to be a source of difficulty for many young students and the question of how best to teach it to the youngsters is gaining renewed interest. Currently, both teachers and students in the United States, Canada, and other parts of the world predominantly use the decomposition (D) strategy, though there might be other student-developed strategies. There is one very important advantage for using D strategy: the ability to demonstrate the regrouping procedure using bundles of sticks, straws and other manipulatives. This has made the strategy very popular, especially following the "meaningful teaching" at the turn of the century (Brownell & Moser, 1949). However, the method appears to have many inherent drawbacks, which adversely affect pupils in their performance on arithmetical tasks involving compound subtraction. The most obvious drawback of the strategy is that it takes too much time for students to learn the prerequisite concepts like place value, expansion, and renaming. Many students end up using "buggy" strategies due to limited preparation in time (Gyening, 1993).

This paper reports on grades 4 and 5 students' learning of the base complement additions (BCA) strategy for solving compound subtraction of whole numbers. This study is an attempt to shed some light on the teaching and learning of compound subtraction by BCA; and to verify if it will promote meaningful understanding of compound subtraction using BCA.

The Base Complement Additions Strategy

The base complement additions (BCA) is a strategy for solving compound subtraction problems by changing both the minuend and the subtrahend. The strategy transforms compound subtraction to simple subtraction by way of base complement(s). By base complement of a whole number n relative to the base b, we mean the positive number that should be added to n to obtain b. In other words c is the base complement of n if $n + c = b$, where b is the base and b, c, and n are whole numbers. The rationale for BCA approach is based on compensation. That is, if the same number is added to both the minuend and the subtrahend the difference is not changed though there is change in arithmetic expression.

Methodology

The study started on January 2000 and ended on August 2001. To know the participants' entry behavior they were first given a set of test items on subtraction to do. This was then

followed by interviews. The researcher met with the participants for 8 contact periods during which they were given instructions on both D and BCA (not at the same time). The duration for the intervention was approximately 35 minutes for each contact period. Fourteen days after the eighth contact period post-intervention tests and interviews followed consecutively. All the tests, instructions and interviews were centered on subtraction, especially that which involves renaming. All the interviews were audiotaped; and the transcribed audiotapes, the test papers and the working papers formed the bases for the data required for the study.

The numeration cards were used for development of the instructional intervention. These were mainly used for developing meaningful understanding of place value, base complements, compound subtraction expressions, and transformation of compound subtraction to simple subtraction expressions. The numeration cards were displayed and named as small (ones), long (tens) and flats (fifty and/or hundred) to help participants understand BCA strategy.

The learning situation was first presented in free play before it became structured (Dienes, 1973). The representations covered concrete materials, followed by image formation of the first representations. Sketches, diagrams, and pictures were used for the second phase of the instructional intervention before the final phase, which was symbolic (Bruner, 1966; Dienes, 1973).

Findings and Discussion

The pre-intervention test and interviews indicated that participants used more than one strategy for simple subtraction but not for compound subtraction. The post-intervention interviews revealed additional strategy they used, for solving compound subtraction, in which they were more confident with. Participants made many errors when working with "borrowing" strategy. But with the introduction of BCA they were seen to make only few errors with respect to multi-digit subtraction computations. Before the intervention participants could not easily determine errors in compound subtraction computations. After learning BCA they could easily look at subtraction expressions and determine whether they are right or wrong and why. These findings give a clear indication that BCA is an excellent tool for computing subtraction that involves renaming; and it makes sense to the participants.

Compound subtraction that involves at least two renaming appears complex when borrowing strategy is applied. It involves a series of subtracting and adding before the actual subtraction is done. Thus, when the student forgets to keep track of the procedure, errors result. With BCA, however, one uses only simple addition and simple subtraction hence calculations become easier for the user.

From the findings of the study the learning of compound subtraction using BCA, in grades 4 and 5, could be conceptually based using manipulatives (numeration cards) for meaningful understanding of place value, subtraction concepts, and the idea of equivalence. From the findings of the study standard algorithms could be meaningfully taught at grades 4 and 5 so that students understand the concepts and reasoning associated with the procedures (Ashlock, 2002). This means paper-and-pencil procedures could involve both conceptual and procedural knowledge at grades 4 and 5.

Conclusion

The learning of BCA offered participants an additional tool for their mathematical toolkit. From this toolkit they chose the most appropriate strategy based on their comfort level. Thus, in grades 4 and 5, students' learning of BCA may expand their knowledge base and improve upon their subtraction skills.

With respect to problem solving and mental mathematics it is recommended that BCA be used as strategy for computing compound subtraction. This is because it helps students to appreciate the notion of balance (equivalence) at their early school ages. Also students get to know that complex mathematics could be transformed to simple forms and solved.

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A CONCEPTUAL ORIENTATION TO TEACHING LINEAR FUNCTIONS

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Mathematics reform efforts begun in the 1980s have made a concerted effort to support different ways of thinking and doing mathematics. Many curriculum developers and educators moved away from isolated facts and procedures, and conceptualized mathematics as a collection of big ideas. Yet, even with the many reform efforts in place for several years now, large numbers of students, particularly students of color and students attending urban high schools, are still failing math. Algebra in particular tends to be the point-in-time when students either fail or drop out of math, leaving students without access to higher-level math courses and the mathematical literacy required for higher education and well-paying jobs.

In a large-scale longitudinal study conducted over four years, Boaler and Staples (2003) discovered a unique group of mathematics teachers in an urban school involved in creating their own reform-based curriculum. Data collected from a series of interviews and tests demonstrates that students who attend this school are experiencing success in their math courses and feel positively about the math classes they take. Tests designed and used by Boaler and Staples across three different schools show that by the end of the second year of high school math, students from this particular urban high school outperform the other two groups of students. These students also demonstrated "significantly higher degrees of intrinsic interest in mathematics" (Boaler & Staples, p.20).

Not surprisingly, Boaler and Staples suggest that the mathematics teachers at this school played a significant role in students' success. In this paper, I describe the results of a series of interviews that were conducted with teachers at this school. I was particularly interested in how these teachers conceptualize the teaching and learning of mathematics, and whether features of teachers' conceptualizations may have contributed to urban students' success in algebra.

For this project, two Algebra I teachers were interviewed over the phone, with each interview lasting between two and three hours. Following the interviews, both teachers sent me specific curricular activities to exemplify something they had discussed over the phone. Teachers were asked questions about their methods for teaching linear functions, their learning objectives for students regarding linear functions and the pedagogical strategies they use to support students' to meet the objectives. To guide my analysis, I used Thompson and Thompson's (1994) distinction between conceptual and calculational teaching orientations. According to Thompson and Thompson, a conceptual orientation to teaching involves a "rich conception of situation, ideas and relationships among ideas"(p.86), while a calculational orientation guides students to focus on application and procedures.

My results suggest that the teachers at this urban school possess conceptual orientations to teaching linear functions and that these conceptions play a key role in students' future mathematical learning and success. Through numerous curricular examples and explanations, each teacher explicitly articulated how her instruction is guided by a system of ideas about linear functions, which is made up of multiple representations and connections between the representations. Students are also expected to develop an understanding of this system, and both teachers provided concrete ideas about how to support students to reach this goal.

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UNDERSTANDING DIVISION BY FRACTIONS: AN EXPLORATORY STUDY OF 7 MATHEMATICS EDUCATION GRADUATE STUDENTS

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This study explores the understanding of division by fractions of mathematics education graduate students. While the question of what students understand about division by fractions has been explored with school students and with intending and practicing teachers, it is not a question that has been explored with graduate students in mathematics education. Looking at such a population is an important issue: Graduate students are often the ones who teach prospective math teachers, so attempts to change pre-service teachers' views on division by fractions can be informed by learning more about how mathematics education graduate students think about this topic area. A brief synopsis of the current literature findings regarding prospective teachers' understanding of division by fractions will be provided. Participants in the present study were seven mathematics education graduate students at a large Midwestern university. To assess participants' understanding of division by fractions, each of the participants was asked to imagine that he/she had to respond to a sixth grader stating the following: "*I know that when I'm supposed to divide two fractions, I have to turn one of the numbers upside down and multiply, but I don't know why all of a sudden it gets changed to multiplication, so I forget which one to turn upside down and I get a bunch of the problems wrong* (Borko et al., 1992, p.202). *Could you help me?"*. Participants' responses were first transcribed, coded and, then, interpreted in the context of existing research. My analysis indicates that despite the participants' strong background in mathematics, only one of them exhibited conceptual understanding of the rationale underlying the "invert and multiply" rule; the conceptually oriented student flexibly manipulated an area model and then used inductive reasoning to illustrate the validity of her argument. Among the remaining six graduate students, five of them preferred to focus only on the formula and the rote application of the procedural steps. It is interesting to note that two of these five procedurally oriented students chose to propose an alternative formula as a means to demonstrate their own understanding; one went so far as to say that "*to remember the algorithm is more helpful than to understand it*". In addition to the five procedurally and the one conceptually oriented students, there was one graduate student who had both a lack of conceptual understanding and also the serious misconception that multiplication makes bigger and division makes smaller. Without underestimating the difficulty of the topic of division of fractions, the analysis of the present study's data has serious implications for graduate mathematics education programs, particularly in terms of addressing deficits in graduate students' subject matter knowledge for teaching.

Technology

A TAXONOMY OF GENERATIVE ACTIVITY DESIGN SUPPORTED BY NEXT-GENERATION CLASSROOM NETWORKS

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Never ask a question with only one right answer.
– Judah Schwartz

Previous work has examined how new theoretical, methodological, and design frameworks for engaging classroom learning are provoked and supported by the highly interactive and group-centered capabilities of a new generation of classroom–based networks. This network-supported interactivity, coupled with generative design, allows the environment of the classroom itself to be thought of as a group-oriented "manipulative" or mediating tool for teaching and learning. Given that this level of technologically-supported interactivity and group-oriented design is new to classrooms, new challenges for teachers, researchers, and curriculum developers relative to how to think about designing for, and working in, these types of environments need to be addressed. We present a taxonomy of generative activity design that has emerged from our work in developing and then working with next generation systems.

1.0 Introduction

Due to the group-focused interactivity and data collection capabilities of next-generation networking, we have a new tool to explore the dynamics of – and designs for – classroom learning. A number of projects supported by the National Science Foundation have responded to the challenge of advancing learning in these highly interactive networks by using mathematical/scientific ideas to organize and analyze classroom activity. This use of domain-related "big ideas" to study group learning has been discussed as "mathematics structuring the social sphere" [MS3] (Stroup, et. al. 2002; Stroup, Ares & Hurford, in press). Many of the network-based projects began by focusing on student learning (Kaput & Hegedus, 2002; Roschelle & Vahey, 2003; Wilensky & Stroup, 1999). Recently, however, significant efforts have emerged that focus more directly on issues related to supporting, and learning from, teachers' developing understandings of how best to design for network supported classroom activity. We suggest that there is a kind of "resonance" between technological affordance and generative forms of teaching and learning. Generative design, as we use the term, develops from and supports the core commitments of the reform movements in mathematics and science education and it is this resonance that we look to advance. Toward this end, we present a new taxonomy of kinds of generative activity useful both in clarifying the relations between kinds of generative activities and in clarifying internal aspects of generative design, especially as supported by next-generation classroom networks.

2.0 Technology and Moving to Group-Oriented Generative Design

There is a significant literature related to generative approaches to teaching and learning. Generative teaching, as discussed by Wittrock, is "a model of the teaching of comprehension and the learning of the types of relations that learners must construct between stored knowledge, memories of experience, and new information for comprehension to occur" (1991, p. 170). What

Wittrock means by the learners' active construction of new "relations" is close to what might be called constructivist teaching pedegogy. Consequently, generative learning in his framework involves students' ability to create artifacts that embody their constructed understandings. In a closely related way researchers from the Learning Technology Center at Vanderbilt emphasize aspects of creating "shared environments that permit sustained exploration by students and teachers" in a manner that mirrors the kinds of problems, opportunities, and tools engaged by experts (1992, p. 78). Teaching involves "anchoring or situating instruction in meaningful, problem-solving contexts that allow one to simulate in the classroom some of the advantages of apprenticeship learning" (1992, p. 78). Although many of these previous theories are generative at the level of the individual learner or even at the level of a small group (Lesh, et al., 2000) there is not enough of a picture of how to structure the cross-individual or cross sub-group learning. Our efforts are directed at extending and reconceptualizing these earlier analyses of generativity to engage the issue of how to design for the diversity and multiplicity of learners' ideas and insights supports the emergence and development of mathematical and scientific reasoning in group contexts.

Generativity as we discuss it below and elsewhere (Stroup, 1997; Stroup, et. al. 2002; Stroup, Ares & Hurford, in press) focuses on design for the group. The emphasis is on supporting the interactions of teacher and students together in a classroom and natural subsets of this classroom-based grouping. Additionally, we are interested in how mathematical and scientific content itself can frame the design of classroom activities and supportive technologies. Content in this sense is an organized body of knowledge developed over time, which while enacted through activity, still retains coherence and structures the group activity. The diversity of forms of student participation – including linguistic and socio-cultural diversity – is seen as the "engine" that drives the emergence and development of content in this group-oriented approach to generativity (e.g., see Ares & Stroup in this volume). Diversity creates the space of possibilities that students and teachers can use to advance teaching and learning.

The group focus of next-generation classroom networks develops from, and is consistent with, this expanded sense of generativity. These systems are typically designed "from the ground up" with the classroom in mind. Rather than constraining the learning experience to be somewhat narrowly individualistic, these technologies support socially situated interaction and investigation. Typically, each student has a device that allows him or her to participate either synchronously or asynchronously in group-oriented activities. Often the devices have local input, display, and analysis capabilities (e.g., those of a graphing calculator) and can interact in the network with each other and even with other kinds of devices (e.g., data-collection tools like Calculator-based Rangers™ motion detectors as used in the *People-Molecules* activity developed by Wilensky and Stroup [in review]). The interactions and emergent results can be projected on a public display space – often in real-time – using a computer or calculator projection system. Using this infrastructure and generative design, the learning trajectories and the processes of knowledge construction are owned by the group itself. Software and hardware work together in supporting this "group-oriented" design. The classroom or group, as supported by the network infrastructure, becomes a kind of participatory manipulative for teaching and learning challenging mathematical and/or scientific content.

A significant number of these networks are about to become widely available and are poised to become a major presence in classroom learning. It behooves us, then, to begin to support approaches to optimizing the generative potential of these networks. Although many of the forms and kinds of activities discussed using the pathways and endpoints discussion below can be done without next generation network technology and, indeed, have been used in pre-service teacher education (cf., Stroup, 1997), the sense is that the highly interactive, and group-focused capabilities of these next-generation systems can support, extend, and add to this kind of activity design for classrooms.

3.0 What are Generative Activities?

Generative activities in group contexts are seen as *space creating* activities that extend the ideas of co-operation and emergent structure to classroom based activity. Learners create a space – or coordinated collection – of expressive artifacts and actions in relation to some shared task or set of rules. The structures that are created or embodied are not determined in advance but are co-constructed by learners as their sense-making evolves and develops. Students can be asked to create objects or outcomes that are the same or alike in some mathematically or scientifically significant sense (see taxonomy below) and these responses can be arrived at or built from the underlying sameness in a wide range of ways. The sameness gives coherence to the task. Generativity requires that the activity produce or give origin to a diversity of responses. This diversity is then used by the group to explore patterns in the responses and in the structural ways in which the responses might be seen as relating to one another, or of-a-whole. Ideally the space of responses will be large enough so that the kinds of behaviors or expressive artifacts students create give the teacher significant insights into the ways the students are thinking about the task. The activity should be "thought revealing" in this sense (Lesh, et al., 2000), and it should also be capable of giving rise to additional rounds of generative exploration and/or detailed investigation.

4.0 Taxonomy of Generative Activities

The following taxonomy of generative activities is organized in terms of pathways and endpoints. Pathways are intellectual and/or behavioral routes for arriving at a given endpoint. Endpoints are outcomes or artifacts created by learners that represent some form of completion of the given generative task (See Figure 1.).

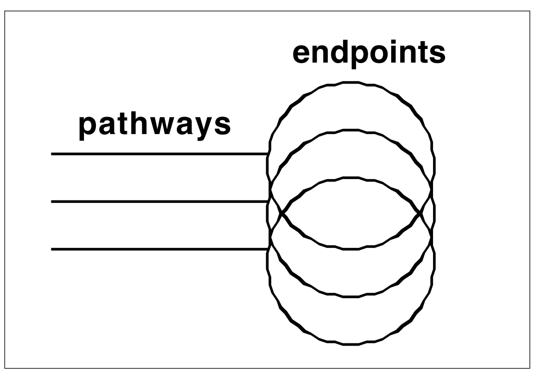

Figure 1. The taxonomy centers on a pathways and endpoints analysis of generativity.

4.1 Nominally Generative

Using a pathways-and-endpoints representation, the form of generative activity shown in Figure 2 is barely, or nominally, generative because the structure of the activity has an agreed upon endpoint and a single pathway to this endpoint. An example would be asking students to simplify the expression $2x + 3x$. The endpoint would be $5x$ and the path to getting to this endpoint would be the application of a specific rule for simplifying.

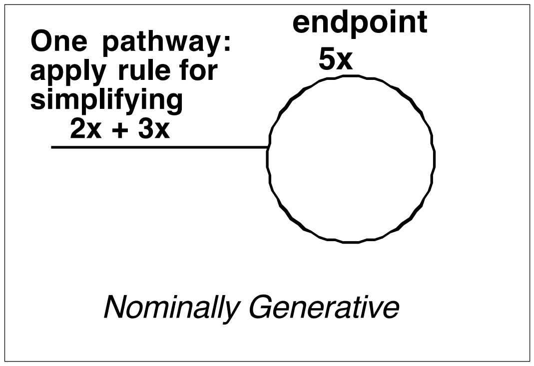

Figure 2. Nominally generative activities have one pathway to get to a single endpoint.

The “space” created by nominally generative activity in a classroom is a discussion of “right” and “wrong” answers and of how to correctly apply the given rule or procedure. Nominally generative activity accounts for much of traditional, or pre-reform-based, classroom practice. As a design approach it is relatively ineffective in putting students’ ideas at the center of classroom discourse and learning.

In our work with teachers we have sometimes talked about nominally generative activity as deriving from approaches associated with one-on-one tutoring. Whatever the efficacy of asking questions like “What is 2x + 3x?” is in the context of tutoring individuals, tutoring-type questions tend to “break” as they are pushed to scale beyond one-on-one situations. In a classroom if we are asking one student the 2x + 3x question, other students are left with little to do or to contribute and this kind of questioning makes nearly no use of the group itself. As the number students moves from a few to a whole class, the limitations of tutoring-type questions becomes more profound; at any given moment most of the students, of necessity, are not included in the core activity. Rather than critiquing the usefulness of nominally generative questions in group contexts, too often teachers simply use the technology of the copy machine to compensate for, and artificially scale to the group, “tutor type” of questions. Individual copies of nominally generative questions are handed to each student (or, similarly, written on the overhead projector or chalkboard), and each student can now “participate” individually in answering the questions, but the group itself has no constructive role. Moreover the teacher is still confronted with the task of figuring out what, if anything, to do with all these worksheet-based or individual responses.

Computer-assisted instruction (CAI) environments, supported by traditional forms of networking, have attempted to address the issues related to teacher load but they do so in ways that preserve the limited focus on right and wrong answers. In addition, network-supported CAI systems make no use of the group itself as a community of inquiry and insight. “Individualized instruction” is somehow understood to mean the learning must take place “alone” and the responses must be evaluated in terms of right and wrong. More recent “cognitive” tutoring environments, while representing significant improvements over traditional CAI, still retain this very limited sense of what individualized means, and what it might represent as a purported ideal of instruction (cf., Carnegie Learning, 2003). Generative design, as supported by group-supportive networking, moves in a very different direction. Individualization is associated with seeing the unique-ness and diversity of each student’s participation as making an essential contribution to the emergent sense-making taking place in the classroom. Space-creating *play* (Stroup, Ares, et. al, 2002; Stroup, Ares & Hurford, in press), not item-response convergence, is a central feature of generative instructional designs.

4.2 Multiple Path, Agreed upon Endpoint

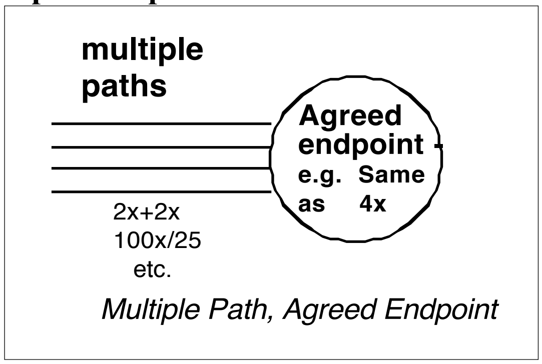

Figure 3. Visual representation of multiple path, agreed upon endpoint task.

Nominally generative tasks can often be made significantly more generative by simply reconsidering the form of the question. For example, rather than asking students to simplify 2/4, ask them to create ten fractions that are the same as 1/2. Rather than asking students to simplify 1x+3x, ask them to find five functions that are the same as 4x. Better use is made of the uniqueness and creativity of each student in the class (everyone is involved in creating new functions) and attention to mathematical structural issues is highlighted. With all of the student responses, or more often a selection of each student's favorite, projected in front of the class, structure-related questions like the following can be asked: "How can all these functions that look so different be the same?" or "If I added another example, how would you know if it was the same as or different from 4x?" Inviting students to extend the patterns they've observed to work in one context to a novel context also highlights mathematical structure. After completing the 4x activity, we've asked late elementary aged students to create functions that are the same as 4sin(x). Attention is now drawn to the *form* or *structure* of mathematics that "works" across contexts.

Asking students to pursue multiple paths-agreed upon endpoint tasks allows a larger mathematical space to be explored than would be the case with nominally generative tasks and supports participation in a way that has the potential to significantly advance the group's engagement with mathematical structure. Additionally, the teacher gets a quick snapshot of where student thinking is. For example, if none of the equivalent expressions involves decimals or negative numbers, the teacher is able to both see the space of kinds equivalence they are comfortable with and areas they may need to explore more. When this happened in some of the classrooms we worked with in Roxbury, Massachusetts the teacher could immediately adjust the flow of the activity in the classroom. One teacher simply asked his students, who were supposed to be familiar with these ideas, "How come?" in a playful/joking way. He then went on to encourage them to include, in the next "round" of expressions, examples of equivalent expressions involving decimals and negative terms.

Unlike the nominally generative tasks described earlier, a final observation we can make based on our work in classrooms is how mathematical creativity, flair, and insight are more likely to be acknowledged and celebrated with this type of task. 8x-4x is certainly acceptable (and was praised by the teacher) as an example of an expression that is equivalent to 4x, but 1,000,004x – 1,000,000x and 100x/25 are seen as more interesting by the students themselves. We know this, in part, because once these examples were projected for the whole class other students quickly worked on creating similar examples to share. *Mathematics* serves to structure the *social* activity of the group – students create, discuss, and share expressions that embody the idea of equivalence – and at the same time, the *social* sense of knowing and legitimate participation serve to structure the *mathematical* activity (Stroup, Ares & Hurford, in press). Sixth graders, within ten minutes of working with graphing calculators for the first time, created these and many other examples and our experience is that the next time an activity like this is done, students want to be the ones creating more interesting examples to "show off" to their peers.

In collaborating with pre-service and in-service elementary and secondary teachers, we began working on this kind of design well before the latest generation of highly interactive classroom networks was developed (cf., Stroup, 1997). Teachers find it useful to think about turning answers ("4x" as in "1x+3x = 4x") into questions (Can you find expressions that are the same as 4x?). Highly interactive networks support this kind of generativity by allowing expressive artifacts to be readily shared, displayed, recorded and aggregated. Students can submit responses anonymously and then decide later if they want to take ownership of a particular solution (Davis, 2002).

4.3 Modeling - Multiple Paths and Endpoints Where Fit with Data is Central

Modeling has the potential to be a multiple pathway and multiple endpoint activity. Learners can create different models that yield distinct outcomes. A central feature of the subsequent conversation is

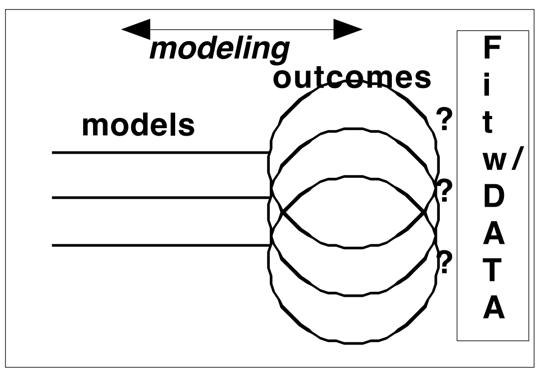

Figure 4. Modeling characterized as series of pathways, endpoints, and comparisons with data (experience).

how well the outcomes of the model fit with the data (whether real or anticipated) or experience (broadly conceived). Unfortunately modeling in classrooms is often pursued as if it was nominally generative. Much of current laboratory work in science classrooms has this collapsed structure. Students are to use a prescribed model (single path) to create computed outcomes that are then mechanically compared to the actual data collected from using tightly scripted "lab" procedures. A similarly collapsed notion of modeling is also what gets discussed when the "application" of a particular mathematical idea is presented in textbooks or classroom presentations. Modeling at its best, however, would have a description closer to that represented in Figure 4. – learners would create a range of models and use them to create model outcomes (implications or predictions). These outcomes would then be discussed in terms of goodness of fit to data (real or anticipated). Structural conversations about the ways in which various models might be similar or distinct can and should occur. In addition, issues related to deciding what it means to fit data can be engaged. The double arrow over modeling indicates that models and outcomes interact iteratively in the sense-making of learners.

As other researchers have noted, in what can be seen as extending aspects of the pragmatists' notion of "truth" to modeling, "models consist of conceptual systems that are expressed using a variety of interacting media (concrete materials, written symbols, spoken language)" and are used to organize our experiences and action in the world (Lesh et.al. 2003, p. 214). While a generative sense of modeling can certainly be carried out without network technology, next generation network capabilities allow the students and teachers to make visible and act on the "interacting media" used to express a given set of models. Whether it is a drawing, a sketch in a network-enabled geometry environment, a finite-difference equation, text, voice, or a Logo program, next generation networks offer the potential for making the machine-based interacting media associated with mathematical ideas the coin of the realm in pursuing generative approaches to modeling. Models can be made more visible and can be acted upon directly in a network space. A perhaps more dramatic outcome, in terms of student learning, may be the ways in which network mediated role-playing (discussed below) begins to make visible to students that useful and informative modeling is often the negotiated product of *groups* of interacting modelers

4.4 Design Tasks – Multiple Path and Endpoints where Satisfaction of Goal is Central

Design tasks are similar to modeling above in that both are multiple-path and multiple-outcome tasks.

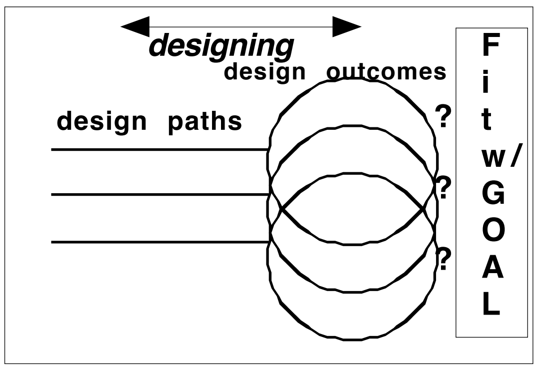

Figure 5. Generative design tasks are like modeling tasks except fit with a goal is central.

The difference is that fit with the goal or design specifications replaces the analysis of fit with data. As is true with modeling, structural issues can be ignored in which case designing comes to be only about learners arriving at particular designs. Unfortunately, when design tasks are pursued in classrooms it is often the case that no larger discussions of the processes of design are engaged. Much of the richness and learning potential of design is lost. When these types of lessons are approached in a more generative way structural issues of design, esthetics and even aesthetics are engaged. This kind of task is closest to what the researchers at Vanderbilt refer to as generative teaching and learning. As noted earlier, teaching comes to be seen as being about "anchoring or situating instruction in meaningful, problem-solving contexts that allow one to simulate in the classroom some of the advantages of apprenticeship learning" (1992, p. 78). As with modeling, network supported design tasks can make the design artifacts and representations public and even interactive. Cross design analyses related to what makes a "good" design is more readily supported and apprenticeship begins to be seen as a shared, group, activity.

4.5 Emergent Group Activity

Participatory simulations (cf., Wilensky & Stroup, 1999) and other forms of mathematical and scientific role playing are examples of emergent group activities. Learners assume *iconic* (like-the-thing) roles in a system and through their interactions create emergent behaviors of that system. Using individual devices learners assume the roles of predators or prey in an ecology simulation, or control individual traffic lights in a simulated city and then work towards improving traffic flow (Stroup & Wilensky, in review). The idea is that the emergent behavior created by these activities

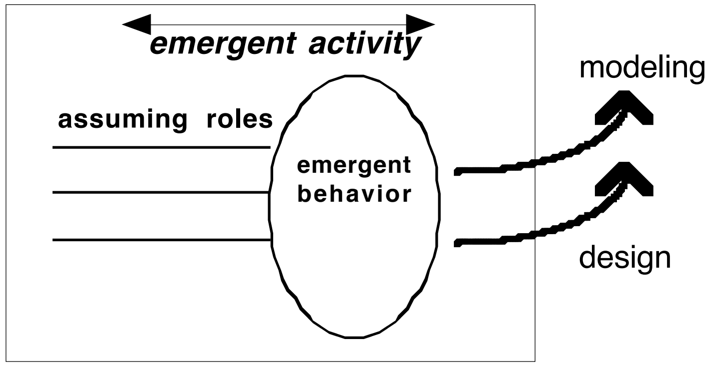

Figure 6. A range of role-playing activities, including participatory simulations, are fully generative.

becomes the object of attention and analysis as modeling or design tasks (see Figure 4 or 5 above). As with problems raised above, if the sole outcome of participating in an emergent activity is that a certain behavior emerges – and there is no subsequent analysis of how the behavior might have developed, how it might be different in another iteration or under different conditions, or how it might be like or unlike other systems and so on – then little learning of a generative or structural nature is likely to occur. However if thoughtfully utilized, next-generation networking can play a particularly significant role relative to emergent group activities and learning about properties of emergent systems in general.

4.6 Explorations of Kind and Quality of Pathways

This is possibly the most challenging of the generative forms to describe. Using the pathways and endpoints depiction, the focus here is not so much on getting to an endpoint as exploring the "quality" of possible pathways (Figure 7). Not only would this kind of exploration involve the many ways, for example, to prove the Pythagorean Theorem, but it would also include broad issues related to how reasoning moves forward, what the nature of development or improvement is, establishing cross-context similarity in the structure of a given system, structural reasoning proper, generalization, reflective abstraction (as it appears in Piaget's writings), and how situated-ness helps to determine the nature of reason and belief. Even the notions of domain and systems theory – as it includes complexity theory – exist as kinds of explorations into the nature of structural reasoning. This attention to forms

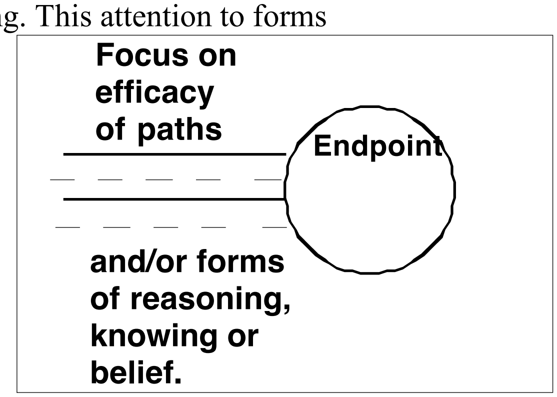

Figure 7. Exploring the kind and quality of pathways is generative.

of reasoning and the quality of pathways, can be engaged mechanically or by rote as is often the case with many students' experience of geometric proof or many students' experience of studying algorithmic design in computer science. But to the extent that these ideas can be engaged generatively, the potential of students attending to the *forms of reasoning* can assume greater significance in group-based learning settings.

The particular power of network-supported capabilities relative to this form of activity comes from making visible multiple instances of a particular kind of reasoning. This form of generativity is focused on rhetoric and logic, broadly conceived, and students and teachers are supported in engaging the senses in which the particular forms of reasoning they use are related. What kind of reasoning, for example, allows us to speak of this expressive artifact "3x+1x" as being the "same" as "2x + 2x"? How is this form of reasoning like or different from the form of reasoning that allows us to say that two pieces of music are both jazz?

5.0 Conclusion

The pathways and endpoints taxonomy of kinds of generative activity is intended to be useful both in clarifying the relations between kinds of generative activities and in exemplifying internal aspects of generative design, especially as supported by next-generation classroom networks. These approaches to generative learning and teaching can be integrated with new forms of functionality supported by next-generation classroom networks and result in significant improvements in mathematics and science teaching and learning.

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DYNAMIC VISUAL AND SITUATED COMPUTER ACTIVITIES DESIGN FOR LEARNING LINEAR FUNCTION

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In this study, the authors integrate the cognitive theories into the Geometers Sketchpad computer software to design five visual dynamic activities for learning the concept of linear functions. Users can record, collect and tabulate the crucial numerical data manually or automatically. The users are expected to generate the algebraic relationship of two variables via repeated experiment and observation.

Key words*: linear function, computer assisted learning*

Purpose of the study

The concept of functions is crucial in secondary school mathematics curriculum (Harel & Dubinsky, 1992; Yen & Law, 1993). The concept of function was stressed and also served as core guideline in secondary and post-secondary math education as specified in the *Curriculum and evaluation standards for school mathematics* and *Principles and standards for school mathematics* as edited by the National Council of Teachers of Mathematics (NCTM, 1989, 2000). In contrast, the Grade Eight math curriculum in Taiwan has introduced the meaning of functions and graphs of linear functions and quadratic functions. However, current curriculum design of function focuses on the description of textual rules and representation of static figure. In the classroom, teachers tend to stress too soon the correspondence of variables in functions and their algebraic representations. In the instruction of functional graph, teachers anticipate that their students learn the most efficient way to illustrate functional graphs. Consequently, students, while learning functional graphs, are inclined to be trapped in the repetitious mechanic calculation and dot-by-dot charting rather than take hold of the concept of function.

Hsieh and Huang (2002) indicated that function originated from an eventual description of causes and effects and served as a kinesthetic sketch of a movement, a description of time in relation to object movement. Thus, teachers should fully provide a dynamic cognitive learning environment so students may freely manipulate various values and objects, record the numerical changes of objects taking time as an axis and further determine the relation of numerical variations upon learning the concept of function. Lu (1988), after analyzing and comparing various curriculum projects in America, denotes the application, practicality, and flexibility of the American secondary math curriculum. American math educators also advocated the authentic situations for introducing mathematic concept and approaches, comprehending math generalities, and inducing student thinking from practical situated scenarios and inferences.

The learning of meaning in math formerly stressed upon the learning of skill and speed, while new learning theory focuses more on the derivation and association of meaning. Even (1993) pinpointed, teachers' major function is to construct a math learning environment for encouraging discovery and construction in the context of constructivistic approach, wherein students are encouraged to discover and ask questions. Knowledge derives from the connection and reconnection of individual experiences, an active construction rather than passive acceptance (Von Glasersfeld, 1990). Teachers have to establish an educational environment helpful for the development of math problem-solving ability so students may form a correct concept image of functional graph (Even, 1993). The task of learning is assimilation, i.e., new ideas are to be arranged into prior cognitive system for reconstructing anew cognitive system. Therefore, texts should also arrange appropriate situations to allow prior cognitive system to accommodate new concepts (Lin, 1994).

Since the practice of traditional experimental instruction in math is difficult and the acquisition of numerical information is time-consuming, the application of math situations for educational and learning is troublesome. Recent computer technology has advanced itself in the integration of information technology into scientific domains, such as the realization of educational policies and pragmatic instruction. Teachers make use of the informational technology to develop instructional tools and interactive courseware units and learning environments to improve traditional instruction. And the enhancement of performance in data processing, particularly the function of charting and imaging programs has effectively elevated the motivation of learning math and promoted the effectiveness of learning outcomes in math. However, static presentation of graphs or problems fail to form an active, constructive learning environment. The dynamic package of Geometer SketchPad (GSP) provides basic tools for geometric graphs and measurements, ruler for graphing, and other features such as recording the processes of graph variations, sequential dynamic transformations, structural retention, specific and general graphing. These features and functions not only produce precise dynamic configuration but also assist teachers with handy, maneuverable learning environment for determining graph property (Lin, 1996).

Consequently, this paper integrates theories of learning and educational design to design with GSP a learning situation that promises student manipulation, observation, inspection, and deduction. The concept of linear function is introduced by simulating real life situations. The design focuses on fetching the numerical values of the figure at any time interval and recording the points of movement to avoid the numerical description of the figure. The built-in function Tabulate is used to record and retain the parameters to be observed. Thus, by experimenting, some observable hypotheses are made to be further verified. In this manner, the dynamic design of this new approach to math education may compensate the outcomes hardly viable in static course instruction and inject new meaning into student learning (Hsieh, 2002).

Hsieh and Huang (2002) had pinpointed the flaws in the design current math textbook on the instruction of function: disassociation from real life and emphasis on the conversion of representation; formalized definition rather than the operability of function; failure in introducing rectangular coordinate before parallel coordinate; less emphasis on the arithmetic operation of function; and finally, concentration on the explanation of arithmetic symbols rather than those of corresponding figures. The designed contents in this paper are viable in the Windows environment and present linear functional graph with GSP software application, wherein each example in a unit is segmented into several pages and linked for student to select, review, or advance to the pages needed for learning. Each page is presented in three modes:

(1) text mode: Problem situations, operation procedures, definitions and conclusions are presented;
(2) numerical mode: Student operate, explore freely the numerical variations in variables, parameters and measurements in situations;
(3) graphic mode: Simulate dynamic graph in problem situation, change functional graph by changing the corresponding values or mouse maneuvering of the animation button.

Instructional materials incompliance with curriculum design, course contents, and students' cognitive development can be realized by integrating functional graph into the computer-assisted learning environment to take advantage of the computer features in animation, computation, and instant feedback. Take GSP for example, exemplary objects allow students to actively manipulate the objects and get instant feedback and observe the relational dynamic changes within a representation on the screen. The design is based upon the construction of cognitive structure rather than math computing skill. Therefore, students can get hands-on experience exploring, concluding, discovering, and verifying with the texts on functional graph. And teachers may revise the instructional design per student need at any time.

Design of dynamic visual situations

The design of dynamic situations for the learning of linear functional graph was based upon the standard of secondary school math curriculum as revised and announced in October 1994 by the Ministry of Education (MOE) of Taiwan, in addition to the reference of educational objectives specified in the Grade Eight textbooks and teacher's Manual. Examples are provided to simulate real life situations and retain merely the main variables associated with the subjects to avoid student distraction due to redundant variables. The examples illustrate diverse representations of linear function and the link and the translation among various representations. The examples allow students to repeatedly drill and practice with different items and freely modify the alphanumerical values. Instant feedback and change of alphanumerical values and tables are in sync with the changes of dynamic simulated configurations to reflect the link and translation among various representations.

Five examples are included in this paper:

1. Volume of water in a water tank (Rf. Fig. 1)

A rectangular-shaped water tank has a square base of two meters long. The volume of water in the tank increases with the depth of water in the water tank. Then

(1a) When the water in the tank reaches five meters deep, what is the volume of water in cubic meters?

(1b) When the water in the tank reaches 20 meters deep, what is the volume of water in cubic meters?

(1c) Observe the change of water volume with the change water depth.

(1d) Let the water depth be x meters, and the water volume be y cubic meter, please specify the relationship between x and y in a math equation.

2. Time and Distance (Rf. Fig. 2)

Kirk attends road running. The runs at three meters per second on average. Then

(2a) How many meters does he run after running three seconds?

(2b) How many meters is he from the starting point after running ten seconds?

(2c) Assume he is y meters away from the starting point after running x seconds, specify the relationship between x and y in a math equation.

3. Celsius vs. Fahrenheit (Rf. Fig. 3)

The measurement of temperatures is in two systems: Celsius (centigrade) and Fahrenheit. The conversion equation between the two is F=(9/5)C+32.

Find the corresponding temperatures in Fahrenheit when temperatures in Celsius change.

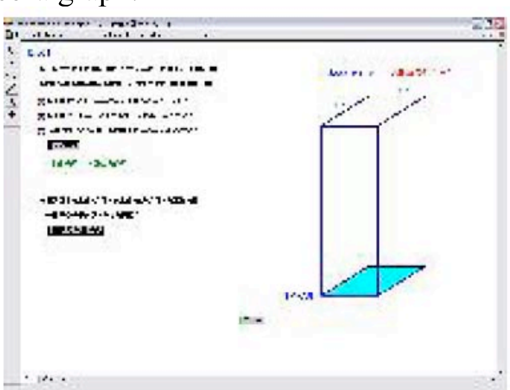
(3a) When the temperature is zero degree Celsius, what is the corresponding temperature in Fahrenheit?

(3b) When the temperature is 30 degrees Celsius, what is the corresponding temperature in Fahrenheit?

(3c) When the temperature is minus ten (-10) degrees Celsius, what is the corresponding temperature in Fahrenheit?

(3d) When the temperature is minus forty (-40) degrees Celsius, what is the corresponding temperature in Fahrenheit?
(3e) Try to change the temperatures in Celsius and observe their Fahrenheit counterparts.
4. Mountain Temperature (Rf. Fig. 4)
We know that the temperature on top of the mountain is lower than that on earth. But do you know the relationship between the temperature on mountain and its altitude above sea level? The temperature at a site is known to drop 0.6 degree in Celsius (centigrade) for an increment of every 100 meters in altitude as y= 26 - 0.6 * x/100.
(4a) Johnny is in a place of 400 meters in altitude. What is the temperature in Celsius at Johnny's site?
(4b) Howard is in a place of 1500 meters in altitude. What is the temperature in Celsius at Howard's site?
(4c) Jimmy is in a place of 1500 meters in altitude. What is the temperature in Celsius at Jimmy place?
5. Heating water (Rf. Fig. 5)
Dave has a stove with three functions: heating (H), warming (W), and cooling (C). Now put a glass of water in the stove and the water temperature will increase three degrees Celsius when the H (heating) button is pressed; the water temperature will remain unchanged each second when W (Warming) button is pressed; and the water temperature will drop by two degrees Celsius per second when the C button is pressed.
Assume the water temperature in a glass is initially 26 degrees Celsius and becomes y degrees Celsius after x seconds.
(5a) Observe the water temperature y in Celsius change after x seconds after randomly pressing the H, W, or C buttons.
(5b) Consider how we should record the water temperature at different intervals.

The afore-mentioned five examples of activities are introduced from real life situations. Each example covers several pages for student links to the learning of the concept pf situated linear algebra graph.



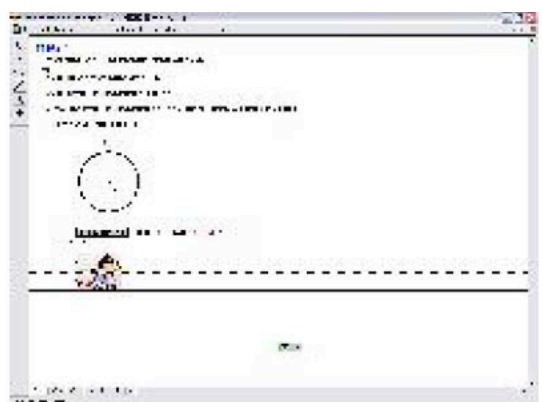


Fig. 1. water level of the tank Fig. 2. Time and distance

Fig. 3. Celsius vs Fahrenheit Fig. 4. Mountain temperature

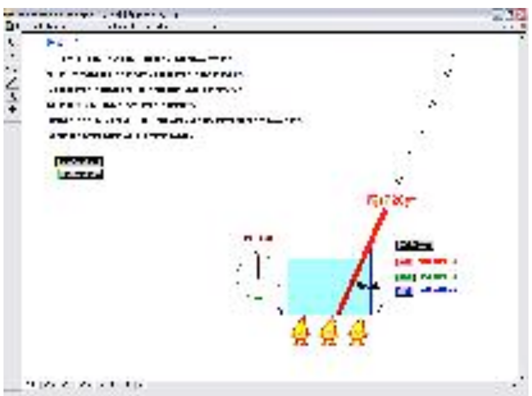


Fig. 5. Heating water

Each example includes textual description of the situation and evident visual presentations for invoking students' similar experiences in life in charting a complete graph while retain the objects in the situations for students to manipulate. Students are expected to explore, induce and construct a complete body of knowledge of the concept of linear functional graphing. Items deviant from real situations or the concept are excluded to avoid distracting students in learning.

In traditional static instruction of function, teachers tend to textually describe an imaginary situation and then demand students to calculate a specific value of a variable. In this design, students open the file of each example and the example presents a problem the same way as the traditional instruction does. For example, students are asked to find the corresponding value of a value (such as volume of water, distance) to a specific value of another variable (such as time). In the example of "water volume in a tank," student will figure out the water volume corresponding to a certain depth of water in the tank. At the moment, students have not actually manipulated any objects in the example. However, if students have prior knowledge of variable and function, they will be able to determine the corresponding relationship (function) between the variables and find the answers of the variable (water depth) and corresponding variable (volume of water) of the problem by substituting the parameter with a specific value. But we cannot optimistically expect that students solve problems so smoothly. In the designed example, students are expected to mouse-click the "water depth" to change the default value, and instant feedback and record of the various representations are provided as specified in the following figures. Students are expected to acquire the knowledge of the interdependence between the two variables that water volume changes with water depth in a container.

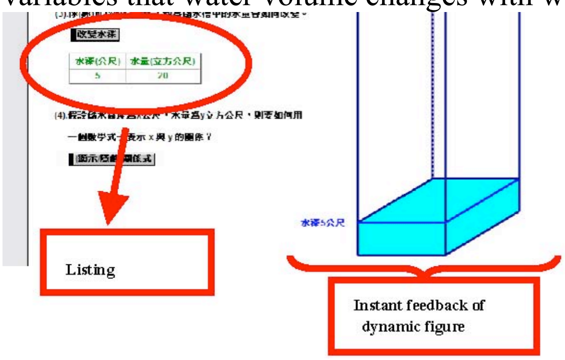


Fig. 6. Visual and numerical record of the procedure

In the activity of "water volume in a tank," students get instant feedback by pointing-and-clicking the animation button to change the depth of water and record simultaneously the various sets of parameter values of the two variables (water depth and water volume). Students can then collect relevant data to help construct the dynamic graphic representation that determines the linear relation between the two variables or understand the meaning the two variables represent (Rf. Fig. 7).

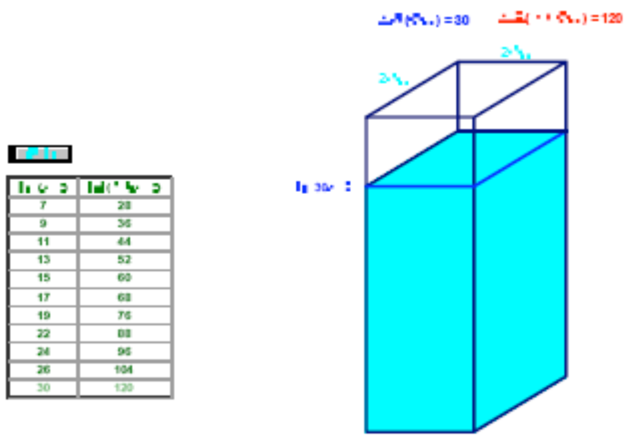


Fig. 7. Dynamic representation of numerical values and changes of the figure.

Research suggested that the concept of parallel coordinate be introduced in the instruction of graphing functions. Goldenberg et al. (1992) indicated, the numerical data of two different variables be charted on the two corresponding parallel coordinates and then be connected through appropriate conversion based upon the respective order pairs and further be charted onto the commonly known rectangular coordinate. Hsieh and Huang (2002) further advocated the gradual transitions from parallel coordinate to bevel coordinate then to rectangular coordinate while maintaining the quantitative relation between the two variables by rotating the two coordinate axes. So students may know the graphic representations of the correspondence of the two variables and form a correct concept of the functional graph and the meaning of the points and the order pairs in charting the functional graph. (Fig. 8)

In traditional instruction, students mechanically select the order pairs that satisfy the arithmetic equation in the static situation (or abstract arithmetic representation) teachers presented and convert them into two points in the coordinate plane, and then chart a functional graph that meets teachers' expectation. In such a learning process, students are unable to freely manipulate the variable in the situation and present the coordination that reflects the value of the variables. However, in a dynamic situation designed with GSP, students take over the maneuverability for learning and knowledge construction. Students feel free to change the variable objects indifferent coordinate system (Fig. 9 and Fig. 10). Such a design allows students to visually perceive the change of values of the two axes (number lines) that represent two systems of variable numbers. Students can self-construct a concise and clear graphic representation of a function with or without teachers' guidance. On page two of the activity "Water volume in a tank," students get the graphic representation of the two corresponding variables of a function in different coordinate systems.

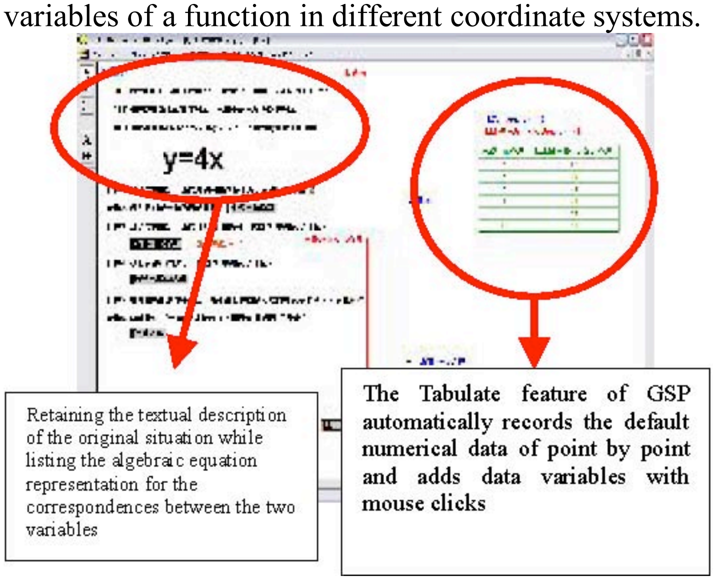


Fig. 8. Process of learning anew. explore situations → record numbers → draw graph → determine relationship

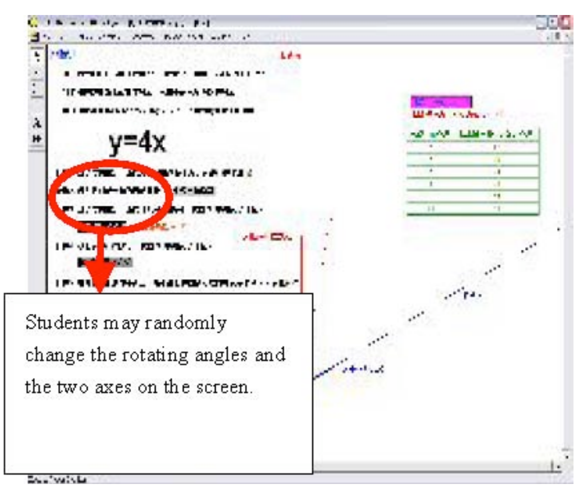


Fig. 9. The bevel coordinate after appropriate rotation of the two overlapping points of the two number lines of the parallel coordinate

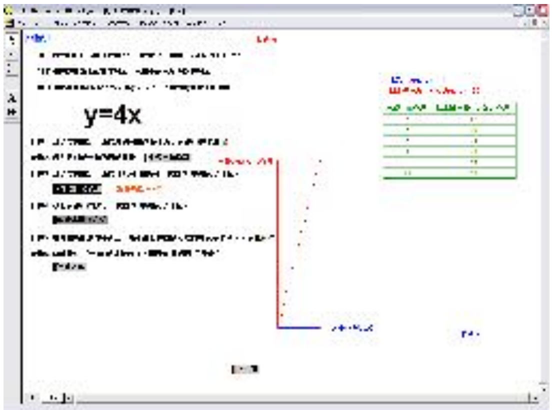


Fig. 10. The rectangular coordinate after rotating the two axes 90 degrees.

When the values of two variables in the bevel coordinate system and rectangular coordinate system change, the values of the two variables are designed to present in the two independent coordinate axes, so students may observe the variations on the two axes and formulate the concept of order pairs. Traditionally, students are demanded per teachers' instructions to formulate the variables into (x, y) and plot the corresponding points onto the coordinate plane. Thus, a point on a coordinate plane represents a specific variable and teachers attempt to present more similar points that represent the generality of variable. Such an instructional practice deprives students of their chances of freely constructing the concept of "order pairs" even students finally understand and accept such a way of presentation.

The points representing the order pairs change their positions with the variations of specific values of the variables in the bezel coordinate system or rectangular system, the coordinate plane records the trail of the movement of the points. Therefore, students may clearly perceive the variations of the variables in the ongoing changes of the situations (fig. 9 and Fig 10) and the implied invariation to gradually formulate the concept of the functional graph and accept that the changing variables originally on two different axes can determine the interdependency of the correspondence with a line (functional graph).

The design also noted the link between graphic representation and representation of other format when different coordinate systems present corresponding graphic representations between two variables. The Tabulate function of the GSP can track the numerical values of the two variables corresponding to the moving point in the coordinate system; therefore, students may perceive the variations of specific values on two independent axes while maintaining a specified correspondence.

As the values of the dependent and independent variables are respectively presented onto two independent number line (axes), students may be misled while observing the correspondence of two variables and charting a more accurate functional graph. The example is so designed that the unit of length of the two axes can be specified by teachers to not mislead students into twisting the functional graph due to the selection of different length unit or their conception of the "bezel rate" in linear functional graph.

In this paper, the activity of each example is designed to be dynamic and is added at the same measure discretely. Teachers may consider the fact that the presentation of discrete values more clearly specifies the interdependency of two variables than the presentation of sequential numerical values. Take the "Water volume in a tank" for example, students will know that depth of water can be constantly varied. The design of increasing water depth at an interval of one meter aims at minimizing the effect that complicate numbers may deter students from constructing the linear correspondence of the interdependency of two points. The presentation of collecting data of water depth and water volume point by point will enable students to self-construct the linear relation of the two variables. And discrete situations are designed to address to a series of authentic situations.

On page three of each example, the situation prompts to the traditional rectangular coordinate (Figure 11), students are asked the same question as in the general textbook to construct a graph of a situated linear function. In example two of "time and distance," page three is illustrated as the following:

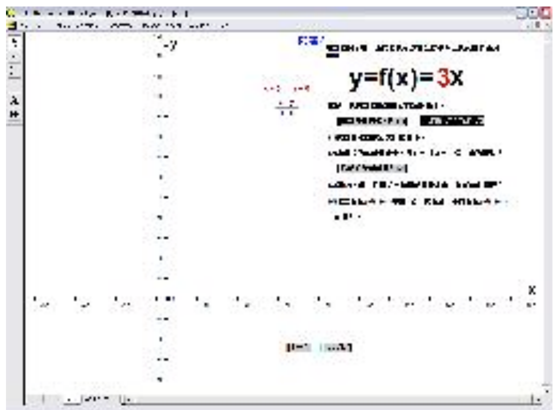


Fig.11. Traditional rectangular coordinate is last introduced

The page as shown above is intended to present the graph of function $y=ax$ and enable students to extract real situations from life based upon prior examples. Students repeatedly practice with various items under teacher guidance to construct the interdependency of linear relation between the variables in the situations given via arithmetic equation and the meaning the equation represents. In case students fail to correctly construct and abstract the arithmetic representation of the function for "$y=ax$ where a is the graph of function for a specified value," the activity returns students to a previous situation to redefine the meaning of the equation compliant with real life experiences.

This page is intended to guide students to clearly and briefly construct the graph of linear function and chart the graph of the linear function with decreasing point-trailing records in the designed activity. Therefore, students will be guided step by step to get to know the most efficient way to draft the correct graph of linear function given the context that students already understand the graph of linear function. (Rf. Fig. 12)

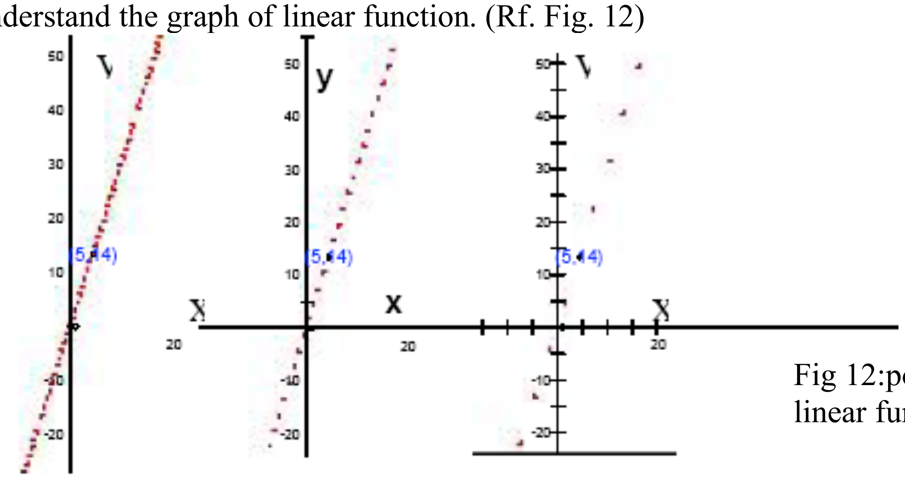
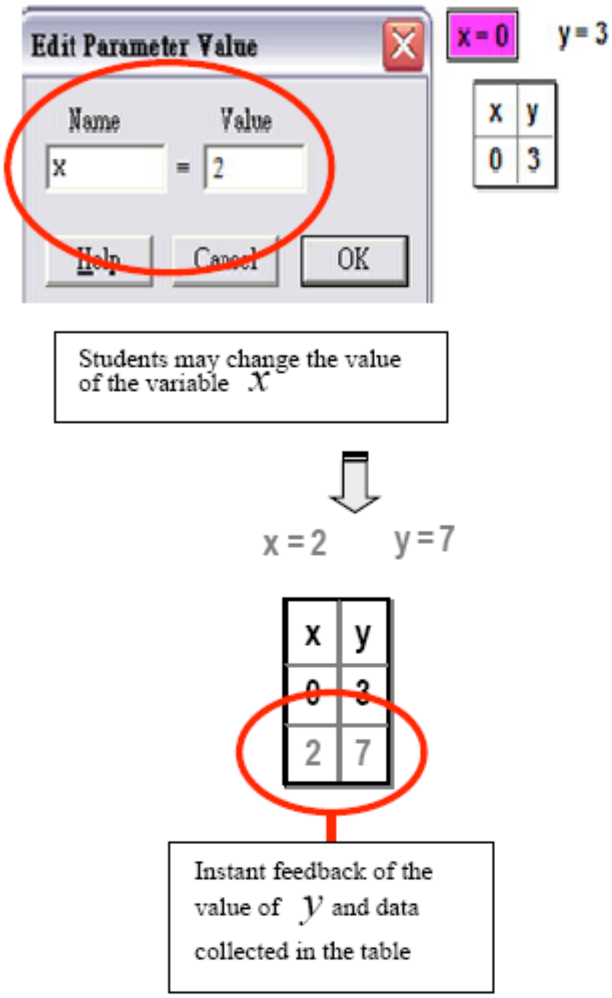


Fig 12:pointploting linear function

Conclusion

In traditional classroom instruction, students have to select a specific value for the variable x and introduce the value into the algebraic representation of function to yield the corresponding value of variable y, if they need to collect the numerical data of a set of two variables of the linear function. The design in this paper simplifies this process. Students get instant feedback of the value of the parameters and collect the numerical data in the tabulation in the situations designed with GSP.



In traditional instruction of linear function, students are soon asked to select two different points in a rectangular coordinate plane that satisfy the algebra of function and are instructed to connect the two points with a ruler to yield said linear functional graph. Students will be asked further to do a similar exercise. The traditional teaching aims at taking a hold of the time span when students have not forgotten the procedure of charting the graph so they may chart the linear function correctly and rapidly for educational assessment. Consequently, math learning is just a process of repeated drill and practice of highly abstract objects. In the design of this paper, students are free to change the values of the parameters of a and b in $y = ax + b$, and chart said linear function. Thus, students not only practice various similar problems for enhancing their conception, but also review the process of prior manipulation and probes when encountering difficulties.

Hence, the design in this paper significantly improves the static design of traditional curriculum and instruction and provides students with a dynamic learning environment to simulate real-life situations and lead students to actively manipulate, prove, observe, hypothesize about, deduce, discover, prove, and further construct solid, accurate and stable concept of linear function.

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TALKING ABOUT PROPORTIONS CONCEPTUALLY: ORCHESTRATING BOTTOM-UP WHOLE-CLASS CONVERSATIONS

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In this paper we address a longstanding concern of reform in mathematics instruction, one that Deborah Ball has synthesized in the question: "How do I create experiences for my students that connect with what they now know and care about but that also transcend their present?" We analyze several episodes of a classroom design experiment on ratio in the context of measurement, with ten 7th grade students in an urban public school in the USA. We discuss the challenge of orchestrating whole-class conversations that (a) can be viewed by students as centering on issues they are interested in discussing and capable of making contributions to, and (b) are situated within a communal learning path that leads towards the development of sophisticated mathematical understandings.

During the past 15 years throughout North America, there has been a strong push to transform mathematics instruction. Within Canada, México and the United States important efforts are made to reform the ways mathematics is taught in schools. In this paper we address a longstanding concern of reform in mathematics instruction, one that Deborah Ball (1993) has synthesized in the question: "How do I create experiences for my students that connect with what they now know and care about but that also transcend their present?" (p. 375).

Previously, Gravemeijer, Cobb, Bowers, and Whitenack (2000) have explained the general principles of an instructional orientation that helped to address this matter in classroom design experiments that lasted up to one school year. Our goal in this paper is to advance the agenda by looking into some of the specific issues involved in following this orientation. Specifically, we discuss the challenge of creating instructional situations in classrooms that (a) can be viewed by students as centering on issues they are interested in discussing and capable of making contributions to, and (b) are situated within a communal learning path that leads towards the development of sophisticated mathematical understandings.

The illustrative case our analysis draws upon involves several episodes of a classroom design experiment on ratio in the context of measurement with ten 7th grade students in an urban public school in the USA. These episodes took place during the first instructional session we had with the students. In this session the teacher was faced with the challenge of engaging students in a whole-class conversation that could be productive with respect to her long-term instructional agenda, and at the same time, would be viewed by students as meaningful and based on their contributions. Meeting this challenge proved to be nontrivial in that students' initial understanding of the instructional situation reflected their previous participation in classroom practices where mathematical success, in many cases, is equivalent to correctly using prescribed algorithms and formulas.

The Classroom Design Experiment on Ratios in the Context of Measurement

The classroom design experiment involved ten teaching sessions conducted with a group of ten 7th grade students in an urban public school in the southeast United States. The school served an ethnically diverse population. The teaching sessions took place twice a week during the students' activity period (i.e. the last period of the school day). A member of the research team assumed the role of the classroom teacher and was assisted on numerous occasions by two additional researchers who also participated in planning the instruction. During the classroom design experiment, the research team tested and revised a conjectured learning trajectory (Simon, 1995) aimed at supporting middle-school students' understanding of proportional relations in the context of measurement (cf. Freudenthal, 1983; Simon & Blume, 1996; Thompson & Thompson, 1996).

Our interest in proportional reasoning was motivated by previous experiences with middle grades students in which they had difficulty in making sound mathematical interpretations of data represented by ratios (e.g., "price per ounce," "income per capita," etc.). Given the importance of phenomena that are commonly organized into ratios in science and social studies, we considered developing an instructional approach focusing on ratio an important aspect of middle-school mathematics.

The data for our analysis consist of videotape recordings from two cameras of the first teaching session of the design experiment. One of the cameras was focused on the teacher and the other on the students. In addition, a set of field notes and copies of student work are part of the data corpus.

Each student was interviewed individually before and after the classroom design experiment. During both interviews, students were presented with situations in which using a ratio to measure a specific lived quality would be pertinent (e.g., using a ratio such as "income per capita" to determine how wealthy different cities are, when knowing their population size and their total income). These interviews revealed that, by participating in the classroom design experiment, nine of the ten students went through an important shift in their understanding about using ratios as units of measure.

Whole-class Conversations as a Context for Mathematical Learning

The instructional orientation we follow when conducting classroom design experiments is an adaptation of the Realistic Mathematics Education model, developed at the Freudenthal Institute in the Netherlands (cf. Gravemeijer, et al., 2000). In the adapted model, we frame whole-class conversations as the main context in which a teacher can generate collective opportunities for students' learning. It is worth clarifying that we do not regard these conversations as having a direct (or causal) impact on students' learning. Instead, we construe them as social events that influence students' learning indirectly. It is by participating in the constitution and evolution of whole-class conversations that opportunities arise for students to reflect on, objectify, and reorganize their (prior) mathematical activity (Cobb, 1998).

In the course of our work as classroom-based researchers as well as instructional designers, we have identified several important aspects that a teacher needs to proactively work on to make whole-class conversations fruitful in supporting students' learning of mathematics. One of these aspects concerns the nature of the discourse in which the whole-class conversations are grounded. We have learned that fruitful whole-class conversations are grounded in conceptual discourse (Cobb, 1998; Thompson, Philipp, Thompson, & Boyd, 1994), which entails not only addressing the ways of calculating, but also explicating the reasons for carrying out those calculations with respect to the problem-situation at hand. Whole-class conversations grounded in conceptual discourse encompass both students' calculational processes and the task interpretations that underlie those ways of calculating and constitute their rationale (Cobb, 1998).

A second important aspect of a fruitful whole-class conversation we have identified concerns the norms of participation. Whole-class conversations render more opportunities for learning when students feel obliged to listen and try to make sense of each other's contributions, and to be outspoken about not understanding others' contributions (Cobb, Yackel, & Wood 1989).

A third aspect of a fruitful whole-class conversation concerns the possibility of all students having a way of participating in the discussion even if participating involves merely being an active listener attending to the discussion and reflecting on what is being said. When a student lacks this possibility, his or her opportunities for learning become severely limited.

The final key aspects of whole-class conversations that support students' mathematical learning concern the nature of the topics of conversations that are addressed in the discussion. These topics need to be (a) seen by students as worthwhile discussing and built upon contributions made by themselves (as a group), and (b) mathematically relevant in terms of the teachers' anticipated (long-term) learning trajectory. The former aspect is crucial because students are much more keen to engage and reflect on a discussion when they view themselves as the (direct, partial, or at least potential) contributors to the conversation. Nevertheless, it is not sufficient for whole-class conversations to center merely on mathematical issues contributed by students. The topics that are discussed also need to have the potential of leading to conversations involving increasingly sophisticated mathematical issues.

Results of Analysis

In this section, we analyze several episodes that took place during the first instructional session of the classroom design experiment. The ten students that participated in this instructional session were a subgroup of a regular 7th grade mathematics class in the school, and were invited by their mathematics teacher to participate in the experiment. The instructional sessions took place in a regular 7th grade mathematics classroom in the school. The students' regular mathematics teacher was present throughout the design experiment as an observer.

Students' Initial Contributions

The teacher's instructional goal for the first session was to engage students in thinking about the proportional accumulation of two quantities (Thompson & Thompson, 1996). At the beginning of the first session, the teacher introduced a problem that focused on the proportional relationship between miles driven and gallons of gas consumed. She explained that normally she would drive 204 miles a week using 9 gallons of gasoline, but because of a new schedule starting the next week she would be driving about 300 miles. She wanted to know how much gasoline her car would consume weekly on this new schedule. While explaining the problem, the teacher drew a double-scaled number line on the board (Gravemeijer, 1998; Streefland, 1984) as shown in Figure 1.

Figure 1. The double-scaled number line

The teacher introduced the double-scaled number line to represent the proportional relations between distance and amount of gasoline. Her intent was that the line would become constituted as a record (of a narrative) of the event that was taking place for the students (i.e., a car covering distance). The vertical lines were intended to represent the endpoints of the two accumulations that were simultaneously occurring as the car was driven (i.e., driving distance and gasoline consumption). The teacher's expectation was that the students would come to view these outcomes as being proportionally equivalent. If this occurred, they would take it as self-evident that were the car to travel twice the distance, it would consume twice as much gasoline. In this sense, the distance between the vertical marks was proportional to the values they represented. For example, the mark 300 was about 3/2 as far from the origin of the line as the mark 204 because it represented a value that was about 3/2 as big (see Figure 1).

Megan was the first student to propose a way of solving the problem. Her approach was based on using cross multiplication.

Megan: I think you can put it in fraction, say 9 over 204 equals…and then compared it to X over 300?

Teacher: What would that tell me?

Megan: Huh…huh…If you make it into a fraction,

Other students: Proportion…

Megan: Oh, proportion, proportion! If you make it into a proportion... If you make 9 over 204 and equals…or… I don't know, let's say equals, X over 300. You simplify the bottom and cross multiply…

Megan's contribution did not take into consideration the graph the teacher had drawn. She seemed to be responding to a question, yet not asked by the teacher, "which calculation should be used to solve the problem?" To the teacher, Megan's approach appeared to be considered by most of the students as a reasonable solution to the problem as no one voiced a different opinion and some tried to further elaborate on carrying out a cross-multiplication.

Although cross-multiplication was a mathematically adequate way of dealing with the problem, the consensus to use it in solving the problem indicated to the teacher an shared notion among the students; that is, this activity was about solving a word problem by identifying and applying a previously taught algorithmic method. Such notion was in conflict with the envisioned instructional agenda that the teacher had in mind which centered on making quantitative sense of the situation (Gravemeijer, et al., 2000).

The main challenge for the teacher then became redefining for the students the instructional activity so that students' contributions could shift to focus on the proportional covariation of two quantities (i.e., distance and gasoline consumption) in relation to the situation at hand. Such goal was paramount in terms of the viability of the conjectured learning trajectory that was being tested in the classroom design experiment and, thus, in the teachers' instructional agenda.

It is worth mentioning that we see these students' initial ways of interpreting the activity as reflecting their prior participation in the practices of traditional school mathematics, where, generally speaking, competence in mathematics is equivalent to correctly using a prescribed algorithm in solving a problem. Thus, the challenge that the teacher was facing can be considered typical when reformed mathematics instruction is initially introduced to students who are habituated to participating in the practices of traditional school mathematics.

The double-scaled number line (see Figure 1) became an important resource for the teacher as she addressed the challenge. In an effort to support the students to reinterpret the problem situation, the teacher told the students that she was interested in knowing what approximate amount of gas was needed for a 300 mile drive before she conducted any precise calculations. She then guided a whole-class conversation focused on generating this estimate, in the course of which she made frequent use of the double-scaled number line.

Teacher: I want to get a sense of what you think the answer is gonna be. How much gas you are going to use? We know when you drive that distance 204 miles [motioning along the double-scaled number line to indicate a distance of 204 miles], you watch the gas gauge you use 9 gallons and you then have to stop at the gas station and put 9 gallons in, right [pointing to the numbers]? Instead of driving that distance 204, you will be driving 300 miles. Do you think you will be using twice as much gas as you used before?

The teacher's efforts to renegotiate the nature of the activity with the students were fruitful. By redefining the activity as "estimating," and by developing a narrative that referred to a gas gauge moving and a gas refill, she seemed to orient some students to view the problem as *real* and, therefore, to view the numbers involved as measured quantities. In addition, the teacher used the inscription to support the development of the narrative by purposefully tracing with her hand the distance from the origin of the line to the mark 9/204 (see Figure 1).

Many students indicated "no" to the teacher's question about whether she would need twice as much gas to drive a 300-mile distance, and Alice concluded by saying "It won't be twice as much, because you are not going twice the distance." The teacher then asked if the students thought the car would use the same amount of gasoline for both the distance of 204 miles and 300 miles, and another student responded:

Amanda: I think it will be more than 9 gallons because if it is 9 gallons for 204 miles, 300 miles is more than 204 miles, so it will probably be more than 9 gallons [pointing to the graph when she mentioned 204 miles, 300 mils and 9 gallons].

Teacher: So you are going further. See I need some more gas in there [pointing to the distance between the 204 and 300], right? Yet it is not twice as far so you are not going to use twice as much gas [motioning with her arm back and forth along the number line]. So you are going to figure out how much more gas you are going to use. Does that make sense? We don't need the exact answer. We can just estimate.

Alice and Amanda's contributions were crucial in terms of the teacher's instructional goals in that they were reasoning about the problem as an experientially real situation involving measured quantities (Gravemeijer, et al., 2000), a premise for productive whole-class conversations from the teacher's perspective. The teacher therefore recast their contributions and elaborated on them as she gestured to the double-scaled number line. By doing this, the teacher attempted to indicate to the students the kind of mathematical contributions she valued. Our analysis of the whole design experiment reveals that from this point on the shift in students' interpretation of the nature of the instructional activities was relatively stable; in the ensuing conversations students seemed more and more inclined to construe the instructional activities as involving experimentally real situations that needed to be resolved quantitatively. Likewise, algorithmic explanations detached from the problem situation became increasingly rare as the design experiment progressed.

It is worth noticing that the teacher might not have been successful in redefining the nature of the activity, had she solely tried to *tell* the students what she wanted the activity to be about. The contributions made by Alice and Amanda became a valuable resource for the teacher in conveying to the whole class the kind of participation that she valued. Importantly, the emergence of these contributions was supported by the introduction of the estimating activity.

Students' Contributions as Resources to Advance the Agenda

As the session continued, the following exchange took place and indicated a shift in at least some students' ways of thinking about the proportional covariation of two quantities. This shift afforded the teacher with new resources to advance her instructional agenda.

Teacher: Ok, so we want to use this information [pointing to the "9/204" mark] to try to get this estimate [pointing to the "300" mark]. Right? Is that what you are saying. Ok. So does anybody want to try that? Ok, Megan.

Megan: Ok, [pointing from her desk at different parts of the graph] since 300 is about 100 more than 204, we need to find out how many gallons of gas you would use for say 100 miles. So if you divide 204 by, in half, it's about 100. So you would divide 9 into half, which would be 4 and a half.

Teacher: Ok

Megan: And so I guess to get… it would be about 9 plus 4 and a half

Megan's approach was to find correspondences based on the proportional relations between the driving distance and the gas consumption. Her approach can be considered a continuation of the type of reasoning supported by the activity of estimating. As she explained her thinking to the class, Megan seemed to reason that since half the distance required half the amount of gasoline, one could find out how much gasoline was needed for the additional 100 miles by halving the amount of gas needed for 200 miles.

The research team had previously documented the emergence of similar ways of reasoning during a pilot study conducted prior to the classroom design experiment, as well as during the pre-interviews of this experiment (Cortina, 2002; Cortina, Saldanha, & Thompson, 1999). The research team conjectured that reasoning of this kind could serve as the starting point of the learning trajectory. Briefly, in the conjectured learning trajectory it was anticipated that the initial whole-class conversations could be grounded in the different ways that students used to organize the proportional covariation of two quantities. Such conversations would latter serve as a basis for further discussions that centered on conducting proportional comparisons of qualities that were dependent on the proportional covariation of two quantities (e.g. compare a car that consumed 15 gallons of gas for 360 miles to another one that consumed 9 gallons for 200 miles to determine which one was more fuel efficient). These later conversations would again serve as the basis to discussions that focused on constructing normalized ratios and using them as measures of specific qualities (e.g., "miles per one gallon" as a measure of fuel efficiency).

In the context of the conjectured learning trajectory, Megan's contribution was valuable to the teachers' instructional agenda in that it proposed a relatively simple way of proportionally organizing the covariation in the accumulation of two quantities (i.e. distance and gas consumption). The teacher hoped that this kind of mathematical activity could become a topic of discussion and, thus, the focus of students' mathematical thinking. She therefore, inscribed Megan's approach on the double-scaled number line and added to it the inscriptions "9 + 4.5 = 13.5" and "4.5/100" (see Figure 2). She then asked the class if they could explain "Megan's way" in an attempt to focus the ensuing whole-class conversations on this way of organizing the proportional accumulation of two quantities.

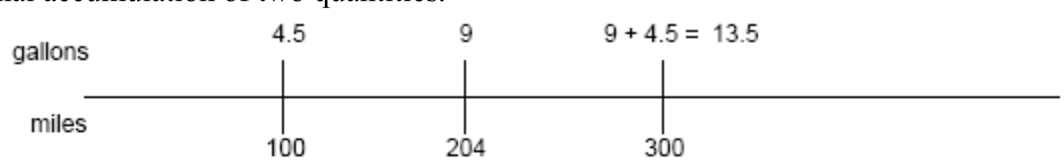

Figure 2. The evolving double-scaled number line

Nicolas: Ok, first she had to figure out how many miles she was using per 100 miles. Right?

Teacher: Ok I got a question for both of you guys [Nicolas and Megan] or anybody else. Why are you trying to find out about 100 miles [pointing to the "100" in the "4.5/100" mark; see figure 2]?

Nicolas: Because it is about 100… 300 is close to 100 miles more, a little less than 100 miles, than 204 miles. So, ah, in order to find bout how much gas you'll be using with 300 miles, you have to find out what 100 miles more is.

Teacher: Ok. So you are taking half of the 200 [indicating the distance between the "100" and the "204"] and then you are just finding out what half the gas is [indicating the distance between the "4.5" and the "9"]. Because that's like an extra half [indicating the distance between the "204" and the "300"]. So if that's 200, an extra half would be 300 [indicating again the distance between "204" and the "300"], so that's got to be an extra half there [indicating the distance between the "9" and the "13.5"]. Is that kind of how you are thinking about it?

By having students give explanations and by referring to the double-scaled number line as she recast aspects of students' contributions that were salient to her mathematical agenda (i.e., half the distance, half the gasoline), the teacher seemed to help the rest of the class make sense of both Megan's solution and the problem situation on which it was based. It should be clear from these episodes that even though the teacher was the one who proactively advanced the instructional agenda and developed the double-scaled number line to include more complicated inscriptions, it was the mathematical contributions made by the students that became inscribed in the double-scaled number line and that constituted topics of discussion in the whole-class conversations.

It is also worth highlighting that the instructional decisions the teacher was making as she orchestrated the whole-class conversations were not only determined by her "on the spot" interpretation of the emerging chain of events, but were also informed by the conjectured learning trajectory that was being tested in the classroom. Therefore, although the teacher's instructional efforts needed to constantly reconciled with the students' emerging contributions, the conjectured learning trajectory served as a conceptual resources for the teacher to anticipate the nature of those contributions, and the mathematical issues that could emerge and become sensible topics of conversation. In the case of Megan's contribution, the learning trajectory allowed the teacher to recognize it as a contribution worth discussing.

The whole-class conversation continued as the teacher asked the students to estimate how much gasoline her car would consume if she drove 500 miles. While doing so, she marked "500" on the double-scaled number line. Rashied proposed to add the 9 gallons that corresponded to 204 miles, to the 13.5 gallons that corresponded to 300 miles. In this way, he said, one could find out how much gas the car would consume for 500 miles (i.e. 204 + 300 = 500 miles [approximately]; hence, 9 + 13.5 = 22.5 gallons). Rashied's approach resembled Megan's in that it was based on putting together "pieces" of known proportional correspondences (i.e. 9 gallons to 204 miles and 13.5 gallons to 300 miles) to figure out the unknown one (i.e. the gas needed for 500 miles).

Rashied was a student who had experienced much difficulty solving similar kind of problems during the pre-interview in which he did not use any kind of proportional strategies. Rashied's contribution to the whole-class discussion exemplifies how whole-class conversations supported students in reorganizing their thinking. Megan's explanation of her approach and the elaboration made by the teacher seemed to help Rashied understand a strategy for finding proportional correspondences.

After all the students indicated they had understood Rashied's way, Alice contributed to the conversation by proposing a different approach. Alice argued that because 500 miles was 5 times 100 miles, they could multiply 4.5 gallons by five to find out how much gasoline the car would consume for 500 miles (i.e. since 5 times 100 miles equals 500 miles, the car would consume 5 times 4.5 gallons of gas). Alice's approach was significant to the teacher's long-term mathematical agenda because it used a proportional correspondence as a *multiplicand* to find a larger correspondence. Similar to how she built on Megan's approach, the teacher asked other students to explain Alice's way and meanwhile, she added to the graph 5x100=500 and 5x4.5=22.5 in an effort to clarify this approach.

As the class continued, the teacher capitalized on the emergence of the two approaches (i.e., Alice's *multiplicand approach* and Rashied's *putting-proportional-pieces-together approach*) to orchestrate a discussion that focused on the different ways of organizing the proportional covariation of the two quantities in question (i.e., fuel consumption and distance covered). Such discussions allowed the students' in the class to reflect on their own and others mathematical activity. This discussion was possible due to the diversity of students' reasoning that transpired in the whole-class discussion, as orchestrated by the teacher.

Key to this success in this instructional session was the bottom-up instructional approach followed by the teacher. She engaged in instruction not with a fixed plan, but with an instructional agenda that allowed her to make adjustments based on her continual assessment of how students were interpreting the ongoing classroom events. From the students' perspective, it seemed to be their contributions that became topics of whole-class conversations, and their thinking that was inscribed on the double-scaled number line. This perception oriented them to use the inscription system as they reasoned about new situations.

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ON THE INSTRUMENTATION PROCESS OF A COMPUTATIONAL TOOL

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This document analyzes the work of a group of university students in their attempts to comprehend the concept of derivative. In particular, we focus on analyzing the type of students' cognitive reorganization that they show as a result of using EXCEL as a problem solving tool. The cognitive changes developed by the students are identified through the connections they establish between systems of numeric, graphic and algebraic representations. The work of the students was oriented trough the discussion of problems that involve phenomena of the rate of change and linear approximation as fundamental ideas to the understanding the derivative concept.

Introduction

The importance of employing different technological mediums in the learning of mathematics has been sustained in the results obtained in different investigations (Santos, et al 2003), wherein the student's understanding of mathematical ideas seems to be tied to the relationships they are able to establish among different representations. Recent curriculum proposals recognize that symbolic calculators and computer software, as well as spreadsheets, can become powerful tools for students to develop numeric, graphic and symbolic representations.

In order for technology to contribute to the exploration of connections, among different representations and as a consequence to the construction of mathematics concepts for the students, the activities in which the students take part in the classroom with the help of technological mediums must be carefully designed and directed by the instructor. Thus, we document the cognitive reorganization shown by first year university students when they were asked to use EXCEL to solve a series of tasks that involved the concept of derivative. The tasks selected in this report focus on the ideas of the rate of change and linear approximation for approaching the derivative concept.

Conceptual Framework

In the process of learning mathematics student's understanding can be enhanced when working with activities that involve the use of different representations. The relationships that students establish among different representation systems can be considered as indicators of change in their cognition. One method of seeing the connections that a student establishes among different representational systems is through the examination of external representations that the subject produces in writing, in speaking or in the manipulation of symbols such as algebraic symbols. By way of internal representations the individual constructs these external representations. Between the internal and external systems of representation, the subject establishes significant relationships, which can be modified throughout the development of the learning activities.

By assuming that individual's comprehension can be exhibited in terms of expressing different representations, the activities implemented in the classroom, employing a cognitive tool such as a Excel can help students in the construction of connections among representations.

The term cognitive tool is used here "as whatever medium that helps transcend the limitations of the mind…in thought, learning and activities involved in the problem solving"

(Pea, 1987 p. 91). In order for the students to utilize Excel as a cognitive tool, he or she should develop a psychological construct. This construct is found to be determined by the actions that are realized with the use of EXCEL during the problem solving process. The interaction of the student with the tool (Excel), along with the combination of representations, knowledge and mental operations, form what is known as instrumented action (Verillion in Guin and Trouche, 1999 p. 201). In this context, we can conclude that the use of Excel as a tool in mathematics learning might be closely related to the students' understanding of the mathematical concepts that underlie the embedded procedures to represent and operate those concepts.

In order to interact with the instrument it is necessary for students to access to mathematical knowledge to understand and employ the commands properly. At the same time it is the instrument that permits the differentiation of mathematical objects from their different representations, thereby promoting better understanding of the concepts.

However, the appropriation of an instrument is not accomplished naturally for all individuals. It is the instructor who must be responsible for the design of learning activities that orient the work of the students toward investigation and exploration by way of the interaction between systems of graphic, algebraic or numerical representations. This then leads to the students' cognitive reorganization (Dorfler 1993).

In activities that incorporate Excel as a tool, exploratory work seems necessary to promote interaction among graphic constructions and numerical and algebraic calculations. The work of the students in these activities must stimulate them to compare results and observe the differences between the work with paper and pencil and that in which they use the tool (Guin & Trouche 1999).

Subject Methods and Procedures

In this study we document and analyze the work shown by university students who worked on activities oriented towards the construction of the derivative concept integrated with the use of Excel as a cognitive tool.

The use of different systems of representation permits us to approach a mathematical concept such as the derivative. Our idea was that the study of the derivative concept can be developed trough learning activities that involved infinitesimal, symbolical, definition, geometric interpretation, physical interpretation and linear approximation approaches (Thurston 1994).

The integration of all of these ideas develops a more sophisticated thought process which generates new concepts and new mathematics structures.

Tasks used in this study can be classified into two groups; a) those that relate quantities by way of the mathematical model of a contextual situation; b) others wherein the mathematical model is given in an activity to realize the interpolation by way of linear approximation.

In each activity the students have the opportunity to identify important mathematical ideas that emerge in the proposed problems: functions, dependent and independent variable, linear functions, the rate of change, the slope of a straight line and the derivative functions.

In order to document the distinct forms of understanding of the fundamental concepts surrounding the derivative, activities were selected and designed so that they could be explored by the students with different mathematical resources; utilizing different forms of representation, these being the point between the mathematical activity of the student and the incorporation of the tool (EXCEL).

The work of the students in the problems was organized in terms of problem solving phases that involve:

Understanding of the problem

Analysis of the information
Identification and exploration of the function
Identification of the mathematical ideas

The activities were given to 25 engineering students from 18 to 19 years of age who were taking a first calculus course. The activities were implemented in bi-weekly sessions of two hours each during the regular semester.

The material and equipment available to students included:

1) For the individual exploration work, the students used a scientific calculator or computer (according to the number available), paper and pencil.
2) In the group work they used scientific calculators, EXCEL (according to availability) and paper and pencil for the written reports.
3) For the discussion phase the group used the blackboard, paper and pencil.
4) The sessions were tape recorded.
5) The work with the calculator or computer was saved on files for later analysis.

During the work sessions the students had the opportunity to use a scientific calculator, Excel and/or paper and pencil. Students worked initially on individuals bases. In this stage they were asked to hand in a written report of their ideas and experiences with the task, explaining and justifying their results. A second stage consisted of work in groups of three in which they exchanged ideas in order to come to an agreement for the solution of the problem. They then gave a written report.

The group work included, in its final stage, a discussion of the related mathematical ideas. The discussion was moderated by one of the members of the group and directed by the instructor.

The problem of the Towers, (Taken from Fey, 1995, p. 73)

The problem provides a formula to calculate the largest distance that can be seen from the top of a building: if h is the height in meters from the point of observation, that is, the distance of visibility in kilometers as a function of the height is obtained from the rule: $s(h) = 3.532\sqrt{h}$. This, students were asked to:

a) Calculate the distance of visibility that can be achieved from the top of different buildings. (See Table 1)

Structure	Locality	Height, (m)	Distance of Visibility, (km)
CN Tower	Toronto Canadá	555	
Sears Tower	Chicago, USA	443	
World Trade Center, N	New York, USA	419	
World Trade Center, S	New York, USA	419	
Empire State Building	New York, USA	381	
Amoco Building	Chicago, USA	346	
John Hancock Building	Chicago, USA	344	
Centrepoint Tower	Sydney, Australia	325	
Texas Commerce Tower	Houston, USA	305	
Allied Bank Building	Houston, USA	300	
Eiffel Tower	Paris, France	300	

Table 1. Buildings and their corresponding height.

b) Make a table with the relationship of the heights to the values of s(h).

c) Figure the interpolation for the determination of the distance of visibility taken from a graph of the function s(h).

d) Determine the height of the building in order to be able to see *x* kilometers.

e) Analyze the change in distance of visibility when the height is increased.

f) Describe the tendency of the changes of the incise d).

Presentation of Results

The following describes the work exhibited by a small group of students (Alexis, Iris and Gina) in the towers problem. The interaction of the group with the problem, the role of the computer and the instructor is documented. Students worked in this task during three sessions of two hours each.

Understanding the problem

In the solution of the problem the students had to identify that the heights of the buildings in Table 1 in meters and obtain the distance of visibility with the formula $s(h) = 3.532\sqrt{h}$ in kilometers. The constant 3.532 included the conversion factor (meters to kilometers).

During the first session the questions that directed students' interaction with the end of generating a common context for the problem were:

How is the answer verified when the formula is in meters or in kilometers?

Must a conversion of units be realized? That is do height and visibility needs to have same dimensions? Do small variations in the height produce significant changes in the distance of visibility?

After a discussion, the students came to an agreement on what they had to consider as the distance of visibility and that given the scale which was employed, a variation of centimeters in the height did not substantially modify this distance of visibility.

During the interaction of the students when discussing their individual solutions, they were confused with the units applied for the height and the distance of visibility. The statement of the problem indicated the height in meters and the distance of visibility in kilometers. The discussion turned about the need for converting the height of the buildings given from meters to kilometers, as is seen in the following:

Alexis: I, for one, think that the question tells you that on a clear day at the height that you are on the tower you can see up to 50 km. Therefore, I see it as a rectangular triangle. In other words, you have a height but maybe your distance is much farther, depending on the height from which you see the distance and that which you can come to see.

Alexis: I think that you have to convert the height into meters, if you use the formula $s(h) = 3.532\sqrt{h}$. When you apply the formula for the first height of 555 meters this will give you a value of 83.20 meters (emphasizes the word meters). And when you convert them you use the rule of three.

Gina: But it would be illogical that we were in a building so tall and if we say that we can only see 83.20 meters, or in other words, it is a very short distance.

These dialogues show that Alexis thinks the distance of visibility and the height form a rectangular triangle (this was mentioned by Alexis), considering how it can be seen in their later dialogues that the variation in the distance of visibility is linear. In thinking of a rectangular triangle Alexis does not take into account the curvature of the Earth and the given mathematical model $s(h) = 3.532\sqrt{h}$.

However, after Gina's participation in the above dialogue, Iris and Alexis revised their ideas and found, without formal verification, that the results obtained using a change in units did not make sense in the context of the problem.

Alexis: Then this formula gives the kilometers you, good.

It is through the dialogues of the students that we find that Alexis initially mentioned that all the factors influence the distance of visibility s(h), including the height of the person. Iris intervened with the comment that it maybe because of this they were working the distance s(h) in kilometers, because this way the variation in the height of the person was not significant.

After this discussion they decided to use the formula without making the unit changes.

Analysis of the information given with paper and pencil

The written registers and the audio tapes demonstrated that when they employed the relation s(h), in general they immediately substituted the values of *h* and directly obtained the values of s(h) for each line of Table 1, as can be seen in this dialogue:

Instructor: What did you do to fill out this column?

Iris: Well, the same, I substituted each one with values using the same formula.

Instructor: And then?

Iris: From the table I took the values from 0 to 100 and made the same formula; from 0 is 0...

For this work the students used the given formula and a scientific calculator to calculate the distances of visibility from different heights. They elaborated a table to register the values they found, however, the calculations were arranged in the form shown in Figure 1 and did not facilitate the analysis of the general behavior of the function.

In order to analysis the data and the way they incorporate the tool EXCEL in the development of the problem it was suggested by the instructor that they made a table with the help of Excel.

Gina's work with paper and pencil is shown in the following Figure 1:

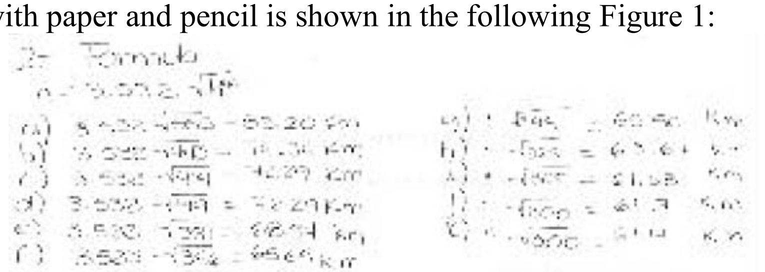

Figure 1. Gina's response to question a.

The first approach of the students with the EXCEL was to work with it as a table to concentrate on the results shown in the above Figure. They showed that Excel can be used to manage numerical functions and columns through the corresponding formula.

The elaboration of a table using EXCEL was an opportunity for the students to think of a more convenient way for ordering the data, identifying the independent variable and the values that must be included in order to analyze the behavior of the model. They talked about ordering the data from the largest to the smallest, from smallest to largest, by intervals, as Gina did (Figure 2), or with the values of the table given in the activity.

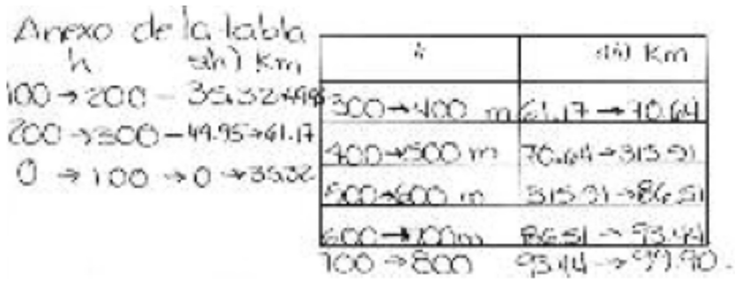

Figure 2. Gina’s table.

Analysis of the information from a table and a graph

Students were asked to make the corresponding graph and respond the questions c and d. Figure 3 represents the work shown by the small group using EXCEL.

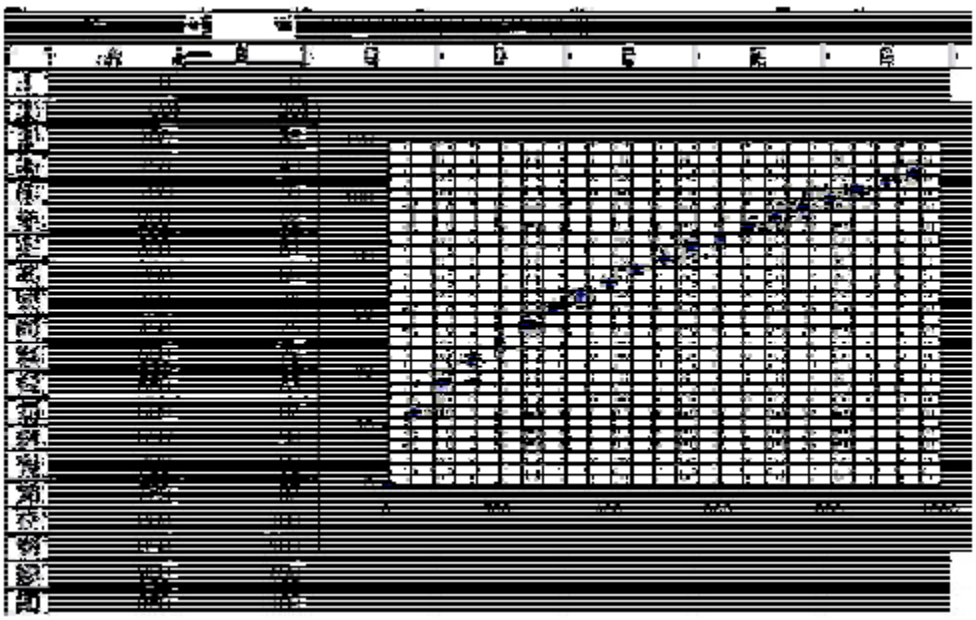

Figure 3. Visibility versus height.

The students observed that in order to calculate the value of the distance of visibility of the question c, it was necessary to realize what they called a refinement of the selected intervals of the height. In Figure 4 this refinement is shown.

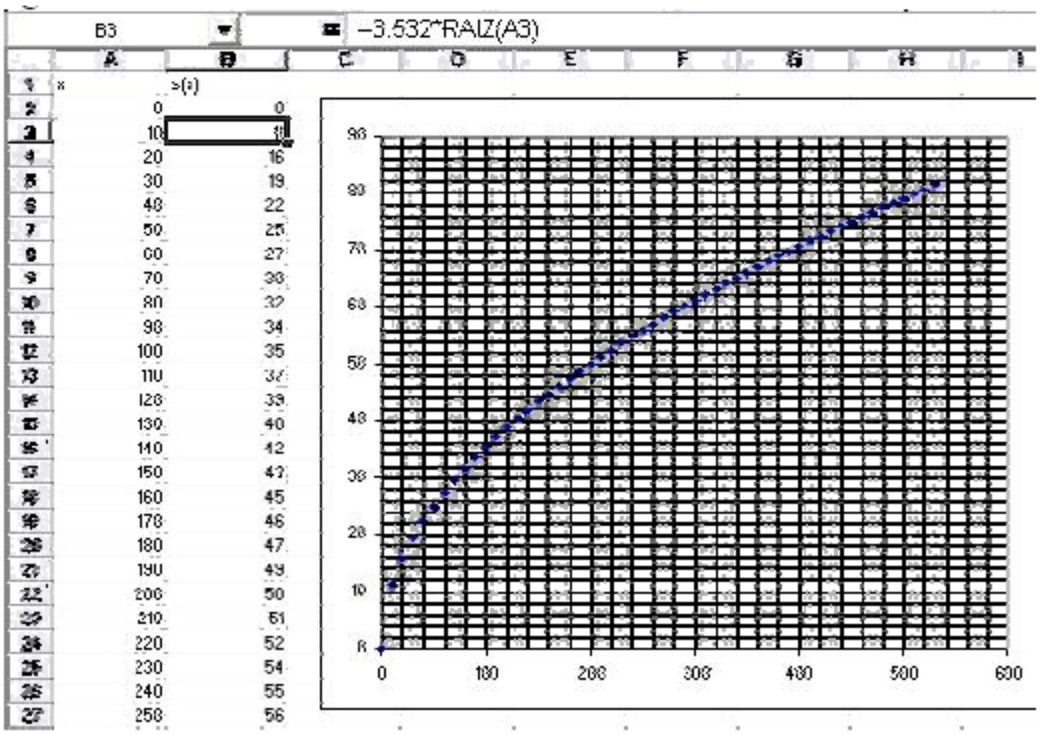

Figure 4. Refinement in the increases of the heights

The student changed the intervals in the heights of the buildings of 50 to 10 meters

This permitted them to consider data interpolation in order to respond to questions c and d.

Identification phase and exploration of the function

The student's work described previously shows that they employed the functions as relations entrance-exit which impeded the realization of a global analysis of the function. Alexis indicated that by increasing the height in intervals of 100 meters, the distance of visibility was increased. Iris observed those of the table elaborated with EXCEL, and pointed out that the distance of visibility did not increase proportionally, as Alexis had said in the phase of understanding the problem. Alexis considered that the function followed a linear behavior and it is from the numerical and graphical exploration with EXCEL that the contradiction between the data obtained in the table elaborated in Excel and the mental model that they had preconceived is taken into account. The analysis that the students carried out with the data obtained in EXCEL was made from the calculation of the average rate of the function, as shown in the following dialogues:

Iris: It increases, but is diminishing because in the last interval of 100 meters it increases 10.Why does it decrease?

Alexis: Listen, if here there are 10 and then if we add from 400 to 500 it is 9. But why? Wouldn't you think that from a higher point the distance of visibility would be greater?

After exploring again the table and the graph elaborated in Excel they concluded that the distance of visibility increases with the increase in height but the function does not follow a linear behavior.

Conclusions

An important research question in this study was "What is the instrumentation process followed by students to incorporate a technological tool in their problem solving approaches?" Results in this study indicate that students' first approaches to the task, based on paper and pencil work, involved the use of isolated knowledge and basically focused their attention to the calculations required in the problem; however, the use of the tool, later, helped them analyze the problem globally and recognize that the behavior of the representative function was different from what they had assumed initially. In this process, it was evident that students went through a cognitive reorganization in which the use of the tool played a fundamental role to not only to visualize the problem as a whole, but also to analyze the same task from diverse perspectives including the examination o cases not included in the statement of the task.

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DECODING COLLABORATIVE LEARNING IN A WIRELESS HANDHELD COMPUTING ENVIRONMENT

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This paper reports on data from a pilot study exploring the use of wireless handheld computers to support middle school students' collaborative learning of mathematical functions. The wireless computer network developed for this study blends multiple linked representations of mathematical functions with role-based student group work to facilitate solving complex mathematics problems. Drawing on an analysis of daily video of student work with these representations on handhelds over a three-week period, I provide a sketch of one group's learning process.

Introduction

This paper reports on data from a pilot study exploring the use of wireless handheld computers to support middle school students' collaborative learning of mathematical functions. The wireless computer network developed for this study blends multiple linked representations of mathematical functions with role-based student group work to facilitate solving complex mathematics problems. The distributed nature of problem-solving work in this networked environment blurs the boundaries between the learning that individual students accomplish as they solve such problems, and the learning accomplished by the student group as a whole. Consequently, my research attempts to account for the ways learning might be observable in the performances of both individual students and small groups working with this handheld computer network. For the purposes of this paper, I will focus my analysis at the group level to examine what these students might be learning about using different representations of functions to solve increasingly complex problems, and how that learning unfolds over the course of the study.

Conceptualizing Distributed Learning

The style of this research is decidedly interventionist, participating in a paradigm of design-based research that endeavors to bridge the research-practice divide by melding carefully planned instructional environments with emergent theories of learning (The Design-Based Research Collective, 2003). Classroom technologies such as handheld computers can only be meaningfully understood in an ecological sense—as thoroughly embedded in patterns of classroom activity and discourse, and deeply intertwined with instructional materials and with mathematical problems and practices (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003). Indeed, educational researchers and technology developers are not the only agents in the conception and deployment of a learning tool; teachers and students also work together to shape the instructional import of classroom computer technologies (Mehan, 1989). To that end, this research seeks to establish a wireless handheld computing classroom environment that takes advantage of some of the most promising features of those devices in order to understand something of the mathematics learning ecology that might emerge.

Two such promising features of a wireless handheld network are particularly salient in the design for this study. First, like its handheld cousin the graphing calculator, a wireless handheld computer can rapidly propagate mathematical objects like functions across multiple representational modes, including algebraic symbols, graphs, and tables. Second, and more provocatively, a network of wireless handhelds can link those representations across multiple devices, so that an equation edited on one device produces a corresponding change in the graph on another device. The software developed for this study links an array of several such representations together across all the handhelds in a server-defined group of four students. These groups of students work together on a curriculum unit that introduces the topic of algebraic functions through the applied context of cryptography. With the help of the handhelds, students use simple polynomial functions to encode and decode text messages. Decoding those messages, in particular, requires iteratively consulting at least two and usually more representations in an open-ended problem-solving process that often proves to be quite complex.

In order to conceptualize the collaborative and computer-mediated code-breaking work of these student groups, I adopt the perspective that cognition is a distributed phenomenon, often extending beyond the mind of an individual to comprise the coordinated efforts of a network of people and artifacts (Pea, 1993). Several recent studies of mathematics teaching and learning in tool or technology-rich environments have demonstrated the utility of focusing their analyses not on individual actors, namely teachers or students, but rather on emergent phenomena such as "classroom mathematical practices" (Cobb, 2002), patterns of classroom activity (Stevens, 2000), or "mathematical thinking practices" in which students and teachers jointly engage (Hall & Rubin, 1998). The shared focus of this work on the emergent and social nature of mathematical thinking and learning informs my efforts in this paper to develop an account of small groups of students collaborating with and through computer tools.

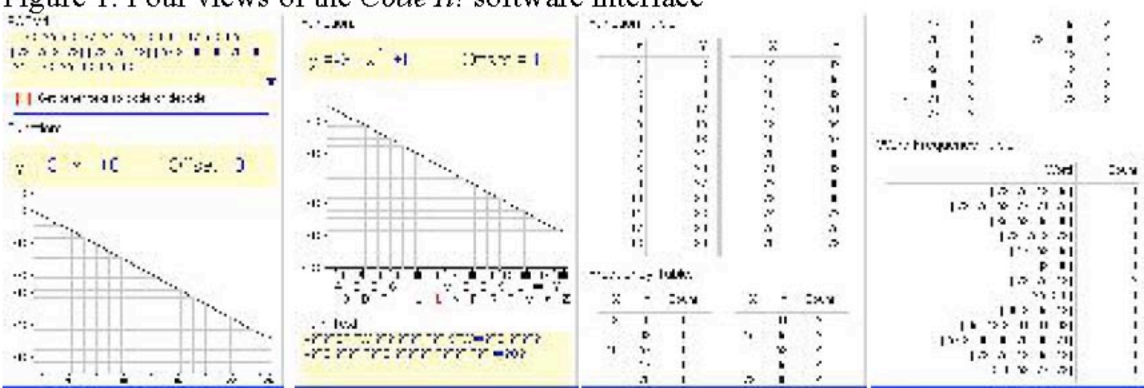
In the context of this study, the relevant artifacts are not just physical tools like handheld computers, but also the various function representations they display. Hutchins' (1995) account of the joint navigational efforts of groups of people and tools guiding large ships characterizes that computational work as the propagation of representational states across an array of media linked to multiple actors. Similarly, breaking codes in this handheld network depends on the coordinated examination of multiple representations of an encoded message that cannot always be simultaneously viewed on a single device. As they work to decode messages, student groups deploy their combined resources in particular configurations—for instance, a student viewing and editing the equation in the software might coordinate with a student observing changes in a graph of that equation. When such a configuration leads to the successful breaking of a code, the students have accomplished an act of problem solving that they would likely not have achieved without one another, let alone without the handheld representations.

A theory of distributed cognition would be incomplete without an account of the potential changes in that cognition that might comprise distributed learning. Cobb (2002) has proposed that tracking "the emergence of new practices as reorganizations of previously established practices" (p. 189) in classrooms where students are jointly engaged in reasoning with tools and inscriptions amounts to tracking learning. In a similar fashion, I characterize learning as emergent moments in the course of collaborative work wherein distributions of students and representations are reorganized into new and more efficient arrays. In other words, if a once-stable pattern of students working with an equation and a graph were to shift into a new pattern including the productive interaction of a table, the stabilization of that new pattern would constitute learning for the group. In the analysis that follows, I seek to identify those emergent patterns of representational use in decoding practice that might characterize the learning trajectories of a collaborative student group.

Methodology

In conducting this pilot study, I worked with a group of researchers and teachers to develop both software and curricular materials for a middle school mathematics unit, and then to pilot the unit in a summer school setting with 120 students in four classes. The students in the study represented a wide range of prior achievement levels in mathematics, and the groups were organized to reflect that range. Modifying principles of complex instruction for heterogeneous groups (Cohen, 1986, Lotan, 1997) to fit our wireless handheld learning environment, we assigned each student in each small group not only a role for dividing labor on various collective tasks, but also responsibility for viewing a different representational mode during decoding activities. Thus, one student might be responsible for recording notes and for reading the frequency tables, another for managing group equipment and for monitoring the function table, and so on. Figure 1 shows how these representations appear in four different views of the handheld software interface. These four views are essentially stacked on top of one another in the actual handheld software interface, and students can freely scroll their viewing window up and down between these different views, though their assigned roles do correspond to the different views as indicated. Of course, the students did not always remain at their assigned representational stations, and both reorganizations and stabilizations in their patterns of distributed representational use often came about precisely because they disregarded these assignments.

Figure 1: Four views of the *Code It!* software interface



Publisher's View | Presenter's View | Equipment Manager's View | Recorder's View

This paper draws on a detailed analysis of daily video of a group of four boys engaged in code-breaking activities with the handheld system. The video data has been organized into a series of "decoding events"; these events commence when the group downloads a new encoded message from the classroom server and begins trying to break it, and conclude when the group solves the code, abandons it in favor of another one, or stops working at the end of the class period. The events range from less than two to more than thirty minutes in length, and over three weeks of decoding activities with the handhelds the group engaged in seventeen such events. As part of my larger project I have transcribed all of these decoding events and coded each student utterance according to four analytic categories. The coding category of primary relevance for this paper is intended to identify which representation(s), if any, are specified by each utterance. The analytic codes that correspond to these representations are summarized in Table 1.

Table 1: Representation Codes

Coded Text	A series of numbers, with brackets to demarcate words, comprising the encoded letters of an encrypted message.
Equation	A “candidate” algebraic function entered by the group’s publisher in an attempt to match the function used to encode the message.
Offset	Adjusts the mapping between the letters A through Z and the integers 1 through 26. An *offset* of zero maps A to 1; an offset of 1 maps A to 2.
Graph	Displays a *graph* of the candidate function in a window fit to the range of values in the coded message.
Plaintext	Shows how the candidate function would decode the encrypted message, displaying a letter when the inverse of a candidate function maps a number in the coded text to an integer between 1 and 26, and a question mark for numbers mapped to anything else.
Inverse	Provides a tabular view of the *inverse* mapping shown in the plaintext.
Function Table	Provides a tabular view of the candidate function mapping of the integers 1 through 26.
Frequency	Shows how often each numerical letter appears in the encrypted message.
Word Freq	Shows how often each bracketed sequence of numerical letters appears in the encrypted message.

Importantly for the analysis that follows, only eight of these nine representations are displayed in Figure 1; the “Inverse” code corresponds to an inverse function table, which was included in an earlier version the software in place of the function table. Midway through the study, a combination of a programming bug associated with the table and confusion on the part of students prompted a redesign in which the inverse function table was replaced with the function table.

Sometimes these representations were identified explicitly in students’ utterances, so that when, for example, a group publisher asks the student viewing the word frequency table if there are any one-letter words in the encoded message, I coded this question as an instance in which the word frequency table had been specified. Similarly, if the group recorder or another student responded to this question with an observation based on the word frequency table, I likewise coded it as such. In other instances I relied on inferences from context, in combination with visual inspection of the video to determine whether the speaker scrolled up or down from their previous position, and with server data about the codes the group was working on. Importantly, I could not always reach a definitive conclusion about what representation, if any, had been specified, and those utterances were not coded.

Patterns of Representational Use

Table 2 provides a window into the evolution of the group’s decoding practice over the course of the study. Because these numbers are culled only from those utterances and actions among group members that clearly indicate use of or reference to a specific representation, the picture they paint is partial at best. Nonetheless, they do tell a story, illustrating the cycles through which the group explored and experimented with new representations and refined its strategies for utilizing those representations as they worked together to break codes. Even as the group developed increasingly efficient ways to decode messages, the codes gradually became more complex, in some key cases leading to struggles that forced the group to explore the array of available representations anew and to generate and refine a new set of strategies.

Table 2: Representation-Specifying Utterances—Group A

Decoding Event	14a	14b	14c	14d	15a	17a	17b	17c	21a	23a	23b	23c	24a	24b	24c	24d	30a
Equation	7	2	5	16	5	9	3	3	22	3	2	1	2	18	6	25	11
Plaintext	4			15	3				4				5			6	13
Coded Text		2		1				2	12	2	6	1			4	4	
Inverse		5	1	41	6				2								
Frequency		3	2	14	3									4	3	8	27
Graph				13		18	1		1	5	6	1		1	1	7	
Offset				3					1	6	2	2	5			5	8
Word Freq				2													
Function Table									1						8	3	17

Each decoding event in Table 2 is identified by a number corresponding to the date in July on which the event occurred, and a letter indicating where each given event fell in the sequence of codes worked on by the group that day. The group's work over these 17 decoding events and three weeks of classes appears to cluster into four fairly distinct patterns of representational use, as indicated by the shading. They appear initially to have favored working with the inverse function table and the frequency table. They soon transitioned into central use of the graph in addition to and then in place of these tables. They next began referring to the graph in regular combination with the coded text and the offset. Finally, they returned to regular use of the frequency table, this time in consistent combination with the function table. Those four patterns are bounded by certain critical events through which the group appears to have learned to break codes in new ways. The discussion that follows combines an analysis of representational use with an account of the decoding events drawn from classroom and video observation to trace these unfolding patterns in greater detail.

On July 14th, the first day they began downloading codes to break, the group referred to a gradually expanding array of representations in the process of trying to decode messages. In examining their first code, 14a, they discussed only the candidate equation and the plaintext message. As they proceeded to events 14b and 14c, they not only made reference to the coded text, which was actually the very first representation visible at the top of the initial log-in screen, but also scrolled down to investigate the inverse function and character frequency tables. Code 14c was the first they successfully broke, and they completed it with relative ease, as indicated by the low total number (eight) of representation-specifying utterances they made before solving it.

By contrast, code 14d proved quite difficult for the boys, and in fact they never solved it. In the process of trying, however, they experimented with every representation available to them (the function table was not added to the decoding interface in place of the inverse until the following week), engaging in particularly extensive discussions of the inverse function table, the frequency table, and the graph. While the frequency table, the offset, and the word frequency table apparently bore little code-breaking fruit and were all virtually abandoned by the group for the next several days, the inverse function table and the graph appeared to take at least temporary hold. In event 15a, a code they successfully broke, the group referenced the inverse function table six times, more than any other representation in this relatively short (58 total utterances, 17 of which specified representations) round of decoding. Even more strikingly, a few days later in event 17a, they referred to the graph in 18 out of 27 representation-specifying turns. Event 17a featured another successfully broken code, and the group appears to have hit on a successful formula involving little more than the graph and the equation, as they dispatched 17b and 17c each in a matter of minutes, and with very little explicit discussion of representations (just four and five representation-specifying turns, respectively).

July 21st occasioned a significant departure from this trend. Event 21a involved a coded message designed by the teacher and entitled "challenge." Indeed, the code lived up to its name, and in their efforts to break it, the group examined seven of the eight available representations. They managed to break it only after a hint from the teacher that the code contained an offset. Prior to this day, students had been working under instructions from the teacher that they were not to write codes involving offsets because they had not yet become sufficiently skilled at breaking codes without offsets. With this restriction removed, the offset became a standard component of subsequent codes, and drew explicit references in most (six out of eight) of the remaining events during the unit. Apart from this addition, the group returned to what had become "normal" practice during the next several events, consistently and exclusively focusing their discourse on the equation, the coded text, the graph and the offset in events 23a, b, and c.

July 24th marks the final major overhaul in standard practice. Prior to the decoding events of that day, the teacher and a researcher together demonstrated an approach to breaking codes that drew extensively on the frequency and function tables. At the same time, the level of complexity of the codes was elevated another notch, as codes featuring not only offsets but also quadratic and cubic functions with double-digit coefficients now became commonplace. While code 24a was only moderately difficult in this regard and the group solved it without utilizing the newly highlighted strategy and representations, code 24b proved sufficiently challenging that the group gave up on it, though not before considering the frequency table in four utterances. Likewise, in event 24c they begin to discuss the function table in detail, referring to it in eight of their 22 representation-specifying utterances before they abandoned their efforts on this code as well. In event 24d they conducted an extensive campaign, expanding their efforts to once again consider seven of the eight available representations before they finally broke the code with the help of a researcher.

By their final day of breaking codes, the group appeared to be settling into a newly standard set of practices that concentrated on the frequency and function tables. With 27 and 17 respective utterances, the boys specified these representations extensively in successfully breaking a complex code. Again, the additional representations that they referred to when they were struggling with codes 24c and 24d were no longer among those they explicitly referenced.

Conclusions

This succession of decoding events suggest a process whereby a group of students first develops expertise in solving problems according to a particular way of configuring representations in a distributed network, then faces a problem of sufficient complexity to render the particular configuration ineffective, and is ultimately forced to develop new configurations of distributed representations and activity. Moreover, these cycles of rediscovery and refinement and their correspondingly shifting networks of representation and collaboration offer glimpses into the process through which the group learns to solve problems in this networked computer environment. More thoroughly understanding this group learning process will require looking within these individual decoding events for a more detailed, nuanced picture. Likewise, making sense of the mathematics these students are learning along the way will require looking within groups to examine the collaborative processes through which they make decisions, and individual students' differential forms of participation in those processes. To that end, my efforts in further work will be directed at resolving this group narrative with an account centered on the distinct experiences of individuals within the group, and similarly detailed analysis of a second student group will also shed comparative light on this learning process.

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UNDERSTANDING AND DEVELOPMENT OF BASIC GEOMETRIC PROPERTIES: INNOVATION IN A LOW-LEVEL MATHEMATICS CLASSROOM

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This contribution presents some results of an exploratory study into the comprehension and development of basic notions of geometry by a group of 28 ninth grade students in a low socioeconomic, urban school in Mexico City. The exploration was based in the development of a series of activities in which the students handled dynamic software of geometry (DSG). The study focus on the students' language when referring to the actions they performed with the DSG. Their descriptions give evidence of a transition toward appropriation of the contents and forms of manipulating the DSG (Verillon and Rabardel, 1995). Geller's (2004) reflections on the use of teaching materials and their connection with the notion of mathematical literacy suggest an explanation about the low impact of introducing the new form of group work (task guides for the activity, using the DSG, articulating the result).

Theoretical Framework

The introduction of technological tools into the learning of math and science has led to developing special theoretical frameworks for research (see Verillon and Rabardel, 1995; Mariotti, 2002). Referring to a tool as an artifact, Verillon and Rabardel pointed out that an instrument results from the establishment, by the subject, of an instrumental relation with an artifact, whether material or not, whether produced by others or by himself. Yet the mere appropriation of a piece of content, a template, or pattern of use only defines the instrumentalization of the artifact. Only a consecutive construction of utilization schemata associated to the artifact in use will permit the establishment of a psychological relationship, an instrumentation or instrumental relationship between the subject and the artifact, along with control over the necessary limitations during the activity. Internalization of the instrument is possible at a later stage (Mariotti 2002: 707), which potentializes individual problem-solving with the instrument.

Besides critiquing the introduction of innovations into the classroom, Geller (2004) has recently identified fundamental tensions between the development of new teaching materials and student cognitive styles. Geller's reflection introduces the concept of mathematical literacy to show the crucial relationship between Math teaching material and the manner in which it is deployed in the classroom.

Finally, as Fennema and Romberg (1999) have noted, the value of oral or written articulation or description of the activities performed, as was required in each of these sessions, resides in the fact that articulation is one of the mental activities that foster mathematical and scientific comprehension.

Objective, Methodology and Results

The exploratory study we report here was carried out in a public ninth grade classroom in Mexico City, located in a low socioeconomic district. The results reported herein are part of a wider project researching the potential technology may have in the development of mathematical communication which could establish baseline standards for justification or proof. Our first conclusions have to do with learning basic geometrical properties through dynamic software of geometry (DSG).

Twenty-eight students, 14 and 15 years of age, participated in four 50 minute sessions in March and April 2003. The room was equipped with eleven computers. The students used the DSG software in each session, following the instructions in the work guides we had prepared. The guides were distributed in the classes and the students were asked to solve the questions or follow the instructions in them.

The corpus we obtained consists of all the worksheets and some video recordings of the students' post hoc explanations.

The following are transcriptions of the student worksheets. Some of the results obtained in this exploration are commented on. What is interesting to note is the language they use when referring to the actions they performed with the DSG[i]. Particular emphasis is placed on the ways they operate the DSG in task resolution.

Session 1

The first DSG task (on March 25, 2003) centered on exploring the menu. Suggestions were given to draw points, lines, and triangles. Revealing student final reports are transcribed here.

Octavio and Priscila:	We can move the vertices, that is, that we have to try to make the most precise triangle possible. And the mathematical properties are met making the triangle and obtaining its angle.
Francisco & Juan N:	We found out how to work the DSG ... and we formed a triangle. Then we put the pointer and pulled a vertex wherever we wanted to and it changed shape, or we can move the [whole] triangle.
Juan V. and Ulises:	1) Recognize or learn to use the PC. 2) Learn to use the DSG. 3) We can make geometric figures with the DSG. 4) We made a triangle. 5) Moving the vertices we can make the figure's shape. 6) We made a right triangle. We obtained the angles, we wanted the computer to figure it for us, it gave us the figure pressing the Measurements window. And we click it in Angle so it would give us the answers, and it gave us the answers clicking on each vertex, and 90°, 32.1°, 57.9°came out.
Iván and Juana:	1) We recognized the items on the menu to be able to see what each one was for. 2) Then we made a triangle, and with vertices we could move it around for different shapes. 3) With the lines we could move the figure wherever. 4) One feature of the DSG is that we can move the geometric objects for to work better. 5) We made a right triangle, measured the angles. We marked the three vertices and the measurement appeared.

Session 2

In the second session, on March 28, 2003, the students were given a series of tasks or problems of construction, for example, "Construct a right triangle in such a way that it remains a right triangle while dragging any of its three vertices. After completing your construction, write up a report describing the triangle you built."

This task is fundamental to progress in the knowledge of and familiarity with the mathematic and educational intent in using DSG. Moreover, it directs the learner toward intervention into the geometric property of perpendicularity, which may be one of the most recurrent topics among all actors, both teachers and students, in public schooling.

This activity also centers on a geometric construction task where the learner must respect established restrictions in the task instructions. These restrictions belong to two distinct fields: one of geometry, where one of the triangle's angles must be 90°, and the other concerning to handle the computerized learning environment, where the property of rightness might be kept when dragging any vertex.

Students are not really expected to solve the construction task successfully. It would be difficult for anyone after just one 50 minute exploratory session of the DSG menu. The purpose for this type of activity is to provide a context in which to discuss geometric construction strategies and to highlight one of the most important properties of DSG use, that being that correct construction from a mathematical standpoint will permit the figure to retain its overall geometric properties while dragging construction elements.

In sum, the students may resolve the task acceptably only when they apply a mathematically correct construction of right triangles or perpendicular segments.

Going deeper into the actual implementation of this task, it should be noted that in every instance the instructor must eventually intervene to suggest that students bring in a mathematical construction of perpendicular segments. If the students had known how to build it, they would have said so at some moment. When the construction process is unknown, as happened in every case, the instructor is the person who explains and carries the task to completion. Yet in the next session, (see below in Session 3,) the students will be obligated to draw perpendicular segments and expected to recognize that property at the conclusion of sequence 3. We will see how the task guide for session 3 provides continuity with geometric constructions and DSG exploratory tasks.

Before presenting sequence 3, first let us comment on the transcription of learner performance in session 2. These are some entries in students' written reports at the conclusion of the second session.

Francisco & Juan N:	No matter how we move the vertices, the angle will always measure 180°, the sum of the three angles, no matter how they change.
Iván and Juana:	Pulling the vertices, it's still a triangle.
Juan V. and Ulises:	When moving the angles, a right triangle is there, and the sum of its angles gave 180°.
Octavio and Priscila:	First we joined up points so that adding its vertices it would add up to 180°and when dragging any of its three vertices [we saw] it still added up to the same total. And we used Calculate to do the addition.

Commentary: In general, their trials at building right triangles in session 2 were accomplished by "feeling their way around." They first built just any triangle, and then pulled at the vertices and measured the angles looking to see whether any of the angles measured 90°. It is interesting to see how the predominant geometric property reported in their written reports does not correspond to the one requested (they wrote that the sum of triangle's angles is 180°). In addition, notice that their constructions do not hold up to the dragging test, that is, dragging any of the vertices would mean they are no longer right triangles.

Sessions 3 and 4

The activity guide for sessions 3 and 4 were designed to continue the geometric construction activities and DSG exploratory tasks, as shown below. The first step was to familiarize the students with *Compass*, one of the DSG's basic construction tools. The learners were guided to construct an equilateral triangle from a given segment. Their goal was to obtain a figure the same as this one.

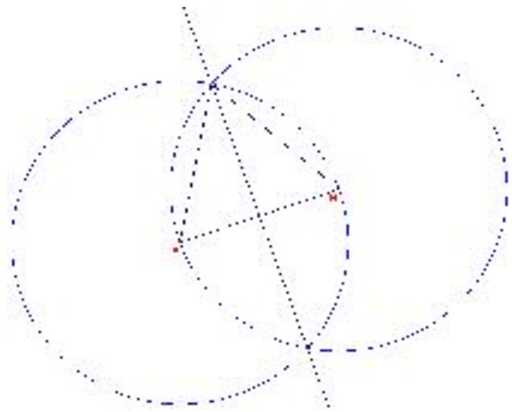

Figure 1

Notice how the task guide included all the steps on how to obtain Figure 1. Then they were prompted to respond to these prompts.

a) What are the triangle's properties?
b) Move vertices A and B. What do you see?
c) Draw the straight line that goes through both points of intersection of the two circumferences.

This line has a geometric property relative to points A and B. What is this geometric property?

At the end of the sequence 3[ii], the students were also prompted to write a report answering some questions based on Figure 2: How would you describe the figure obtained? What elements are in the figure, and how are they related to each other?

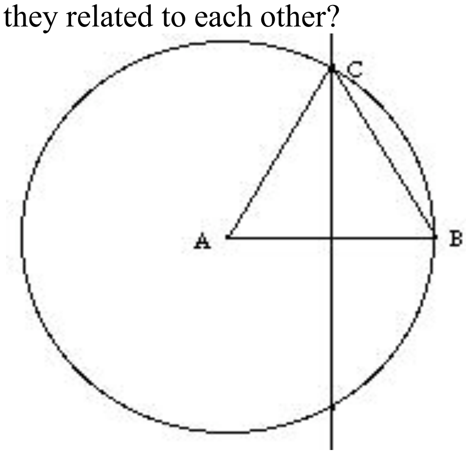

Figure 2

The student reports are reproduced here.

Octavio and Priscila:	It's still an equilateral triangle, and the circle passes through sides C and B of 60°.
Iván and Juana:	1) The parts in Figure 1 are a circle, an equilateral triangle, and a perpendicular line, and points A, B, and C. 2) On point C and B they join at the circumference. 3) The perpendicular is the equilateral triangle's axis of symmetry and it has the same distance

from point A and point B and C is further away.

Francisco & Juan N: 1) We could arrive at the second figure from the first, hiding the second circle. 2) [We can see] A circle, a triangle, and they are related by mean of the vertices.

Commentary: The reason for asking about the elements in the figure was to detect recognition of perpendicularity between the segments constructed, either as an important result or simply a known product of the geometric constructions already made in the sequence of activities. Note how some student answers already have an incipient mode of referring to the objects and geometric objects, as in the case of Iván and Juana. Nonetheless, most of the students tried to use mathematical notations presented in the guide — although in a rather confused manner. For example, they only used one of the points to refer to a side, or only side to refer to an angle, without mentioning the perpendicular relationship between the segments.

In summary, the majority of the students were unable to adequately name the vertices, sides, and angles of a triangle. Only some of the pairs reported the perpendicular line in the Figure that was a result of their construction.

It is likely that these learner descriptions give evidence of a transition toward appropriation of the contents and forms of manipulating the DSG (Verillon and Rabardel, 1995). Overall, however, the learners are still far from an instrumental use of the DSG in obtaining more significant mathematical results, as in an explanation or proof of possible geometric properties. This appears to echo the criticism Geller (2004) has of introducing classroom innovation, as well as to agree that the application of the mathematical literacy concept clarifies the relationship between the use of a teaching material and how it is deployed in the classroom. Geller's reflections suggest an explanation for why the impact of introducing the new form of group work (task guides for the activity, using the DSG, articulating the result) was so small when using technology to develop math communication and standardizing student explanations and validation in the classroom.

Some Preliminary Conclusions

The interest for this exploration is two fold. One issue has to do with the students' attainment at the end of their basic education. Student responses give reasonable evidence of the state of teaching Geometry in Mexican public schools. This 9th grade (*tercero de secundaria* in Mexican Spanish), is the final year of basic education in Mexico, unlike the U.S. *ninth grade* which represents the entry into high school and the opening of a new round of studies. So our results are truly worrying because at the conclusion of their basic school years, the majority was unable to articulate almost any basic geometric properties, as in the case of perpendicular segments. Mexican students are demonstrating that they have only learned to recognize some geometric figures without noticing the possible restrictions for constructing or controlling the drawing.

The other matter this research looked into was how students were able to progress in building the minimal units of mathematical language necessary to refer the results from their geometric basic constructions. This may be due to transition through an incipient stage of instrumentation in this task, and still toward to appropriate the contents and forms of handling the DSG (see Verillon and Rabardel, 1995).

Finally, one possible explanation for so low impact introducing innovation in the classroom is suggested from Geller's (2004) reflections on the use of teaching materials and their connection with mathematical literacy of the population studied. Geller's contribution point up the possible impact technology has in the development of mathematical communication, and the latter's contribution toward standardizing explanation and proof in Math class.

Acknowledgement

The research presented here was partly supported by the National Council of Science and Technology (CONACyT) of Mexico # 38432-S.

Endnotes

[i] Translator: Clumsy writing in the transcriptions reflects the originals.

[ii] Here you have a copy of the complete sequence 3 of activities (sessions 3 and 4):

Step 1

Turn you computer on. Select the *Cabri II* software icon. This time we're going to explore the *Compass* construction tool.

a) Draw a segment. To do this, choose *Segment* from one of *Cabri*'s toolboxes, and double-click on the screen.

b) Once you've drawn your segment, label its endpoints with the names A and B. To do this, choose *Label* from one of *Cabri*'s toolboxes, come up close to the segment's endpoints and click over each tip. Then click on the first one, (i.e. the left end,) and you will see a small square appear, using the keyboard support the Caps key and the A key. You'll see that this end of the segment is tagged with the letter A. Proceed in the same fashion to tag the segment's right end with the letter B.

Step 2

Construct an equilateral triangle using the segment you drew. Use the segment you already have and the *Compass* tool to obtain this figure.

---Insert Figure 1---

a) To do this, follow these steps. Select Compass from one of *Cabri*'s toolboxes, click on the A end of your segment and then click showing the edge of your segment. You'll see how you obtain one of the circles you want to draw. Follow the same steps to draw the other circle, except that this time you'll click on endpoint B and again show the edge of the segment AB.

b) Select *Point* from one of *Cabri'*s toolboxes, and indicate the point of intersection between the two circles. Tag it with letter C. Now select *Triangle* and click on A, B, and C. Answer: What are this triangle's properties?

c) Move vertices A and B. What do you see?

d) Draw the straight line that goes through both points of intersection of the two circumferences. This line has a geometric property relative to the points A and B. Answer: What is this geometric property?

Step 3

Once you've drawn your equilateral triangle, hide the two circles that helped you through the construction process. Use *Cabri*'s *Hide/Show* tool for this. After you have selected *Hide/Show*, click on each of these auxiliary circles.

a) Immediately go to use *Compass* and construct a figure like the one given below. To conclude, write a report that answers these questions. How would you describe the figure you obtained? That is, what are the elements in the figure and how are they related to each other?

---Insert Figure 2---

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LEARNING IN A COMPUTER ALGEBRA SYSTEM (CAS) ENVIRONMENT

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Introduction

This study is a qualitative case study focusing on the question "What are the processes of learning in a Computer Algebra System (CAS) environment for college students learning calculus?" The study is designed to research the impact on student learning of a new software available for mathematics education. The research aims to provide insight into the nature of learning in a technology-rich environment.

Motivation for the Study

Calculating technology in mathematics has evolved from four-function calculators to scientific calculators to graphing calculators and now to calculators (or computers) with Computer Algebra System (CAS) software. The advent of CAS software, which can do a great deal of the problems in a standard algebra or calculus text book at the push of a few buttons, truly represents a quantum leap in technology. The community of mathematics educators is in the throes of a gr eat debate as to whether this is one of the most exciting or most frightening developments in the history of mathematics education (Kutzler, 2000; Waits & Demana, 1999) as mathematics educators struggle with the implications of having software in the classroom which can, for example, expand and factorise algebraic expressions, solve equations, differentiate functions, and find anti-derivatives. For centuries now the emphasis in algebra and calculus curricula has been on the development of technical skills through practicing algorithms: making of students efficient machines for performing the algorithms now automatised by CAS. Machines with CAS capability have, therefore, brought into focus fundamental questions about the purpose of mathematics in school, and the nature and content of the mathematics curriculum. The use of CAS in education is still relatively rare but the growing body of research and the interest of such organisations as the National Council of Teachers of Mathematics (Cuoco et al., 2003) suggests that its extended use is imminent. It is important, therefore, that there be a firm research base for the implementation of CAS capable technology in schools, colleges, and universities which has addressed the processes of learning and the conceptual development of students learning using CAS as well as recognising obstacles to learning with CAS.

Since the wide availability of CAS in the late 1980s, research on its impact on mathematics education has developed in two main strands. The first has concentrated on showing the effectiveness of technology in supporting the learning of specific topics most of which are part of a traditional curriculum (Judson, 1990; Mayes, 1995; Palmiter, 1991). The second strand examines the question of what a technology-enhanced curriculum should consist of, suggesting new topics (cryptography, chaos theory, etc.), suggesting changes to the order in which topics are introduced and suggesting assessments which emphasise problem solving ability over technical skill (Heid, 1988; Herget et al, 2000; Kokol-Voljc 1999).

There is, however, a gap in the research in the area of investigations into the process of student learning and their development of concepts while using CAS. The importance of investigating these processes lies in understanding the impact on learning so that an appropriate balance can be found in the integration of technology into teaching ensuring that the merits of traditional methods are not lost and that the merits and demerits of CAS use are fully understood. The aim of this research is to address this gap by investigating, as a case study, a group of students taking a college-level mathematics course in which a CAS (in this case called *Mathematica*) is at the core of the course design. A case study approach will be taken so that the research can focus intensely on students' work and discussions in class to capture the processes of their learning and their conceptual development in calculus. The research question is:

What are the processes of learning in a Computer Algebra System (CAS) environment for college students learning calculus?

Theoretical Framework

This research employs two theoretical frameworks through which to approach student learning while using CAS. The Rotman Model of Mathematical Reasoning (1993) is used as a macro-framework for the place of technology in the learning of mathematics. This framework is useful for addressing the question of the effect of technology on learning by positioning technology in the activity of mathematical reasoning. The framework also provides a lens for interpreting the role of experimentation in the learning of mathematics and the relationship between technology and learners. The Pirie-Kieren Model of Mathematical Understanding is used as a micro-framework and as a lens through which to interpret and analyse specific learning episodes as they take place in the classroom. This frame for the analysis of learning episodes helps in the interpretation of the process of learning in a CAS environment as well as answering questions about formalisation of mathematics in a CAS environment and questions of students' conceptions of mathematical objects and mathematics as a subject. The two frameworks together provide a vehicle for understanding learners' mathematical activity, reasoning and development in a CAS environment across a period of time.

Design of the Study and Methods

The research is a case study of three students in a Calculus & *Mathematica* (C&*M*) class as individuals and in a group. This number of students was chosen because it is the natural group size in the class. Since much of the work in the class is group work so it is appropriate to study an entire group although the analysis will consider the individuals within the group as well as the group as a whole. The primary data is audio tape and video capture of computer screens of the group's discussions and collaborations in the C&*M* classroom. To provide layers of context for the case study and for the purposes of triangulation, the video and audio data is supplemented by interviews with the students and their written work for the class. Also, a grounded survey was administered to all students in the C&*M* class. An approach grounded in the traditions of qualitative research provides the best opportunity of describing, analysing, and understanding the specific impact of learning in a CAS environment on student conceptual development. As noted above, analyses which are largely quantitative in nature, focussing on increase in student achievement have been done in similar contexts in the past (Judson, 1990; Mayes, 1995; Palmiter, 1991) and do not provide sufficient insight into students' specific conceptual development or the particular impact of CAS on the learning process. This research focuses, not on the outcomes of student learning in a final exam, but rather on precisely these processes of learning and student conceptual development.

Analysis and interpretation of the audio and video data, through the Rotman Model of Mathematical Reasoning (1993) and Pirie-Kieren Model of Mathematical Understanding (1994), is the primary source of answers the research question. The Rotman Model is used as a macro-framework for understanding the place of technology in the learning of mathematics. The Pirie-Kieren Model is used as a micro-framework in which to make a detailed analysis of the work and discussions of the students as they take place in the classroom. The two frameworks together provide a vehicle for understanding learners' mathematical activity, reasoning and development in a CAS environment across a period of time.

Data Presentation and Analysis

This section consists of a detailed analysis of a learning episode using the Pirie-Kieren model and some more general conclusions framed by both the Rotman and the Pirie-Kieren models.

An Example

Find the highest point on the graph f[x] = e^(-x^2) (2 + Cos [x] + (Sin[x])/2). Is there a lowest point on the graph?

Perhaps prompted by the instruction to find the highest point on the *graph* the students begin their analysis by plotting the curve:

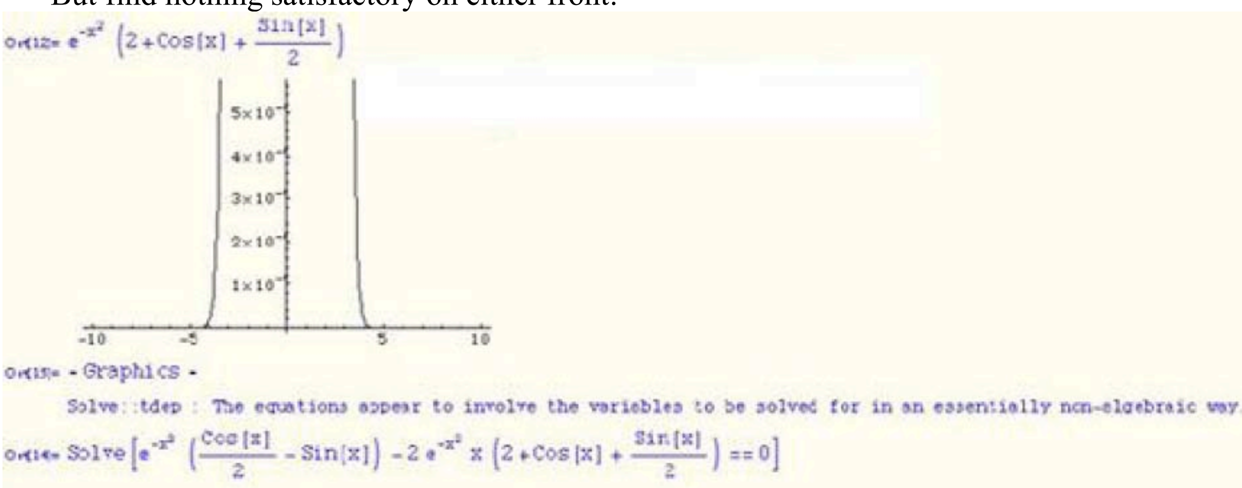
A: OK. Is there a lowest point on the graph?

C: I don't know. Let's plot it and find out.

They decide, as an afterthought, to attempt a formal analytic solution:

C: Let's solve for the maxs and mins here as well.

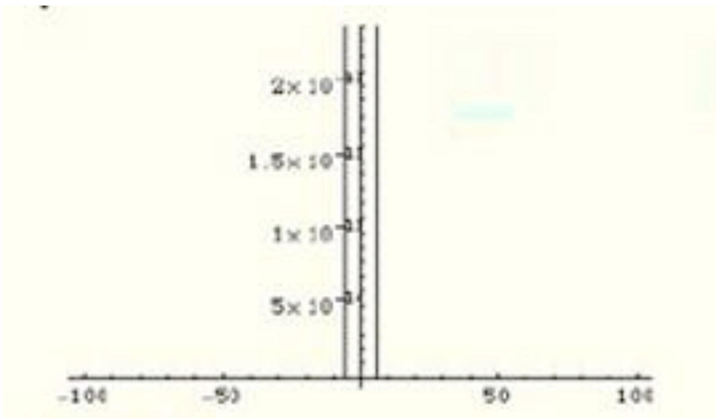
But find nothing satisfactory on either front:



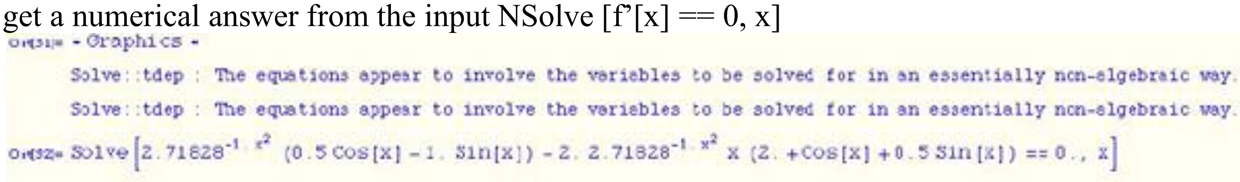
(silence)

A: Did you put the "Solve" function in wrong or is something that's too complex for it?

Thus the analysis begins on a formal level but the students find that *Mathematica* is unable to help them find an algebraic solution. They descend to the Image Making level in order to get a picture of what the solution might be. The students try to make a better image by changing the lower and upper limits of the graph from –10 to 10 to –100 to 100.



This doesn’t help in finding a maximum point but rather than making more images the students switch tactics to trying to get *Mathematica* to find a solution again. This time they try to get a numerical answer from the input NSolve [f’[x] == 0, x]

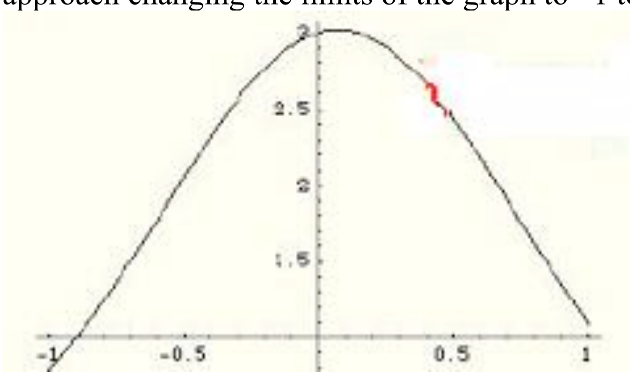

Out[31]= - Graphics -

Solve::tdep : The equations appear to involve the variables to be solved for in an essentially non-algebraic way.

Solve::tdep : The equations appear to involve the variables to be solved for in an essentially non-algebraic way.

Out[32]= Solve$\left[2.71828^{-1.\,x^2}\,(0.5\,\mathrm{Cos}[x]-1.\,\mathrm{Sin}[x])-2.\,2.71828^{-1.\,x^2}\,x\,(2.+\mathrm{Cos}[x]+0.5\,\mathrm{Sin}[x])==0.,\,x\right]$

They give up on the “Solve” function in any of its forms at this point although they will return to a numerical approach at a later stage. For the moment the students return to a visual approach changing the limits of the graph to –1 to 1.



It should be clear to the students now, at least what an approximate solution to the problem is since they now have an excellent representation of the function. However, they fail to recognise this and they then get progressively worse images, with limits of –5 to 5; –25 to 25.

The Image Making approach has again failed but the successful representation of the function inspires a move upwards helped by the Property Noticing by C of the values of the gradient function:

C: Try “f’(0)” and see if that works.

[Output: 1/2]

T: Go to the right and see if it’s still increasing.

C: f’(1). Wow.

T: Put a 1.

[Output –2.38879]

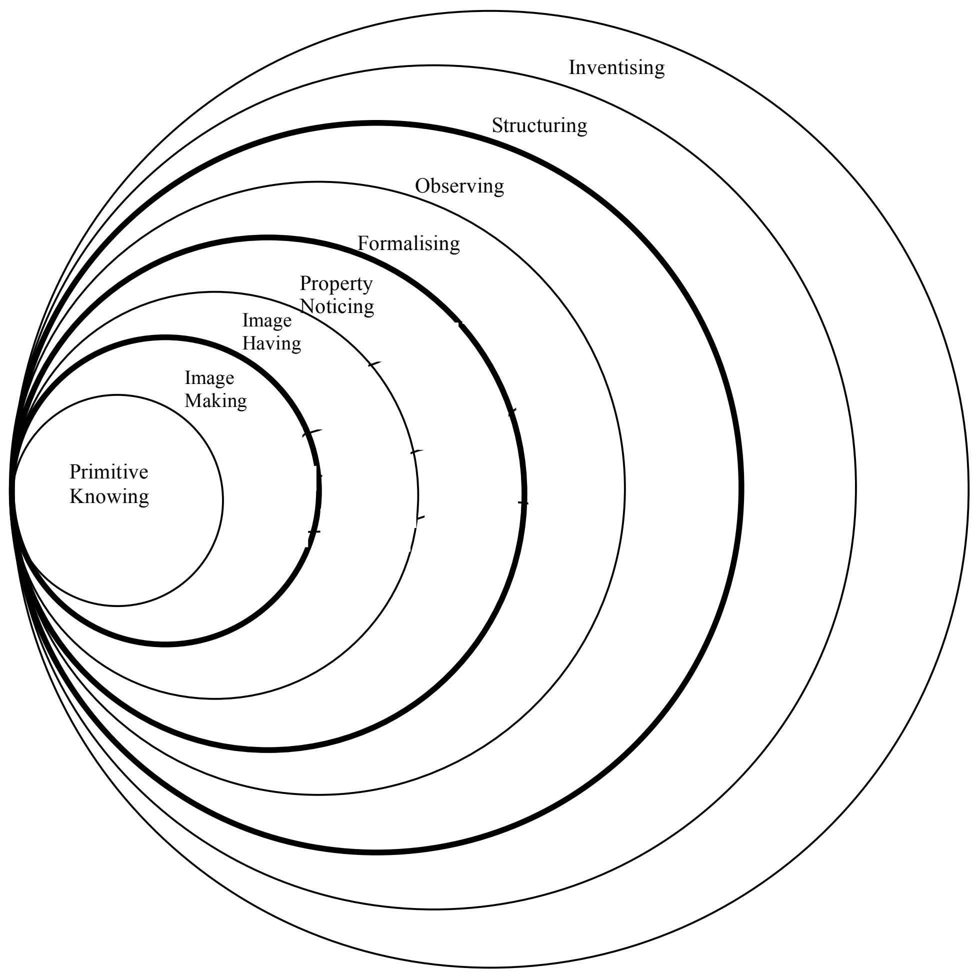
C: Yeah. So it’s decreasing at 1.

T: So it’s between 0 and 1.

C: Well, it’s between 1/2 and 1. You’re right 0 and 1. Closer (as A tries other values).

This exchange represents the mathematical breakthrough of the episode. C has noticed the property of the relationship of the value of the gradient function to the maximum value of the function. The noticing of this property is constrained by, or informed by, the formal problem of finding the maximum value of a function. The students continue using better approximations culminating in a formal numerical solution correct to 15 decimal places. They spend quite a lot of time on finding better approximations and don’t seem to have a good notion of how accurate an acceptable answer is. B speculates on whether the answer they are converging on is the square root of a “nice” decimal like 0.5 without offering any algebraic/analytic suggestion as to why this should be so.

A Pirie-Kieren map of this learning episode is as follows:



We see in this episode that in Week 7 of the 10 week course the students are able to implement graphical, numerical, and algebraic solution strategies but that there is a lack of sophistication in how they use *Mathematica*. We see this in the creation of less successful graphical representations after they have made an excellent representation, as well in the lack of a good sense of the acceptable level of accuracy of answers and in the speculation on “coincidence” in the decimal representation they achieve.

Further Themes

Framing of Technology

The introduction of *Mathematica* to the students as well as the logistics of using *Mathematica* had a considerable influence on the students' progress and behaviour during the time of the study. The students were very slow to make a transition to simply regarding *Mathematica* as a calculating tool/calculator which happened to sit on a computer. This is evidenced by their reliance on calculators or by hand calculations in preference to *Mathematica* for drawing certain graphs or performing certain symbolic manipulations. The reasons for this seem to be two-fold: the instructor/facilitator of the class, and the set up of *Mathematica* in notebooks.

The instructor/facilitator was not very familiar with *Mathematica* and had not used it in a pedagogical context before. At the beginning of the quarter he did not introduce the students to the capabilities of *Mathematica* as a calculating tool either by relating it to calculating experience they had (arithmetical calculations, graphs of functions) or exploring new possibilities such as simplifying expressions or solving equations. A consequence of this deficit was that the students took some time to understand what exactly *Mathematica* is and how it can be used. The other issue holding the students back was that the content for the course was presented as an, in effect, an interactive electronic text. The set up was that in any given section the students would have some questions with some sample code which the students ran and then composed their own code to answer the questions. This restrictive format meant that students were, generally, reluctant to use *Mathematica* in the same way as they would use a blank piece of paper any more than they would do calculations by hand on the pages of a textbook. They tended to wait until they had what they considered to be a correct strategy before they would employ *Mathematica*. This problem lessened somewhat as the quarter passed as can be seen in the episode analysed above which took place in Week 7 of a ten-week quarter.

The overall consequence of the framing of the technology was that the students' use of the Computer Algebra System was clearly hampered by the way the technology was presented to them at the beginning of the quarter and how it was set up for them to use throughout the quarter.

Experimentation

Waits and Demana (1999) and Kutzler (2000) have argued for the importance in CAS as a tool for reintroducing experimentation into the learning of mathematics. As we can see, for example, in the episode above there is a clear difference between awareness of strategies to use in experimenting to find an answer and sophisticated implementation of those strategies. Students were able to employ analytic/algebraic, graphical, and numerical strategies in attempting to solve the given equation. However, the lack of sophistication shown in the students' lack of recognition of when they had a good graph is noteworthy as is their "blind" application of the numerical strategy to get a numerical approximation to 15 decimal places without giving any consideration to what might be a reasonable answer. This lack of sophistication is all the more noteworthy since, as noted above, it took place in Week 7 of a ten-week quarter.

During the quarter it was also the case that the students' willingness to experiment was extremely limited. There were no instances in the ten weeks of the study of students failing to solve a problem and using the power of *Mathematica* to solve a related but simpler version of the problem to gain some insight into the nature of the problem. This constrained experimentation is in many ways related to the framing the technology discussed above since the students never made the move to considering *Mathematica* as a calculating tool independent of the set of problems they were asked to solve in the *Mathematica* notebooks which constituted the content of the course.

Conclusions

This study aims to present close readings of learning episodes to provide, so to say, existence proofs of student learning in a CAS environment. As well as providing examples of this learning the broader conclusions of the study are (i) that the framing and introduction of technology at the beginning of an instruction period impacts crucially on student behaviour and use of technology throughout that period and (ii) that while students will naturally experiment in a CAS environment intervention is probably required fro them to develop sophistication in their experimental behaviour and strategies.

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COMMUNICATING MATHEMATICS ONLINE–COMPARISON OF EXPERT AND PEER TUTORING

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One purpose of this study was to better understand how tutors answer students' online questions by examining the discourse on three asynchronous mathematics help Web sites. This summary report is an overview of results based on data gathered in relation to the following questions: To what extent and how do tutors in online help model students that they interact with? Are expert tutor's answers different from peer tutor's answers for similar questions? How are they different?

Research in the pedagogy of tutoring points to the strategies that human tutors use when they teach mathematics: They mostly follow "curriculum scripts" (McArthur, Stasz & Zmuidzinas, 1990; Putnam, 1987); and may use factors like (a) the sense of student knowledge, (b) errors, and (c) student responses to queries to fine-tune their behavior (Fox, 1993; Schoenfeld, 1992). Depending if they do inquiry or remedial tutoring, "students and tutors engage more in the discovery of new concepts" (McArthur et al., 1990, p. 202) or in the repair of students' misconceptions.

The task of tutoring mathematics effectively, seem much simpler in face-to-face than in computer or even online tutoring. One of the techniques that designers of intelligent tutoring systems try to incorporate, for example, is intensive student questioning as a support for knowledge growth (Self, 1990, p. 4), but such strategy seems not applicable in asynchronous online communication. This certainly supports the claim that "for teachers, traditional professional knowledge is not sufficient to deal with the deep changes in learning, teaching, and epistemological phenomena that are emerging" (Balacheff & Kaput, 1997, p. 495) as a result of progress in computer and communication technologies. Among important topics for future research, Balacheff and Kaput recommended analysis of the complexity of the tutoring task in the context of telepresence, and specification of the tools needed by the human distant tutor.

This study, consequently, intended to define and explore the characteristics of asynchronous online mathematics help environments by, among else, investigating the tutoring habits of expert and peer tutors and looking for evidence that these two categories of online tutors teach differently.

The study was organized so that it looked into the online tutorial discourse with the purpose to also distinguish what and how do tutors in mathematics online help teach, and how helpful are their answers. Since mathematics help sites allow for written communication among people who do not know each other, analysis of discourse on the sites provided the only clue into the nature and relative success of the tutoring exchanges. However, in order for the researcher to get an insider's view on mathematics online help, five expert tutors also took part in the study.

Definitions

Student in this study is any person who poses one or more questions on mathematics help Web site.

Peer tutoring is tutoring by volunteer learners on the Web site. According to Goodlad and Hirst (1989, p. 13), "peer tutoring is the system of instruction in which learners help each other and learn by teaching." Here it is essential that the peer is someone who has the same status as other visitors to the site.

Expert tutor is an experienced, qualified person who is recognized by the administration of the Web site as such.

Theoretical Framework

Based on the constructivist approach (LeJeune & Richardson, 1998; Jonassen, 1995) mathematics online help sites are the environments where students may be taking active role in their learning. Before they pose the questions they may go through the process of self-diagnostics, and after they receive the answer, they may go through the process of self-explanation, which are both important strategies for learning (Chi, 1998). However, based only on questions, while missing all the visual and many other clues about the student, it is very difficult for the tutors to come up with the proper model of a student.

Taking socio-cultural perspective, mathematics online help sites provide support for a student in knowledge building within a community of learners. Tutor is there to help students by using scaffolding techniques and, as a more knowledgeable peer, to support transformation of potential into actual in the student's zone of proximal development (Vigotsky, 1978). Online communication on public help sites also provides visitors with variety of topics presented in argumentative form and in semiformal mathematics language. Therefore, they agree with Resnick's (1988) recommendation that mathematics should be taught as if it were an ill-structured discipline: a domain in which multiple interpretations, arguments, and debate are called for and natural. Furthermore, online help sites have important role as meeting places for people who want to participate in mathematics discourse.

Methods

The mixture of quantitative and qualitative research methods were used here. The discourse on the mathematics help Web sites was analyzed according to the Verbal Data Analysis (Chi, 1997) as a method of quantifying qualitative data. Taxonomies of tutorial discourse developed by researchers in intelligent tutoring systems (Graesser, Person, & Huber, 1992; Shah, Evens, Michael, & Rovick, 2002) were adapted and extended to fit the data thus resulting in developing Taxonomy of Online Tutors' Answers (Martinovic, 2004, p. 66). This categorization was done according to (a) what they teach, (b) how they teach, (c) do they hedge, and (d) how helpful are their answers (Martinovic, 2004; Graesser, Person, & Huber, 1992).

After the data were first categorized, the methods of descriptive statistics and statistical tests for categorical data provided for the numerical characteristics and comparisons between different classes. More qualitative approach was applied on the transcribed interviews with online tutors and their tutoring logs, where field notes were summarized on a grid with categories. Furthermore, the examples of particular questions and answers from the Web sites were compared to general observations.

Data

Data for the study were collected from online contributions on three purposefully selected, public mathematics help sites that offered help on a voluntary basis; written reflections of five expert tutors from two Web sites; and interviews with them.

Records of online student-tutor communication were sampled from the Web sites that differed in the type of tutoring. This was expected to provide the greatest variability between the discourses on these sites. The first site that offered both expert and peer tutoring (mixed site) was used in the initial phase of the study, where some of the instruments were developed and tested. The other two sites, which each offered one exclusive form of tutoring (peer or expert), were used for data collection in the main part of the study. All three sites made available archives of previous student-tutor correspondence. From each of the sites the corpus of 200 randomly selected threads of communication that spanned over one year were selected and categorized. Overall, there were 838 tutors' answers, with particular sample sizes given in Table 1.

Table1: *The Number of Tutors' Answers Collected From Three Web Sites*

Mixed Site (PT)	Mixed Site (ET)	Peer Site	Expert Site
n = 212	n = 53	n = 341	n = 232

The information about the expert tutors (their email addresses) was obtained through two mathematics help Web sites (one of which was also used for the discourse analysis), that offered only expert tutoring. Out of 19 contacted tutors, seven agreed to participate while five actually provided all the data. This sample covered the following scope: Both genders—male (2), female (3); different regions/countries—Central America (1), USA (3), Canada (1); various professions—educators (3), students (2); and a range of mathematics disciplines they tutored—grades K-12, Calculus 1-3, Probability and Statistics, History of Mathematics, just to mention a few. They all had experience tutoring mathematics face-to-face and were able to point to the differences with respect to tutoring online. Their experience in tutoring online varied from six months to eight years, and the number of questions they answered varied from eight to thousands. Out of five tutors, two found the site accidentally, through browsing; the other two became volunteer tutors after learning about the site from somebody already active on the site; and one joined the site on the suggestion of her mathematics teacher.

The tutors were asked to keep logs of their online communication with students where they recorded their thoughts about the tutoring process. The tutors kept the tutoring log of their tutoring activities for the period of two months or five online questions answered, whichever came first. Tutor's log included among else (a) a question that was posted on a help site, (b) tutor's personal notes regarding how he or she perceived the question, (c) the answer, (d) tutor's reasons for choosing specific strategy for answering the question, and (e) resources used to get the answer (books, notes, Web sites, email, consultations, or else).

The same tutors were also interviewed, thus providing relevant information necessary for obtaining fuller understanding of the research topic. This was accomplished through the semi-structured telephone interviews. Participants were interviewed over the phone once during the study period; after they sent back their logs (one did not submit the logs). The interviews had about 20 questions each and were approximately 45 minutes long. The interviews were audio taped and transcribed.

Each interview started with the structured questions, but then moved toward open-ended questions. As semi-structured, it provided the combination of objectivity and depth (Borg & Gall, 1989, p. 452). The interviews covered: (a) the demographic information, (b) general information about online mathematics tutoring experience, (c) probes regarding modeling a tutor (what clues are they looking for in the question, what are their habits in answering online questions, and other), (d) comparison to face-to-face tutoring, and (e) a general view of tutoring task in online help. As a result, the questions were (a) descriptive, about respondent's perceptions of some aspects of tutoring mathematics online, (b) structural, regarding how respondents organize or structure their perceptions of answering mathematics questions online, and (c) contrast questions, which show how respondents distinguish between online and face-to-face tutoring tasks.

These different instruments for gathering the data and combined methods of inquiry insured the best blend of structured as well as unstructured techniques for investigating such complex topic.

Results

The study showed that peer tutors and expert tutors teach differently, and that online expert tutors use tutoring strategies that are specific for this medium.

It was evidenced that online expert tutors try to adapt their tutoring strategy to those students whose questions they choose to answer. Their global strategy is set by the policies of the Web site. They select questions first on the basis of a mathematical discipline/topic, and then according to the time when the message was posted. But after they select a question, they use whatever clues are available to model a student.

Analysis of tutor logs and their interviews showed that online expert tutors looked for the following information regarding a student:

1. *Age, level in school, and background knowledge*—To find out how much they can assume the student knows, or if the student has a sound understanding of basic concepts.
2. *Attitude towards mathematics*—To determine if the student is perceptive, investigative, open for exploring, interested and engaged; or frustrated, just wanting to turn in the homework.
3. *Amount of effort invested in the problem*—To determine how much guidance the student likely needs.

Expert tutors therefore approach the questions holistically in order to determine the students' background and knowledge, their motivation, and how far they came in resolving the issues regarding their questions. They use this information in order to determine how they are going to answer, given that they follow certain constraints, one being the "we are not doing homework for you" policy, and the other related to the efficiency of their assistance. In their interviews all expert tutors expressed concern regarding the difficulty of modeling a student in asynchronous online help.

The summary of data from tutors' logs showed that expert tutors use a breadth of information to make up for the lack of depth of information about a student which is available in other educational settings. Through that process, they decide to what extent they are going to be helpful; which words to use; if they are going to follow the student's reasoning or their own; if they are going to provide more than one problem solving routine; which of the scaffolding techniques they are going to use; and if they are going to satisfy the student's requests.

Tutors seem to be aware of a possible mismatch between their perception of the student and a student, as they sometimes use Multiple Answers (more than one version of a solution/answer). Those were present in about 2%-3% of the peer tutors' messages (on the peer tutoring and mixed site respectively) and in about 7%-13% of expert tutors' messages (on the expert and mixed tutoring site respectively).

When students provide at least some background information, such as: age, grade, textbook they use, and some self-diagnostics related to a problem, tutors can fine tune their answers to match a model of the student. Expert tutors certainly do that, but the observation of answers on the peer site suggests a different conclusion. Peer tutors focus on solving a mathematical problem in a student's message. They approach it as if they themselves would be doing the exercise from the textbook. They rarely attend to anything else in the question and are not constrained by any factor other than time. Peer tutors obviously take some pride in quickly answering a question, because all questions on the peer tutoring site get a first response much faster than on the expert tutoring site. On the peer tutoring site, 75% of the threads received the first answer the same day they were posted and the average response time on the same site was three times faster than on the expert site.

When a student provides a solution and asks for verification, expert tutors use the mal-rule approach as defined in Spiers (1996) thus diagnosing student's errors and misconceptions. If the student's solution contains an error, expert tutors consider if they are going to follow the student's reasoning, or they are going to give their own, as an alternative. Taking into account the model of the student, expert tutors decide how helpful they are going to be. All experts mentioned in their interviews that for them the most difficult decision is how much information to release without actually doing the complete problem for a student.

It seems that expert tutors treat each student individually. Sometimes they create a model of a student by using their own tutoring/teaching experience or they apply the approach similar to one recommended by Self (1990a, p. 112): "Avoid guessing—get the student to tell you what you need to know."

Peer tutors, on the other hand, often provide answers without any explanation, therefore focusing only on "doing the problem" and letting the student use it in whichever way they want. Such answers reached 25% and 34% of peer tutors' messages on the Web sites with mixed and with peer tutoring respectively.

As a consequence of often lacking a model of a student, online tutors hedge in their answers. Across all sites, except on the peer tutoring site, hedges related to the model of a student took more than one-tenth of all hedges. The relatively high percentage of hedging regarding a model of a student among peer tutors on the mixed site can be explained by them being influenced by the expert tutor.

Are Expert Tutor's Answers Different From Peer Tutor's Answers For Similar Questions? How Are They Different?

Experts are much more verbose than peer tutors. The average number of words in tutors' answers on the expert tutoring site was about three times that of the tutors' answers from the site with both peer and expert tutoring, or about 2.5 times greater than in the answers from the peer tutoring site. Furthermore, peer tutors are much more task-oriented than experts who seem to like to provide guidance and teach general rules.

On mathematics online help sites expert tutors use scaffolding techniques of rephrasing, analogies, and summarizing. They often use elaboration as external self-explanation (Chi, 1998) when they explain something to students. Expert tutors also use probes to gauge and provoke student's comprehension. They prefer to teach general rules and language of mathematics, and rather provide hints than complete work in their answers.

Peer tutors pay more attention to procedures, and if they find a question incomplete to the point that they cannot repair it, they brush-off the student (refuse to answer). They teach students how to communicate online and what the elements of fair play are on the Web site. Their instructions do not go beyond the scope of the question, thus predominantly using concise explanations.

So, while peer tutors are found to be mathematics problem-driven and focused in their answers, expert tutors instead follow general rules of efficiency and honesty. As evidenced in this study, experts see themselves as part of a global educational system and encourage students to do the work on their own, not only because it benefits the students' learning, but because it is more honest.

When acknowledging students, peer tutors are gracious, while experts are more direct. Peer tutors, more often than experts, brush-off students and they do it specifically in cases when the questions are not clear. Experts, on the other hand, may refuse to do part of a question because they follow a "not doing homework" policy or because they believe that student can finish the rest of the work on their own. Expert tutors are mostly torn between deciding how much help they should provide, so that it is efficient for them and helpful to a student; and where they should stop, so that they prevent student of using tutor's work as their own and allow student to learn by doing.

If one excludes rhetorical hedges, peer tutors overall hedge more in their answers than experts. Peer tutors hedge more regarding the accuracy, completeness and form of their answers while experts are more concerned if their answers are clear, appropriate and helpful to the student. This again supports the conclusion that peer tutors focus on a mathematical problem in their answers, while experts focus on a model of a student.

This study also investigated the frequency of the first person pronouns (i.e. "I" and "we"), which "indicate the author's personal involvement with the activity portrayed in the text" (Morgan, 1996, p. 5). The frequency of the first and second person pronouns was much lower in the peer tutors' answers than in the experts' (Martinovic, 2004, p. 159). Both experts and peer tutors used the second person pronoun more frequently than any first person pronoun, which "may indicate a claim to relatively close relationship between author and reader or between reader and subject matter" (Morgan, 1996, p. 6). The fact that expert tutors use "we" much more freely than peer tutors probably suggests that they were speaking with the authority of the mathematical community (Morgan, p. 5).

Although, judging from their answers, peer tutors' expertise varies greatly, and in some cases the quality of their answers may raise to the level of expert tutors' answers, there was a general feeling that these two categories of tutors differ in many ways.

Some differences between peer tutors' and expert tutors' answers to students' online enquiries are summarized in Table 2.

Table 2: Comparison of Peer and Expert Online Tutors' Answering Habits

Peer Tutors	Expert Tutors	Evidence
Use telegraphic style.	Use more verbose answers.	Discourse on Web sites.
Focus on mathematical problem.	Approach the question holistically.	Discourse on Web sites. Interview data and tutors' logs.
Usually solve the problem.	Rarely solve the problem themselves.	Discourse on Web sites. Interview data.
Establish a close relationship with a student.	Speak with the authority of mathematical community.	Discourse on Web sites.
	Show personal involvement with a subject (mathematics).	
Provide detailed work.	Provide guidance.	Discourse on Web sites. Interview data.
Pass over syntax errors, incomplete forms and unclear questions.	Teach the language of mathematics and general rules.	Discourse on Web sites.
Show no constrains.	Are constrained by the rules on the site and conflict of professional interests.	Discourse on Web sites. Interview data.

Overall, peer tutoring sites may be used differently than expert tutoring sites. If one needs a quick answer to a specific, typical question, peer tutoring sites are certainly more beneficial. If one needs advice, reinforcement of ideas, or an answer to some esoteric question, expert tutoring sites are more useful.

Conclusions

This study builds on the findings in the areas of pedagogy of tutoring, computer-mediated communication and mathematics learning. It gives an insight into this relatively new educational practice of providing mathematics help online to the general public, with the possibility of implementation in other educational practices. As such it fits the goals of PME-NA by providing for deeper and better understanding of the psychological aspects of teaching and learning mathematics online and the implications thereof.

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TOOLS FOR THOUGHT AND COMMUNICATION

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Based on sociocultural theory (Rogoff & Lave, 1984; Vygotsky, 1978), a case study of a fifth-grade classroom was conducted investigating how the teacher used calculators in mathematics instruction. Data sources included field notes from classroom observations, audiotapes of 110 mathematics lessons, and student work. The teacher introduced a keystroke-based language as a tool that facilitated social activity and communication. The keystrokes were used as referents for writing, discussing, and negotiating mathematical ideas. As students participated and interacted in that social activity, the keystrokes went beyond the role of a communication tool to take on additional roles in developing higher cognitive functions such as planning, reflection, analysis, problem solving, and writing.

Purpose

The purpose of this paper is to present the idea that calculator keystrokes have unique value as a tool that can be used to (1) facilitate social activity, (2) communicate mathematical thinking and (3) facilitate the development of students' mathematical thinking and higher psychological processes such as planning, reflection, analysis, problem solving, and writing.

Over the past 25 years, research has pointed to the need for studying how the calculator affects the learning process, and how a teacher can use a calculator to mediate the learning process (Hembree & Dessart, 1992; Shumway, White, Wheatley, Reys, Coburn, & Schoen, 1981; Suydam, 1979). Yet, a careful search of the literature reveals that these issues have not been given the attention they warrant. Lampert (1991) argues that the teacher has the responsibility to find "language and symbols" which students and teachers can use to enable them to talk about the same mathematical content. Current research has not yet investigated the use of calculator keystrokes as a language that can be used to communicate mathematical ideas, and to create joint knowledge and understanding in mathematics classrooms. Moreover, talk that mediates social activity externally is eventually used as a mediator of cognitive activity internally (Moll, 1990; Vygotsky, 1978). The research findings presented below demonstrate how one teacher used calculator keystrokes to mediate social activities in a fifth grade classroom that resulted in students using those keystrokes in thought.

Theoretical Framework

Learning begins as a social activity, situated in an environment (i.e., the aggregate of the surrounding objects, conditions, and influences) as well as in the context of interactions (i.e., reciprocal exchanges) with people (i.e., teachers and students) and cultural signs and tools (i.e., curricula, calculators, manipulatives), and guided by a more-experienced other (Mercer, 1995; Resnick, 1988; Rogoff & Lave, 1984; Vygotsky, 1978). During this social activity in a mathematics classroom, students encounter a "talk" or discourse focused on mathematical tasks, which is spoken, written, drawn, and gestured. The context and nature of that talk influences what students learn. One of the most consistent and reliable research findings on teaching and learning is that students learn what they are given opportunities to learn (Hiebert, 2003). Some may assume that teachers using the same mathematics curriculum provide the same opportunities for student learning. However, this is not the case. Social activity varies from classroom to classroom, even when teachers use the same mathematics curriculum with the same mathematics tasks. Kilpatrick (2003) explains, "Two classrooms in which the same curriculum is supposedly being 'implemented' may look very different; the activities of teacher and students in each room may be quite dissimilar, with different learning opportunities available, different mathematical ideas under consideration, and different outcomes achieved" (p. 473). Furthermore, Henningsen and Stein (1997) argue that mathematical tasks are central to student learning.

In other words, the nature of mathematical tasks and the talk focused on those tasks influences what students learn. As a result, the importance of language and communication in the learning process in the mathematics classroom has gained considerable attention in recent years (e.g., Steinbring, Bartolini Bussi, & Sierpinska, 1998). Conolly (1989) explained that language plays a powerful role in "...the production of knowledge, as well as the representation, of knowledge....[in] forming meaning...[and in the acquisition] of personal ownership of ideas" (pp. 3-4). Figure 1 displays three critical elements of social activity – environment, talk, and interaction – that facilitate learning (Chval, 2001).

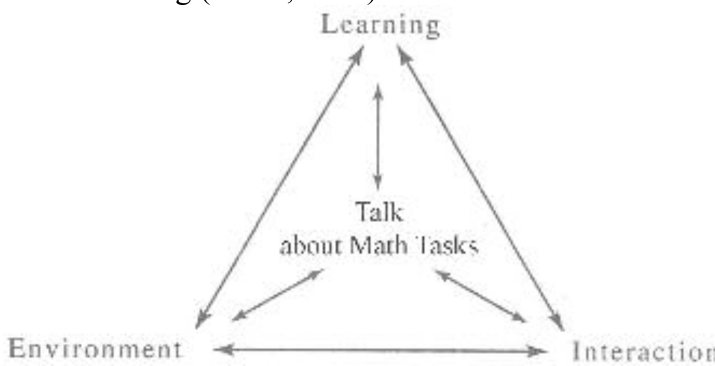

Figure 1. Learning begins with social activity

Vygotsky (1978) differentiated between learning (i.e., acquiring knowledge and skills) and development of psychological processes (e.g., abstract thinking or reasoning). He argued that instruction and learning precede the development of higher psychological processes and that the development of higher psychological processes in turn impacts future learning. He explained that the talk that mediates social activity externally is eventually used as a mediator of cognitive activity internally (Moll, 1990; Vygotsky, 1978). In other words, through the external use of talk, the learner internalizes its meaning and in turn, uses that meaning in thought.

The discussion and analyses also draw on the work of Kress and his colleagues (e.g., Kress, Jewitt, Ogborn, and Tsatsarelis, 2001). In this work, communication refers to all meaning-making systems, or "modes"; these are organized, regular, and socially specific means of representation (Jewitt, Kress, Ogborn, and Tsatsarelis, 2001). Learning can be seen as a "…process in which pupils are involved in actively 'remaking' the information and messages (or complexes of 'signs') which teachers communicate in the classroom. In this way learning…is the pupils' 'reshaping' of meaning (signs) to create new meanings (signs)" (Jewitt et al. 2001, p. 6). Learning, therefore, is rooted in a dynamic process of sign making, but one which is devised, organized, and used according to social needs and practice (Halliday, 1985). Inherent in this conception of learning is that "…meaning arises as a consequence of choice and that meaning is multiple (Jewitt et al. 2001, p.6). In other words, when we make meaning, we choose to speak, use gestures, make drawings, or use whatever resources available to communicate or represent our meaning, and we use multiple modes simultaneously. The discussion below illustrates how one teacher uses calculator keystrokes as one mode to facilitate the meaning-making process.

Methods

This discussion grows out of a case study of Sarah, an extraordinary fifth-grade teacher of second language learners (Chval, 2001). This case study was conducted during the 1998-99 academic year in a school that was situated in a low-income neighborhood in one of the largest public school systems in the United States. In 1998, the school reported 96.8% of the students as low-income, 96.9% as Hispanic, and 46% as limited English proficient. At the time the data were gathered, Sarah had a self-contained fifth-grade class of 24 students.

Data

Sarah's classroom was observed 60 times during the 1998-99 school year including five times in Week 1, three times per week in Weeks 2-6, and one to two times per week for the rest of the school year. An additional 59 mathematics lessons, which the researcher did not observe, were audiotaped. A careful record of what happened in the classroom was documented by compiling field notes, audiotaping mathematics instruction, and collecting artifacts such as student work in the curriculum materials and samples of student writing.

Three sets of audiotapes were transcribed. "Set 1: Beginning of the Year" consisted of the first twelve mathematics lessons (August 27 through September 16). "Set 2: Second Semester" included one lesson selected at random from each month, January through May. By reviewing fieldnotes and student work, October was identified as the critical period for studying the use of the calculator keystrokes. By November, the students' problem-solving strategies, as demonstrated by the use of calculator keystrokes, were already developed and sophisticated. As a result, a third set, "Set 3: October," including the 25 lessons from September 17 through October 31, was transcribed. Each lesson was broken down into a hierarchy of episodes and then individual episodes were coded (Stubbs, 1981). The transcriptions of Sarah's talk were broken down into a hierarchy beginning with mathematics versus non-mathematics. The mathematics talk was further sorted into whole-group versus small-group instruction. The whole-group sections were further sorted into calculator-related and non-calculator-related. Through a constant comparison of the whole-group/calculator-related episodes, the main objective was to look for common patterns in the teacher's talk (Glaser & Strauss, 1967). Each teacher statement was coded. Coded notes were then analyzed for recurring themes.

Discussion

Sarah introduces the term, "keystrokes" in the context of a specific machine: the scientific calculator. In utilizing the calculator, Sarah uses the term in two distinct ways: (1) to denote the striking of a calculator key; and (2) to speak or write the symbol representing the calculator key. For the purposes of this discussion, two patterns that emerged from the data will be described: (1) keystrokes as a means for communicating and facilitating social activity and (2) keystrokes as a means for developing higher psychological processes.

Sarah requires students to present keystrokes both verbally and in writing so that they can be used to communicate and discuss mathematical thinking. In Sarah's classroom, using keystrokes to discuss and negotiate meaning is more important than using the calculator to compute an answer. The following excerpts from the transcripts illustrate how Sarah establishes these practices. See (Khisty & Chval, 2002) for a discussion of Sarah's pedagogic discourse.

Sarah: On your paper write the keystrokes that you would need to put into your calculator to find the area of that rectangle. Don't take the calculators out, I didn't say that. I asked you to write the keystrokes.

Sarah: Your keystrokes are very important to me because they tell me what you are thinking. I cannot be inside your head. Oh, unless I open his head. [class laughter] I can't do it. But if I see your work, I know what you are thinking.

To help establish the context and demonstrate the typical use of calculator keystrokes during the second half of the year in Sarah's fifth-grade classroom, a sample problem and solution is provided in Figures 2 and 3. Sarah drew the figure on the board, asked her students to work in groups to calculate the area of the figure, and then circulated to assess student progress. Note that the problem requires working with area of semicircles and triangles as well as applying the Pythagorean Theorem.

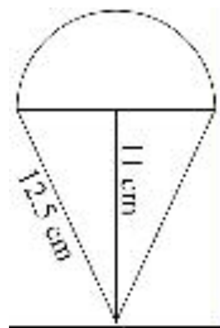

Figure 2. *Problem posed to Sarah's students*.

After a short time, Sarah asked for a volunteer to write the keystrokes on the chalkboard and explain his/her meaning for the first part of the problem. Marisa volunteered. Sarah then asked for a second volunteer to continue where Marisa left off. Violetta volunteered to write and explain the second part of the solution (see Figure 3).

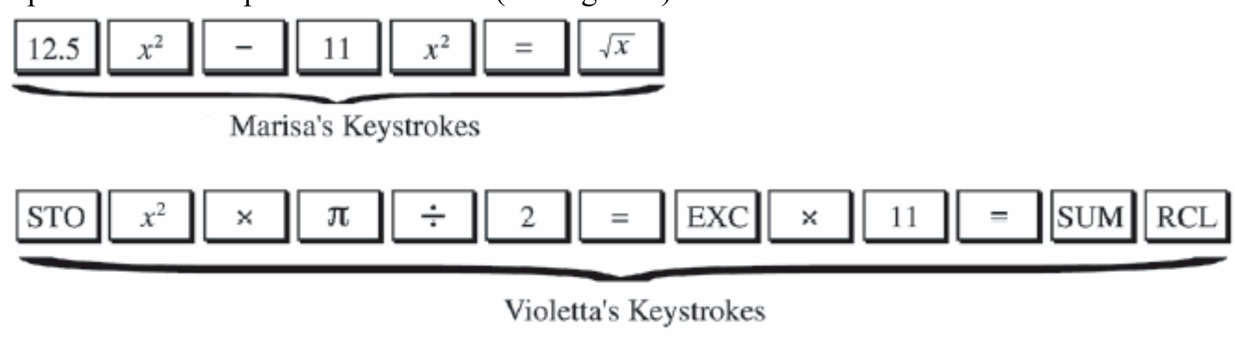

Figure 3. *Solution presented by two of Sarah's students*.

Sarah used the calculator keystrokes as a "common" language, one which was central to the classroom discourse, to mediate interactions in small-group and whole-class discussions. This "common" language was especially important to Sarah's students who were learning English as a second language. Students presented keystrokes both verbally and in writing to communicate mathematical thinking, initiated conversations and posed questions to their peers and teacher using this appropriated language, and analyzed their peers' solutions and articulated corrections and modifications.

As students participated and interacted in the classroom's social activity, the keystrokes were appropriated and internalized by the students and as a result, the keystrokes became "tools for thought" in the sense that students thought in keystrokes. They used the keystrokes to plan and create strategies to solve mathematics problems, to analyze the strategies of others, and to create and articulate a process of reasoning to justify their solutions. In fact, Sarah's students became so proficient at thinking and problem solving with the keystrokes that they no longer needed the calculator itself. Sarah's technique for using keystrokes in a typical mathematics problem follows. It should be noted that Sarah never explicitly articulated this technique, but rather it was the process she used in every observed lesson (with the exception of Step 5). For more discussion of Sarah's use of writing in the mathematics classroom, see (Chval & Khisty, 2001).

Step 1: Students write a plan for solving the problem, using only keystrokes.
Step 2: Students use keystrokes to communicate plan.
Step 3: Students break down sequence of keystrokes into components and the meaning of each component is discussed.
Step 4: Students listen to presentation of keystrokes, analyze keystrokes, and make a decision concerning agreement with that solution.
Step 5: Students write a narrative of how to solve the problem.
Step 6: Students suggest alternative keystroke sequences including more efficient ones.

Through this process, Sarah used keystrokes to facilitate the development of the functions of planning, problem solving, reflection, analysis, and writing—functions that will aid future development of higher psychological processes. For example, Sarah used calculator keystrokes to develop the students' "planning function" or the ability to plan for the solution of a problem (Step 1). Not only must students write their keystrokes; they must be written before the students touch the calculators. In an important variation from common teaching practice, Sarah does not use keystrokes only as a record of what a child already pressed. Vygotsky (1978) argued that "once children learn how to use the planning function of their language effectively, their psychological field changes radically" (p. 28). The keystrokes, as used in Sarah's classroom, serve as a way of communicating and displaying students' thinking, but more importantly serve as a way to create a specific plan for solving a problem before the students touch the calculators. Vygotsky (1978) explains:

> Using words (one class of such stimuli) to create a specific plan, the child achieves a much broader range of activity, applying as tools not only those objects that lie near at hand but searching for and preparing such stimuli as can be useful in the solution of the task, and planning future actions. (p. 26)

As utilized by Sarah and her students, keystrokes are another type of stimuli that are useful in organizing students' thoughts in the planning process. Once the students developed a plan for solving a problem, Sarah used the keystrokes as referents for discussing mathematical ideas. As students presented sequences of keystrokes to the class, they were challenged to communicate their thinking by writing the keystrokes on the chalkboard and verbally explaining the solution. Often times, students presented part of the solution as in the case of Marisa and others continued the solution as in the case of Violetta. Sarah used this public process to help students clarify their thinking and build meaning. Now that the keystrokes were on the chalkboard for all to see, every student had the responsibility to read them and analyze them. Even if a student had written a different sequence on his/her paper, s/he was responsible for understanding other solutions presented by peers. This included recognizing invalid methods, suggesting alternative methods, or determining more efficient methods. In this process, the students used the keystrokes as objects of reflection and analysis. Although some educators may think that Sarah's emphasis on keystrokes reinforces an algorithmic way of solving problems, analysis of student work indicated students often generated 5-8 different solution strategies for many of the problems. This strongly suggests that students were not memorizing and applying procedures for solving problems.

Conclusion

Sarah's teaching provides an example of how to utilize the potential of the calculator as a tool to enhance mathematical learning and development. She demonstrates how keystrokes can be used to promote and facilitate social activity. She introduces the idea that keystrokes can be used as a common language to mediate interactions so that children can negotiate mathematical meanings. Most importantly, she teaches how keystrokes can be used as "tools for thought" to facilitate higher psychological processes such as planning, reflection, analysis, problem solving, and writing.

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THE INTERPLAY BETWEEN TECHNOLOGY DESIGN AND STUDENTS' CONTROL OF PROBLEM SOLVING

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This research examines the ways in which the design of a technology tool may support students' abilities to control their problem solving. We use the constructs of ascending and descending control (Gallo, 1994) and Underwood et al's (in press) design principles for educational applets to conduct a fine-grained analysis of four pairs of students, each working with a preservice teacher, to solve a problem using a java applet. Our results indicate that design principles that encourage the use of features that provide students with status feedback of their problem solving can have a positive impact on students' control decisions. In addition, the design of a tool can influence how students manipulate objects, and this can affect whether a feature supports students' problem solving.

Purpose and Perspectives

In the past twenty years, a greater emphasis has been placed on students' mathematical problem solving as reflected in standards for school mathematics (National Council of Teacher's of Mathematics, 2000). Within the same time frame, there has been an increase in the development of technological tools that are used in mathematics classrooms for a variety of purposes, one of which is to facilitate students' problem solving. Clements (2000) makes a case for how computers can enhance students' problem solving by providing an environment to engage in playful exploration, test ideas, receive feedback, and make their understanding public and visible.

Building on the work of Polya (1957), Schoenfeld (1985) created a framework consisting of four factors that affect students' abilities to solve problems: resources, heuristics, control, and beliefs. In a technological environment, the resources available to students include those made possible by a particular technology tool (e.g., various representations). A student needs knowledge of how to use the technology tool well as knowledge of the mathematical objects that are represented. Thus within a technology environment, it is necessary to consider both the mathematical and technological resources available for students. The presence of a particular technology tool also affects the heuristics a student may use during problem solving, because a tool may afford or constrain certain actions that influence possible heuristics.

As students are solving problems, they not only need to implement heuristics and utilize resources, but they need some mechanism to evaluate their progress, so that they are aware of and critically examining their decision-making. We believe mechanisms for control can be influenced by the design of technology tools such that students can use resources in the technology tool as means for reflection that result in control activities. By analyzing students' control throughout their problem solving with technology, we can ascertain how the design of a particular tool is influencing students' control to adjust their use of resources (mathematical and technological) and implement or modify heuristics.

Gallo (1994) makes a distinction between two types of control, ascending and descending, that are exhibited by problem-solvers as they are working toward a solution.

Ascending control is exhibited when a student's problem solving activities lead to a new interpretation of the problem or sub-problem. Descending control is exhibited when a student's mental model of the problem is used to guide goal-oriented activities. The constructs of ascending and descending control are useful for analyzing students' implementation of heuristics and resources as evidenced in each action of their problem solving process that is ascending to or descending from a new interpretation of the problem. It is also interesting to consider the instances when students transition from one type of control to the other (i.e., ascending to descending, descending to ascending). Several researchers have drawn upon the notions of ascending and descending control in their work with students using technology tools (e.g., Arzarello, Olivero, Paola, Robutti, 2002). We expect that the constructs of ascending and descending control can be used to characterize students' uses of features found in tools. The types of features that are incorporated in the development of a tool are often guided by a set of design principles.

We view the development of design principles for mathematical technology tools as an iterative process that, while initially informed by research (e.g., Clements & Battista, 2000), may be refined based on results of field-testing and analysis. The development of some tools may occur without an explicit set of design principles in place, especially in today's ever-changing technological landscape with capabilities of rapid development and dissemination via the internet (e.g., java applets). However, if these tools are carefully designed, it may be possible to identify best practice principles that can then inform future design of other tools for similar purposes. Through a six-step process, the IDEA (Identifying Design principles in Educational Applets) project analyzed existing mathematical applets[1] and generated design principles in three different categories: motivation, presentation, and support for problem solving (Underwood et al, in press). Our research analyzed the extent to which nine design principles (DPs, Table 1) from Underwood et al's problem-solving category supported students' control while they were using one of the mathematical applets and problem tasks (Figure 1).

For their birthday, the Carp triplets received 26 tropical fish: 13 females and 13 males. They discussed ways to divide the fish among their three tiny backyard ponds. Angel said, "I want the same number of male and female fish in my pond." "Okay," said Molly. "I want three times as many males as females in my pond." "Then I want twice as many females as males in my pond," Gar replied. Is there a way to put all 26 fish into those three ponds, while giving each triplet what he or she wants?	 Interactive Java Applet
How many male fish and female fish does each triplet get in his or her pond? Describe the work you did to find the solution.	

Figure 1: Fish Farm Task (mathforum.org/escotpow/solutions/solution.ehtml?puzzle=40)

Design of Technology Tool

The Fish Farm applet (Figure 1) was created with a tank on its left side with 13 male and 13 female fish that the user could "drag and drop" into any of the three ponds. The representations (iconic fish, pie graph and ratio count) are linked. As a fish is dropped into a pond the ratio count and pie graph representations are "updated." However, this link is unidirectional because a student is only able to manipulate directly the iconic fish and observe changes in the other representations. The ratio count and pie graph are intended to display the current status of the part-part and part-whole relationship between males and females in the pond that can facilitate a better understanding of the problem and engage students in thinking about how to adjust their strategy. In addition, it is intended that the ratio counts may alleviate having students count fish in the ponds and promote a transition between reasoning part-part and part-whole about the ratios.

Methods and Findings

The data for the current project was collected as part of a larger study on preservice teachers' (PSTs') learning to facilitate students' problem solving (Stohl, in press). Each PST worked with a pair of eighth grade students (age 13-14) on two different occasions from below average ability-grouped "Math 8" classes. The PSTs were juniors in a class focused on learning to teach mathematics with technology. The PSTs and students were videotaped to capture the computer screen as well as social interactions. For the current study, we analyzed the work of a sample of four pairs of students each interacting with a PST on the Fish Farm problem (Figure 1).

It appeared that all design principles except DP1, DP5, and DP9 were implicitly followed by the creators of the Fish Farm applet (see Table 1). DP1 was not followed because a history of students' actions and intermediate results are not recorded. However, DP2 was followed because a student could choose to begin their work on the problem with any fish in any pond, thus allowing for a variety of initial strategies (DP3).

Each videotape was viewed while the researchers made notes about how each design principle appeared to be supporting or not supporting the type of control students and PSTs were exhibiting. Transcripts were analyzed as we mapped out each pair of students' problem solving path, noting ascending and descending control trends as well as when students switched between the two types of control. We then coded when each design principle seemed to support (positive) or not support (negative) four types of students' use of control: ascending, descending, a transition from ascending to descending and a transition from descending to ascending.

If a design principle was followed, a tally in the "+" column (Table 1) indicates that students and teachers were making use of some aspect of the technology and this use allowed them to effectively exert control. A tally in the "-" column indicates that students were not utilizing this aspect of the technology and thus their ability to effectively control their problem solving is hindered. If a design principle was not followed, a tally in the "+" column indicates that students and teachers were making use of the principle by using other means besides the technology (e.g., recording history of results on paper) and there was a positive effect on their problem solving. A "-" tally indicates that the violation of the design principle was having a negative effect on the problem solving.

Design Principle	Actor	Ascending Control		Descending Control		Switching from A to D		Switching from D to A	
		+	-	+	-	+	-	+	-
DP1 Provide a history of actions *not followed*	Teacher	1	0	0	0	1	0	0	0
	Student	0	0	3	0	2	0	0	0
DP2 Allow multiple entry points *followed*	Teacher	0	0	1	0	1	0	0	0
	Student	1	0	8	0	0	0	0	0
DP3 Allow multiple approaches/strategies *followed*	Teacher	3	0	2	0	0	0	5	0
	Student	6	1	2	0	6	0	2	0
DP4 Use multiple representations *followed*	Teacher	2	0	2	0	0	0	8	0
	Student	1	1	4	6	5	3	2	7
DP5 Give students opportunity to predict *not followed*	Teacher	1	0	0	0	0	0	0	0
	Student	0	0	0	0	0	0	0	0
DP6 Reward thoughtful strategic use *followed*	Teacher	1	0	0	0	0	0	0	0
	Student	1	1	3	0	2	0	0	0
DP7 Provide closure *followed*	Teacher	0	1	1	0	0	0	1	0
	Student	1	2	1	3	0	0	2	0
DP8 Provide status feedback *followed*	Teacher	5	0	6	0	0	0	15	0
	Student	10	0	8	4	6	0	8	2
DP9 Programming accuracy *not followed*	Teacher	0	1	0	0	0	0	0	0
	Student	0	1	0	8	0	0	0	2
Totals DP1-9		**33**	**8**	**41**	**21**	**23**	**3**	**43**	**11**
Totals DP1-8		**33**	**6**	**41**	**13**	**23**	**3**	**43**	**9**

Table 1: Interaction between design principles and students' and teachers' control.

The tallies in Table 1 allowed us to look across the work of the four pairs of students to characterize an overall effect of the design principles on the use of control in students' and teachers' problem solving. Overall, the nine design principles had a more positive effect in every type of control exerted. For several design principles we noticed a difference in whether the design principle was supporting control for students versus PSTs (e.g., DP4). DP9 (Program applet with needed accuracy) showed only negative effects on control, as we did not code positive instances of DP9. We assumed that when DP9 was followed, every control action made by students and teachers was positively supported by the accuracy of the technology and that the technology added value to students' problem solving. Discounting DP9, the final row of Table 1 indicates that the other design principles (1-8) had only 31 combined instances of negative effect on control, in comparison to the 140 instances of DPs 1-8 having a positive effect on control.

Looking within control type, the design principles were almost always positive in a transition from ascending to descending control. The most negative effects of the nine design principles occurred during descending control when students were trying to implement a strategy. However, there were also a large number of positive effects (n=41) on students and teachers' control in descending control. Usually a design principle was supportive of teachers' and students' control when they switched from descending to ascending control. In the switch from descending to ascending, there were also many positive instances (n=43) of design principles supporting control. The design principles were also mainly positive (n=33) in ascending control. Thus, as students and teachers examined results from descending control actions and began to tweak a strategy or build up to a new model of the problem, the design principles were overwhelmingly positive (n=76) rather than negative (n=19).

Each of the nine design principles had their unique contributions to students' and teachers' control. A brief description and illustration for one design principle (DP3) is provided to elaborate on the characterizations we developed from the cross-pair analysis. Descriptions for all design principles are provided in Stohl & Hollebrands (in review).

DP3 (Support multiple approaches and strategies) may have supported students' use of a variety of heuristics and mathematical resources available to them and allowed students to use different strategies to solve the problem. As a result, more students were able to meaningfully engage in solving the problem and obtain a solution. The analysis of the data revealed that students did employ a variety of strategies. These strategies included a base rate "round robin" strategy, the use of equivalent ratios, and the coordination of the number of fish in the ponds and tank.

The base rate round robin strategy involved placing a minimum number of fish in each pond to satisfy its ratio while students were exhibiting descending control. For example, a student placed 1 male and 1 female in Angel's pond, 3 male and 1 female in Molly's pond, and 1 male and 2 female in Gar's pond and repeated this procedure until there were not enough fish to satisfy a pond. This contrasted with an equivalent ratio strategy in which students, again exerting descending control, placed a number of male and female fish that satisfied the ratio for that pond. For example, a student placed 3 males and 3 females to satisfy the 1:1 ratio for Angel's pond. They then moved on to one of the other two ponds placing a number of fish that satisfied its ratio. Upon examining these results, they then focused on the remaining pond, which sparked a transition from descending to ascending control. However, when students focused only on the number of fish in each pond, they often reached a juncture in their problem solving where they could no longer proceed and needed to reexamine and adjust their strategy. Some students used a strategy that coordinated placing fish in each pond to satisfy its ratio while attending to the number of fish remaining in the tank and determining whether the ratio for another pond can be satisfied was described. This strategy was exhibited in quick alternations between ascending and descending control.

The inclusion of the multiple approaches design principle supported students' implementation of and changes in a strategy throughout all aspects of ascending and descending control. This design principle also supported the control exerted by teachers as they questioned students and helped them refine their strategies, and in some instances, actually suggest a particular strategy to implement. The ways in which the PSTs facilitated students' problem solving was examined in detail by Stohl (in press). In only one instance, this design principle did not support students' problem solving. In this case the ability to use multiple approaches to solve the problem led to the implementation of a random trial and error approach, occurring in an ascending control trend, which did not assist students in making progress toward a new strategy or solution.

Discussion

We found that certain design principles appeared to facilitate students' exertion of ascending or descending control or transitions between the two. In particular, the availability of status feedback seemed to enable students to continue to work on the problem without reaching an impasse. This finding is similar to the results reported by Hillel, Kieran, and Gurtner (1989) who claimed that in a Logo-based microworld, students did not become stuck while solving a problem. The feedback from the Logo environment, as well as feedback in the Fish Farm applet in our study, often led to another spontaneous action that kept students' problem solving process proceeding. In our study, students had the advantage of receiving feedback from the computer as well as a teacher. However, designers of technology tools cannot assume that this type of support is always available and should include substantial opportunities for students to receive status feedback from within a technology tool (DP8) that can promote active reflection on goal-oriented activity and reinterpretation of a problem when needed.

Through the four different modes of control, the representations in the applet played a role of support for the students. Many have suggested that multiple representations (DP4) may enable students to focus on different aspects of a mathematical idea (DuFour-Janvier, Bednarz, & Belanger, 1987). The designers of the Fish Farm tool included the pie graph and ratio count in anticipation that students could use these representations to reason about the part-whole or part-part relationships. However, we found that students did not choose to capitalize on the availability of these representations while solving the problem (Figure 1). Rather, students tended to focus on a single representation – the iconic fish that they are able to manipulate directly. This is consistent with the findings of other researchers who report that even when multiple representations are available to students, they focus only on a single representation (e.g., Yerushalmy, 1991). The focus on a single representation occurred most often when students were exerting descending control or switching from descending to ascending control. In these cases students appeared to generate a strategy with the iconic fish and execute that strategy without explicitly referring to the other representations in the applet. It seems that the pie graph was unfamiliar to these students and may not be an effective representation for them to control their problem solving.

It is possible that the absence of students' self-initiated uses of multiple representations was influenced by the design of the tool. This applet has a single access point for manipulating the representations. The only way students can interact with the applet is to manipulate directly the iconic fish, the same representation students mainly used in their problem solving. It might be that this design decision influences the strategies students implement, and the representations they use as they execute control.

The design principles we used are focused on small technology tools (e.g., java applets) designed to allow exploration of a particular mathematical problem. However, we feel these design principles would also benefit designers of more open-ended software for mathematical problem solving. We do not believe that there is a single optimal set of design principles which will completely specify features that will make an educational applet or software application "work" for every student. However, we are excited about our current process as a possible way to inform the design of software intended to promote problem solving and for other researchers to use our analysis techniques to analyze students' control of problem solving with technology tools.

Endnote

1. The java applets examined were developed to accompany middle school mathematics problems (EPoWs), and implemented as part of the NSF-funded grant (REC #9804930) *Educational software components of tomorrow: A Testbed for sustainable development of interoperable objects for middle school mathematics* (*ESCOT).*

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THE EFFECT OF LEARNING OBJECTS ON MIDDLE SCHOOL STUDENTS' MATHEMATICAL THINKING

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This study compares grades 7 and 8 student performance when completing guided investigations with and without access to learning objects that facilitated mathematical investigation. Students performed equally well with or without access to the learning objects. The findings are consistent with other studies where care has been taken to ensure that tasks with access to technology and tasks without access to technology are similar.

A persistent characteristic of mathematics instruction appears to be teacher exposition and student mastery of mathematical procedures for getting correct answers. Typically missing from many mathematics classrooms are higher level thinking activities that engage students in investigation, conjecturing and sense making (McGowen & Davis 2001a; McGowen & Davis 2001b; Romberg 1992). A recent 'solution' to this problem comes in the form of learning objects facilitating higher level mathematical thinking.

Learning objects are a subset of computer software. They are typically web-based, and they are unique in their focus on small interactive units that do not requite students spend a lot of time learning how to use them. Computer-based representations may offer a some advantages for mathematics learning, such as making abstract concepts concrete and manipulative (Lester, 2000; Vosniadou, 1996), supporting higher order thinking by reducing cognitive load (Connell 2001; Kieran, Boileau, & Garancon 1996; Lajoie, 1993; Surgue, 2000) and thus help support conjecturing and hypothesis testing, and improve student mathematics understanding and performance when aligned with constructivist philosophy (Connell 1998). Well designed learning objects may also positively affect the 'cognitive ecology' of mathematics learning environments, perhaps changing how teachers and students view and do mathematics (Borba & Villarreal 1998; Confrey 1993; DiSessa 2000; Levy 1993).

However, some research also indicates that attempts to change classroom practice "are minimally effective, in part because teachers filter what they learn through their existing beliefs" (Gadanidis, Hoogland & Hill 2002; Norton et al 2000; Stipek et al 2001) and teachers assimilate new ideas without substantially altering existing beliefs that drive their practice (Cohen & Ball 1990; McGowen & Davis 2001a; McGowen & Davis 2001b; Norton et al 2000). Computer technology continues, for the most part, to be used at low levels and does not meaningfully engage students with the subject matter (Becker 1994; Clements 2000; Mariotti 2002). Teachers continue to doubt the potential of computers to enhance mathematics learning (Norton et al 2000).

In the study presented below, we explore middle school students' thinking with learning objects that facilitate mathematical investigation.

The Study

The study involved 8 seventh grade and 8 eighth grade students. In each grade, 4 of the students were male and 4 were female. These were high achieving mathematics students, most ranking mathematics as one of their two most favourite subjects. The students were identified through the middle grades resource teacher at the school. The study intentionally sought a homogeneous group of students. This has the advantage of reducing the number of factors affecting the results of the study, thus focusing more clearly on student mathematical thinking, and its relationship to the use of learning objects. The disadvantage, of course, is that the results of the study will not apply as broadly.

The goal of the study was to explore how students engaged with guided investigations – whether they succeeded in completing them – and whether they performed differently when they had access to the learning objects. Each student performed two investigations: one on probability and the other on linear functions. Each investigation had two versions: one using a learning object and the other using a hands-on investigation that mimicked the learning object investigation. The intent was that the learning object version and the hands-on version of each investigation would be as similar as possible, in terms of the type of mathematical experience provided for students.

The problem presented to students through the probability learning object was to design a winning strategy for matching the frequency of possible sums when 2 dice are rolled 36 times. In the hands-on version of this investigation, students used pencil, dice, grid paper (similar to the grid in Figure 1, with the numbers 2-12 at the bottom of the grid), and bingo chips. They chose a strategy by marking squares using their pencil. Then they rolled the dice 36 times and recorded results using the bingo chips.

The problem presented to students through the linear functions learning object was to find 5 ordered pairs that match the given rule (of the form "the sum of my coordinates is …..."). Students were also asked to make conclusions about patterns that they noticed. For the hands-on version of this investigation, students used pencil, a grid, and small black buttons to mark points on the grid. It should be noted that, unlike the probability investigation, the linear functions investigation is not part of the mandated curriculum for seventh and eighth grade students (Ontario Ministry of Education 1997). Although students in grades 7 and 8 have to know how to solve linear equations, such as $ax = c$ and $ax + b = c$, by inspection and systematic trial, they do not formally deal with linear functions until grade 9.

Students participated in structured task-based interviews (Goldin 2000) which typically lasted for one hour. The task-based interview is useful as a qualitative assessment tool, offering "the opportunity for research-based inferences about students' achievement of higher and deeper mathematical understandings" (524). Each of the students was presented with the probability and linear functions investigations described above, one of which involved the use of a learning object and one a hands-on activity. The order in which the two investigations were presented to students, and whether it was the first or the second investigation that involved a learning object, was varied. A common set of questions and prompts were used for all interviews. Students were offered help in some cases. For example, some of the seventh grade students were not familiar with negative coordinates. However, no help was offered on the core problem of each investigation. All interviews were conducted during school hours, in a small conference room in the students' school. In addition to completing the investigations, students were interviewed on their perceptions of the investigations and how these activities compared with their classroom activities.

The interviews were tape-recorded and transcribed. Field notes were taken during and following the interview, recording the interviewer's observations of student performance, attitude, and body language. A content analysis was conducted of the transcribed interviews and field notes (Berg 2004). The content analysis focused on three categories: student performance on the probability and linear functions investigations, the role and effect of the learning objects, and the nature of mathematics instruction experienced by the students.

Results

The results of the study are summarized below. The summary is presented in the three categories used to organize the content analysis: student performance, the learning objects, and mathematics instruction.

Student Performance

All students successfully completed the probability investigation (with or without the learning object). By 'successfully' we mean that the students were able to generate and explain an appropriate winning strategy. Approximately one-third of the students started their investigation with the correct prediction, one-third with a prediction that was close to being correct and which they fine-tuned through trials, and one-third with clearly incorrect predictions that they corrected through trials.

All students successfully completed the linear function investigation (with or without the learning object). That is, they were able to predict the nature of the linear function graph from its equation and they were able to explain how the graphs and the equations related to one another. When asked to predict what the graph of the five points would look like (before actually plotting any points), students gave a variety of responses. Some thought the points would form a diagonal line. Others thought "there would be a pattern, but I don't know what." Some had no idea. At the end of the investigation, most students were also able to generate coordinate rules that would cause the graph to tilt in a different direction, such as $x - y = 2$. There was no observable difference in performance between students based on their grade, based on the investigations they completed, or based on whether they used a learning object or a hands-on activity. The only exception was that some seventh grade students needed help remembering, or learning for the first time, how to plot ordered pairs with negative coordinates.

Most students said that they did not find the investigations challenging and this was consistent with observations of their performance. Some found them challenging at the beginning. "But when I got the hang of it, what it was asking me, then it was okay." However, many of the students seemed challenged or hesitant when talking about their understanding. "I can't put it into words, really. I just get it." Although it was evident from their actions that they understood, they experienced difficulty in describing or explaining their understanding.

The Learning Objects

All students responded positively to the learning objects. For example, most used the mouse without hesitation, often doing so in anticipation of the activity, especially in the probability investigation. With the probability learning object, most students became very involved and experimented and tested strategies without prompts. With the linear functions learning object, however, students never moved on to the next game without first being instructed to do so. It appears that the probability learning object drew students' attention much more than the linear functions learning object. There was no observable difference in student engagement between investigations using the learning objects and investigations using hands-on activities. When asked whether they preferred the hands-on or the computer activity, most students did not identify a preference. "See, the paper and the computer, there's no real difference. You have the mouse and you have these little buttons. I'd say they're just about the same." Another student said, the computer "didn't really help me learn more than on paper."

On average, the hands-on investigations did take longer to complete. However, the length of investigations varied significantly. How long investigations lasted seemed in part to be affected by the ‘personality’ of the student – how they engaged with specific aspects of an investigation and how talkative they were.

Mathematics Instruction

For the most part, students’ description of mathematics instruction they experienced in school involved teacher exposition and student practice. The teacher “explains it and we get to work,” said one of the students. “We don’t really do experiments. It’s more of the seat work, with the writing down on paper, the questions in the textbooks … We usually take up homework from the day before and then she’ll tell us what we’re doing, and we do it for the rest of the period pretty much.” Very few students gave any examples of investigations or. When these were described, they were often in the form of teacher demonstrations.

Most students described math as very “structured.” As one student commented, “Math is usually strict. You have to do this, this way, because everybody’s already found out the right way. English you can do more improvisation, your style of writing and stuff. So it’s a lot more open.” Another student said, “Math and science you don’t really have to be too creative, because it’s like there. You either know it or you pretty much don’t.” Some students seemed accepting of this ‘reality.’ “You need some subjects with structure. So I think it’s okay.” A passive role for students is consistent with students’ difficulty in communicating their understanding.

It should be noted that in addition to the 16 students already mentioned, 5 students took part in a smaller pilot study, one of which was in grade 5, two in grade 6, and two in grade 7. The methods and results of the pilot study were identical to the ones with the other 16 students.

Discussion

It is not surprising to us that all students in the study successfully completed the probability and linear functions investigations, with or without access to the learning objects. First, because the students in the study have a history of succeeding in mathematics and enjoying the subject. Also, we have similar anecdotal evidence from having tried both activities in variety of whole class and small group settings. For example, we have used both activities in a low-achieving grade 6/7 split class, starting with the hands-on version, where students worked in pairs, and ending with learning object version as a whole class activity (using a data projector to display the learning object). Also, we have used the hands-on version of the linear functions investigation in two fourth grade classrooms, where students worked in pairs in a guided investigation very similar to the one in this study – and we have noted similar student success (Gadanidis in press). What is perhaps surprising is not that students succeeded with the investigations but rather that such investigations are apparently uncommon in their mathematics classrooms.

In a study of pre-service mathematics teachers’ lesson planning (with and without similar probability and linear functions learning objects) we noted that pre-service teachers often hesitated to use investigation activities due to a variety of factors, including (1) personal beliefs about what students can do and how they learn, (2) classroom management concerns associated with offering students opportunities to investigate, and (3) the depth of their personal mathematical knowledge (Gadanidis, Gadanidis & Schindler 2003). The pre-service teacher tendency to plan lower level mathematics activities is consistent with the types of classroom activities typically experienced by the students in the present study. However, it is interesting that most pre-service teachers noted that, as students, they would have enjoyed most the “actual hands-on experimental part” and the “individual discovery,” as this is “the part where you’re having the most fun.”

It should also not be surprising that students performed equally well with and without access to the learning objects, as care was taken to ensure that the guided investigation tasks with access to technology and tasks without access to technology were similar. Similar findings of minimal effect due to technology were found by Kim, Savenye & Sullivan (2003). Clark (1983; 1995) and Clark & Surgue (1995) suggest that it is the method of instruction – how technology is used – rather than the technology itself that makes the most difference. Studies on the effect of technology on learning can easily be confounded if the instructional model used with technology is different from the instructional model used when the technology is not present.

Conclusion

Our study focused on using learning objects in a guided investigation setting with students who experienced success in mathematics and liked the subject. Further research might focus on a more heterogeneous group of students or on the use of more sophisticated learning objects, such as the Dice Explorations learning object shown in Figure 1. Although we plan to pursue both of these research routes, we are also becoming increasingly interested in aesthetic aspects of mathematics teaching and learning. Both the students in the present study and the pre-service teachers in our preceding study (Gadanidis, Gadanidis & Schindler 2003), identified aesthetic associations with investigating and understanding mathematics. Similar findings are reported in related studies (Gadanidis & Hoogland 2003; Gadanidis, Hoogland, & Hill 2002; Gadanidis, Hoogland, Jarvis & Scheffel 2004). What are the aesthetic associations made by teachers and students in relation to mathematical activity that involves higher level thinking (with and without learning objects)? How might the aesthetic be used as a guide for teachers attending to pedagogical directions that travel beyond mathematics as the learning of procedures?

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THE VARIATION IN THE UNDERSTANDING OF THE LITERAL AS A VARIABLE

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This paper reports the advances of a research project concerned with the understanding of the variation*, the* variable *and the* symbolization *of variables in a* functional relationship. *We used the dynamic geometry software* Cabri-géomètre *to design activities to explore geometric figures that operate as icons and indexes of the variation; such figures point to a symbolic representation and understanding of the variable. Current findings of this study show that these geometric figures considerably help beginning students of algebra to understand the variable and its symbolization.*

Introduction

The transition from arithmetic to algebra requires the learning of new concepts and demands the acquisition of new abilities; in this transition, students encounter many challenges and difficulties. Several studies point out different problems that students face when they start learning algebra, for example: limited interpretations of the *equal* sign (Kieran, 1981; Kieran & Sfard, 1999); difficulties to solve equations with unknown factors at both sides of the equation (Filloy & Rojano, 1989); false conceptions of what the literals mean to the unknown factors or variables (Kieran, 1985); or difficulties to comprehend the variable (Fujii, 2003; Moreno & Santillán, 2002).

Researchers of educational mathematics have used computers and calculators with the intention of enhancing and enriching the teaching and learning of algebra. For example, the concept of variable has being treated using the computer program *Logo* (Noss, 1986), and electronic spreadsheets (Rojano, 1996). In addition, new approaches to the teaching of algebra promote *functional thinking* based on computerized technology (Olive, Banton & Kaput, 2003).

In Mexico, the use of letters to represent formulas of areas and perimeters is introduced, without many complications, in fifth grade. However, when in seventh grade, letters are employed to represent variable quantities, the students face serious problems to accept and understand the symbolization of numbers with a letter. In that direction, we explore the potential of the dynamic geometry software*, Cabri-géomètre,* to enhance the beginner students' understanding of algebra, particularly, the notion of *variable* in its functional relationship and symbolization. We study the mediation between mathematic objects and their learning as a specific tool, and, attempt to address the question 'what mechanisms interplay in the understanding of the variable in dynamic geometry environments'?

Theoretical Framework

All forms of human activity are mediated by tools. Vygotsky (1978) gave meaning to such affirmation, broadening the notion from the physical tools to signs. According to Vygostky, both physical tools and signs are characterized by a mediating function in which "the sign operates as an instrument of psychological activity in the same way a tool operates as an instrument of work". Building upon Vygostky's idea that introducing a new tool in the activity alters human behavior, Wertsch (1993) argues that the tools that are utilized to carry out an activity affect the thoughts and ways of thinking (p. 32). Based on the previous sentence, we assume that introducing new tools in classrooms will open new options to the treatment of the teaching and learning of mathematics; that is the case of the dragging effect, a tool of dynamic geometry that makes possible the generation of a treatment to study both the notions of variable and variation. With the construction of simple figures and by dragging some of their elements, the connections between the variables that form the structure of the figure are made explicit.

The geometric figures function as referents of the variation and the variables, and as icons or signs where the relationship between form and meaning is based on similitude. The labels, comments and measures added [using the software tools] to the figures, may work as indicators of the variation and variables (that is, as signs) when the relationship form/meaning is based on contiguity. In that way, the computerized environment provides with two elements of the Peirce's triad of the sign (Peirce, 1987). The symbol, the third element of Pierce's triad, can only have an indirect relationship with the object it represents, but as indirect as it can be, the relationship is not arbitrary, it is conventional and it supposes dialectical relationships as icons and indexes.

That linkage, which we believe is inherent to the triad (icon, index, symbol), helped us plan the activities of the experimental study. An activity is constituted essentially by: a) the construction of a figure and actions over some elements such as dragging, measuring, and labeling, and b) communication processes supported by certain key words that designate the actions (drag, measure, etc.), and, key terms such as variation and invariance. These words, that have very precise meanings in the environment generated by the software, constitute the core of an elemental system of communication to exchange and build meanings around the actions, and take us closer to the symbolization and understanding of the variable and the variation. That is the hypothesis supported by this project. Because the keywords are rooted in the actions, when mediated by the tool, keywords function to describe the actions, thus, transcending the communication level and entering the level of internal activity, rearranging it, giving meaning to the actions and constructing meaning.

Exploratory Study

The participants of the study are 20 seventh grade students 12 to 13 years old, initiating the learning of algebra. For many students this means their first exposure to and handling of computers. Such a circumstance requires a brief period of training. To help students familiarize with the use of the software, it is necessary to learn the basic commands, and, in this particular case, understand the meaning of keywords such as: drag, variation, and invariant. Following a set of five activities, the students, guided by the instructor, start using these words to communicate and think about the actions executed in the computer.

During the training period, the instructor also introduces the words: variable, relationship, function, dependence, interval of variation, etc., which acquire precise meanings through the actions mediated by the software. At the end of the training the students work in pairs for another five sessions, performing activities oriented to highlighting the variation and invariance of some elements of the figures. These activities guide students to respond to the questions 'what elements are varying?' 'What remains constant?' 'What causes variation or invariance?' 'What is the dependence between these two variables?' Once the students write down their answers in practice sheets, each pair of students read their answers to the class and discuss the appropriateness of the answers, finally, the class decides which answer is the best. The terms mentioned above are used constantly during the verbal process.

Part (i) of figure 1 shows a basic construction of the first two activities. A moving point (p) is dragged over a segment of a given length. The central part of the figure, (ii), is obtained by transferring to a straight line the lengths Ap, with a center on A, and pB with a center in p. The lower part of the figure, (iii), is constructed by transferring the two lengths with center on A. Part (ii) is constructed after performing several actions with part (i), with this, other actions are performed, and only then the lower part of the figure is constructed. The students add the label p to the moving point and the comments A and B at the ends of the segment. The measuring command is applied to the lengths Ap, pB and AB.

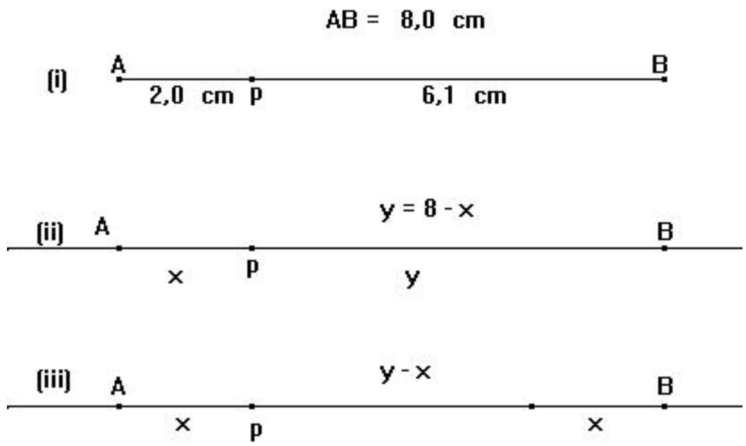

Figure 1

The different values are updated when the moving point p is dragged. The students take notes, discuss their observations and agree on them. For example, considering the higher part of figure 1, an activity consists on obtaining a relationship between the segments. The students start producing verbal statements of the kind, "the length of Ap and pB is the same as the length of segment AB;" "the length of AB plus pB does not change, it is constant," "the addition of segments Ap and pB does not change, it is… constant." These statements, socialized in the discussion, are strongly linked to the manipulation of the objects shown in the screen, to the interaction between the figures and the text (labels and comments.)

The first expressions articulated by the students are of the kind, "3.4 + 4.7 = 8.0". When the students discuss their answers they realize that " there are many correct equalities," then, they set out, "the expression must consider numbers that change." This discussion takes them to expressions such as: 'Ap + pB = AB', or 'Ap + pB = 8'. The instructor asks 'what do Ap and pB represent'? Some common answers are "the numbers that change," and "all lengths of the segments." With guidance from the instructor, the students obtain the expression "x + y = 8". Surprisingly, one pair of students expressed at the end of one session, "an addition of variables equals an invariant!" In figures similar to parts (ii) and (iii), expressions such as y – x, y = 8 – x, x = 8 – y, etc. are discussed.

The activities have been designed to help students in the transition from the graphic-visual level (the figure), to the textual (signs-letters,) going through verbal statements from the actions over the figures to the enunciation and representation with literals of the actions. The purpose is to provoke a process of transformation in which, the symbols, as representations of the objects and the actions over such objects, are internalized.

In this study, we used segments of straight lines, rectangles, triangles and circumferences. We have explored the relationships between segments, arc of circumference, between angles and areas, and between perimeters and areas. In the second part of the study, currently in progress, we introduced tables and graphics as referents of the variation and the variables.

Discussion

Considering the part at the top of figure 1, the initial construction is simply a segment and a point placed on it. This "naked" figure is the basic referent that works as icon of the variation, as an image of the variation of the moving point, since this is the object that varies initially and changes position. Later on, by labeling the ends of the segment as A and B, and the moving point as p, the variation is referred to the segments with name and order: Ap, pB and AB. By naming them they become distinguishable, that leads to associate the variation to the variation of the lengths of these named segments This is the first approximation to the symbolization of the variables and to the construction of their meaning.

Immediately, all segments are measured and their values are associated to a label with the equal sign, for example: AP = 2.8. If p is dragged, the values of Ap and pB are updated "constantly" while AB remains unchanged. At this moment, the modification of the values of Ap and pB can be taken as an indicator of the variation, as an index of the variation and the names Ap and pB acquire a greater sense as variables and, AB as constant.

The transition from the icon to the index starts by measuring the segments and by observing how dragging p modifies their values. Such transition, however, finishes only when the students accept that these measures, the numeric values, take the place of the variation, and point back to the variation. The association of the numeric values to the segments, on the one hand, and the dynamic drawings, on the other, allow us being the context in which the "other" is given meaning and vice versa. Such an articulation is the basis of a comprehensive process of the variation and the representational role of the variables.

The substitution of Ap with x and pB with y is a simple change of designation of the same object, but these new names now refer to the (ii) and (iii) parts of figure 1 and, although the only possible movement is dragging p from (i), there appear new operations that reveal other properties and relationships. Here, the questions that guide the activity are: 'what is the condition so that y – x equals zero'? If in (i) x = y, 'what happens to figure 1 (ii)'? 'what are the maximum and minimum values of x and y in figure 1(i)'? 'what happens at (ii) and (iii) if x is maximum'? 'what happens at (ii) and (iii) if x is minimum'? When y is minimum at (ii), 'what happens to x in (i)'? Each team reads their answers to the class after completing these activities; differences and questions are discussed. Finally, it is proposed the modification of the initial length of the segment AB and repeat the previous activities. These actions as well as the discussion around them, the verbalization of the written responses and the discussion of the best expression nourish the signs with meaning.

Dragging an element of a given figure is the basic action in the dynamic geometry environment; it generates transformations at one or more elements of it, thus, causing the attention and originating the perception of the variation. Students, of course had much earlier, previous to the use of the software, notions linked to the variation [perhaps vague and fragmented images], but, the variation has already been experienced in the every-day life. The construction of the meanings of the variation and the variable is based on the perceptions provided by the tool as well as on the preexistent images and notions. The decisive step in the understanding of the variable, however, seems to be located in the process of the internalization of the represented actions, the verbalizations and the actions with the signs.

How is the internalization manifested in this case? The signs associated to the actions translate into the terrain of the ideas. The following dialogue shows that such interpretation is plausible. Estela (E), a 13 year-old student is presented with a situation linked to figure 2. The instructor (I) suggests the following:

(I) "Move and drag point p and tell me what changes"
(E) "Ap and pB; the total length changes" (she points at the segments Ap and pB)
(I) "Who depends on whom?"
(E) "*Ap* and *pB* depend on *x*"
(I) "Can you describe the existent relationship?"

After some instants of thinking, the student wrote in the blackboard the expresión: S= Ap + pB and says, "But Ap and pB depend on the variation of x" The instructor asked the student 'what does the expression $S = Ap + pB$ tells you'?

(E) "The lenghts change, they are variable, and the sum changes, the sum is a variable too"
(I) "What do you have in mind with that expression?
(E) "When p is dragged, the lengths change and the addition of lengths changes. The values of Ap and pB are varying and their addition is also a variable."

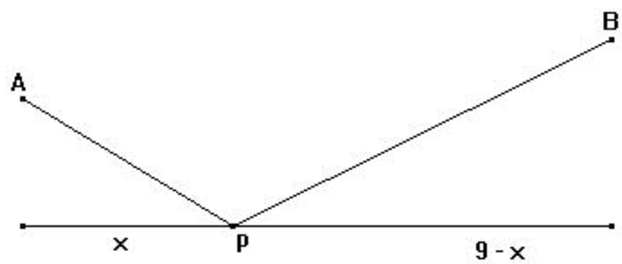

Figure 2

The external signs, the icons (the figure) and indexes (the values that are changing), give sense to the symbols (Ap, pB, y – x, y = 8 – x, etc.), and, from the level of the material actions, the signs reflect on the level of the ideas. In another example, a female student that participated in the study was interviewed one month after the activities ended. Ursula (U) a 13 year-old student was presented with the next problem: "if the cost of each photocopy is 50 cents, 'how much has to be paid for 13 photocopies'?

(I) "How much do you have to pay for 13 photocopies?"
(U) "Six pesos and fifty cents"

After asking other questions to the student, the instructor formulates a new situation.

(I) "can you write down a general relationship to any number of copies?"

After thinking for a brief period of time, the student writes on the blackboard the expression: x (0.5) = y

(I) "What does 0.5 mean?"
(U) "The cost of one photocopy"
(I) "What does the letter x mean?"
(U) "The number of photocopies"
(I) "And, letter *y*?"
(U) "The total cost of all photocopies?"
(I) "What is in your mind when you see this expression?
(U) "If *x* changes, so does *y*, both are variables, but *y* depends on the variation of *x*"
(I) "How do you interpret the expression: y = 0.35x?"
(U) "The cost of the copies is 35 cents. If I want 10 copies, I pay 3.50 pesos."

The ability to generalize and to abstract, associated with the symbols, indicates that the external activity, with the signs, is now in another level (internal). The variables are describing the variation, they represent it.

Conclusion

It is not easy to understand something through a representation; to perceive the changes in different registers and connect them requires learning. Even the perception of the variation assisted with computers faces difficulties. In that sense, the software of dynamic geometry supports the perception of the variation providing with different forms of representation that gives the user the possibility of experimenting and discovering the structural relationships of a drawing. With the appropriate design of activities, these structural relationships may turn into relationships between variables, where, the icons and indexes enrich the meaning of the symbols. The use of computers requires training to familiarize students with the use of the software, but, once the student has become familiar, s/he has access to a new tool that provides new approximations to the traditional subjects of mathematics.

The present exploration of the variable and the variation using the software of dynamic geometry, indicates that the perception of the variation is part of the process of learning the concept of variable in a functional relationship. To perceive the variation and to associate it with the numeric values seems to help students to make verbal and to symbolize the relationships between the variables.

Even though the symbolizations that are shown are elemental, they work, and are one way to support the students understanding. Of course, we continue to explore the potential of “dragging,” along with the design of more focused activities, to support the processes of the symbolization and understanding of the variable, which, will continue to be, despite of technology, a real problem for many students.

We have not treated the variable, in this work, as a generalized number or as an unknown factor, because we speculate that there are other tools that are more appropriate to treat those aspects of the variable. We assume that there are more appropriate tools than others depending on the task to be performed. For example, we can hammer with pliers, but the action is not as practical, the activity performed with that tool may be described as inappropriate and the consequences may be catastrophic. In the same way, the spreadsheet may be used as a tool to study the variation and the variable but it may well not be the most adequate one. Attached to the appropriateness of tools to perform certain activities, there are also words, that is, particular language terms that describe the specific actions of the tools and are key to employ these tools to think.

Words such as: cell, column, copy, and paste, are linked to spreadsheets and describe well the activity in that particular environment, and for that reason they work better to think about the actions performed there. In that sense, the dynamic geometry software is an appropriate tool to show the variation, describe it, as well as think about and develop it’s correspondent symbolization.

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EVALUATING THE INCORPORATION OF COMPUTATIONAL PROGRAMMING ACTIVITIES FOR MATHEMATICAL LEARNING IN MEXICAN SCHOOLS

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In our presentation we will present results from an on-going research evaluating the use and mathematical learning benefits of the implementation of programming activities, using Logo, into the Mexican secondary school mathematics curriculum. Since 1997, the Mexican Ministry of Education has been sponsoring a national project, known as EMAT (Teaching of Mathematics with Technology), aimed at incorporating, through a constructionist approach, computational technologies to the secondary school mathematical curriculum (children aged 12 to 15 years old), in order to enrich and improve the current teaching and learning of mathematics in Mexico.

The project has incorporated results from international research in computer-based mathematics education to the practice in the "real world". In its first phase (1997-2000), the project researched the use of Spreadsheets, Cabri-Géomètre, SimCalc, Stella and the TI-92 calculator, with over 10000 students in that first period (see Sacristán & Ursini, 2001). Despite some difficulties, the impact of the first phase of the project was very positive and the use of some of those tools (Spreadsheets, Cabri and the Graphic Calculator) has been expanding all over the country. The project was groundbreaking in changing the role of the teacher and the traditional passive attitude of children and created an irreversible change that will allow technologies to be incorporated into the Mexican school culture, hopefully in an adequate way.

Despite its success, both national and international advisors pointed out that there was still the need for some form of expressive –e.g. programming— activities, on the part of the students. Thus, since early 2001, a new research phase was undertaken to explore the integration of Logo programming activities into the project. Although in recent years classic Logo programming activities have been left behind in favor of "more modern" tools (other software or Logo-based environments such as LCSI-Microworlds or Imagine), after long and very careful considerations, we opted for this type of activities. This is because we wanted to place emphasis on the *mathematical* learning that can take place by an adequate use of the language, within a carefully structured pedagogical environment.

One of the most important aspects on which we placed emphasis was the pedagogical model encompassing the programming activities. Hoyles & Noss (1992) have pointed to the importance of the setting when implementing mathematical activities with technology. Thus, much of the philosophy and pedagogy underlying the design of mathematical microworlds (see Hoyles & Noss, ibid) was incorporated into the project, although we were constrained by having to comply as much as possible with the present Mexican mathematics national curriculum. We put emphasis on changes in the classroom structure and teaching approach and have developed an extensive amount of worksheets for structuring mathematical microworld activities covering the different themes of the 3-year secondary school mathematics curriculum. We believe that all the above pedagogical aspects are essential for students to benefit from the programming activities.

Much research has been done in many countries, particularly in the 1980s and early 1990s, regarding the advantages for mathematical learning of Logo programming activities. Much of this research is of the qualitative kind using case studies or involving small numbers of students in specific learning environments (e.g. Watt & Watt, 1993) or experimental situations, whereas larger ones have shown inconsistent results. Thus, the large amount of research related to Logo programming in no way undermines the need to carry out a new research in this area (in fact, Clements, 2002, has pointed to the importance of researching curricula after it is produced); this is even more relevant considering the large scale of the project, the different culture examined, and the careful pedagogical model and setting for the programming activities.

In order to evaluate the possible effects of the incorporation of the Logo programming activities, we have been carrying out a multi-level analysis, that started on a small scale 3 years ago and has gradually been expanding, combining both quantitative research (comparing study and control groups) with field observations and interviews. In particular we have evaluated: (i) The ways in which the student and teacher materials are used; (ii) teacher's performance during the Logo sessions; (iii) children's attitudes and, most importantly, (iv) children's mathematical performance both in standardized tests and through their academic scores.

During the academic year 2002-2003, we researched 21 academic groups (7 groups for each of the 3 secondary school grades): 13 study groups involving some 650 students and 8 control groups (aprox. 400 students), with 12 teachers, in two schools. The study groups each had a 50 min. session per week devoted to the Logo programming activities. The control groups didn't, but otherwise followed the same curricular structure, with the same teacher as the study groups.

Among the results of that phase, we have observed a significantly better performance of the 1st and 3rd grade study groups particularly in questions involving algebraic expressions (use of variables) and the area of geometrical reasoning. In particular, first-year students scored much better than the control students in pre-algebra questions and in questions related to the use of symbols and operations (10% better), as well as in proportionality problems. Second-year students didn't show an overall significantly better performance over the control students, but they did in questions related to angles (12.5% better) and proportionality (11% better). In 3rd grade, the study groups were much better in the use of variables (16% better) and also better in geometry questions.

We have, however, also found a strong correlation between our evaluation of teachers' performance during the Logo sessions (teacher's class preparation, their understanding and use of the proposed pedagogical model, their experience working with technology, etc.) and children's performance in math tests as compared with the control groups of the same teacher. In particular, study groups whose teacher rated poorly in our evaluation of his/her performance, didn't score better than their control groups. This clearly points to the importance of the role of the teacher and the implementation of the materials.

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CONNECTING THROUGH E-JOURNALS: BRIDGES BETWEEN DISCOURSE COMMUNITIES

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As educational accountability and revenue shortfalls predominate, educational administrators and policymakers are questioning the link between educational technology use and improved student achievement. Our society's growing dependence on technology has a tremendous potential for providing incentives for using technology in educational reform. While remembering that technology does not drive change, research needs to provide a basis for determining how technologies can support or impede student learning.

Purpose for the Study

The purpose of this classroom-based study (Bolin, 2003) was to explore the changes in a college algebra class as wireless laptop computers were integrated into the learning environment. The study aimed to provide insight into the communication patterns that evolved as the students used the new technology to communicate with their professor and each other and how student participation with the electronic communications contributed to their sense making of mathematics. By examining students' reflective responses to e-journal prompts, I hoped to provide information that might assist other teachers as they struggle to enhance communication and understand student conceptual sense making while potentially offering insights into a more dynamic curriculum with communications, meaning, and inquiry more centrally located.

Research Perspective

To assist in understanding the complex nature of constructed realities, qualitative researchers interact and talk with participants about their perceptions (Glesne, 1999). From the interactionist perspective, I viewed the important role of language or discourse as a social practice (Sierpinska, 1998). As an interpretive, naturalist researcher, I assumed that my research dealt with multiple, socially constructed realities that are complex and inseparable into discrete variables.

I followed Tobin's (2000) interpretive research design within the classroom-based research perspective. Principles of an emergent design that necessitates ongoing analysis and interpretation permitted the study to adapt to changing events of the classroom.

Research Methods

The focus of this study was on the communication patterns that evolved as electronic technology was integrated into the college algebra class. The goal was to explore possible contributions technology made to students' sense making of mathematics.

During the semester-long study, 47 students enrolled in two college algebra classes submitted weekly journal entries through e-mail in response to questions about their mathematical understandings and their attitudes and feelings about mathematics.

Data Sources

Data sources included surveys, e-journals and e-mail, discussion board, and in-depth interviews of 12 students purposefully selected from the two classes. E-journals and e-mail are the focus for this paper. Weekly prompts asked students to describe and connect ideas and concepts encountered in the classroom with their understanding of the concepts and connections to the world outside of the classroom.

Results

While the e-journals supported the communication between the student and the professor, it also encouraged students to take ownership of their learning through their expression of ideas, concepts, and methods in their own words. Students were encouraged to write openly about problems they were having in the classroom, their needs because of different learning styles, and the successes they felt.

E-mail and e-journals provided the opportunities to give individualized feedback and to extend classroom conversation, but it also enabled interactions that could never occur in the classroom because of time, conversation flow, interpersonal dynamics, cultural influences, and language barriers (Kirkley, Savery, & Grabner-Hagen, 1998).

Conclusions and Points of View

The communication patterns were changed through the added dimension of the internet, providing additional opportunities for reflection. The use of E-journals supported and encouraged hermeneutical (Davis, 1997) and multi-perspective listening and expression.

E-journals provided an avenue for students to write as they reflected upon conceptual understanding and participated in the discourse community of the class, bridging the discourse community of mathematics. Additionally, the study supports the assertion that students' writing provides a starting place for change as teachers reflect upon how their students learn and think about mathematics.

As the electronic technology was integrated in the college algebra classes, communication among the students and me evolved into different patterns than I had ever encountered before when teaching this same class. The use of the wireless network that students had access to on their computers made wireless-electronic communication a commodity that could be accessed anywhere on campus and anytime of the day or night.

Electronic communication is rapidly being integrated into the classrooms of the world. The

U.S. government's *No Child Left Behind Act* is mandating requirements for the implementation of various technologies, training for teachers, and calling for states to wire their schools to the Internet. Computer access is necessary for students entering our universities today. "As researchers, we need to provide the designers of learning environments with guidance on how to best position these technologies with the overall course and communication structure" (Kirkley, Savery, & Grabner-Hagen, 1998, p. 232).

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INTERDISCIPLINARY USES OF GRAPHING CALCULATORS IN MATHEMATICS AND SOCIAL STUDIES

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Purpose

Recent reports on the use of technology in public schools suggest that tremendous gains are being made regarding the development of technology infrastructure and general uses of technology in schools. Significant sums of money are being spent to develop a national public technological infrastructure to support technology integration in schools. Given the overwhelming sums of money being spent and the public attention that recent technological innovations have received, mathematics and social studies researchers expect technology to invigorate many aspects of education (Berson, Mason, Diem, Hicks, Lee, and Dralle, 2001; Garofalo, Drier, Harper, Timmerman, and Shockey, 2001).

One area that has greatly benefited from the development of educational technologies is the interdisciplinary study of mathematics and social studies. The graphing calculator, which in recent years has become virtually ubiquitous in mathematics classrooms, has served as a bridge in helping mathematics and social studies teachers find common instructional ground. Despite the emerging importance of the graphing calculator as an interdisciplinary resource for mathematics and social studies, very little research has been conducted on problem solving using calculators in social studies classrooms. This proposed scholarly inquiry would help to fill that void.

The broad goal of this project is to develop a general understanding of how graphing calculators can be used in social studies classrooms. Specifically, we investigated teachers' ability to teach using graphing calculators in interdisciplinary contexts, with emphasis being placed on the impact this type of instruction had on students' performance. The specific research questions included:

- How can graphing calculators be used to teach mathematics and social studies content, particularly in economics classrooms?
- What resources help teachers and students engage in meaningful mathematics and social studies instruction?
- What are the implications for student learning when using graphing calculators?
- Can integrated instruction using graphing calculators promote collaboration between social studies and mathematics teachers?
- What are the implications for teachers' instructional planning when graphing calculators are used in social studies classes?
- Can the interdisciplinary teaching approach to mathematics and social studies satisfy the state and national standards for each discipline?

Perspectives

A recent meta-study of 43 research reports in mathematics education indicates that students who used handheld graphing calculator technology with curriculum materials supporting its use had a better understanding of how to solve problems and interpreting graphs in applied contexts than those who did not use the technology (Burrill, 2002). The use of graphing calculators in social studies helps teachers empower students to construct more personal understandings of the social problems and issues that frame the content of social studies (Drier and Lee, 1999). The instructional use of graphing calculators represents a unique opportunity to enhance the quality of mathematics and social studies instruction.

Professional organizations in mathematics and social studies are encouraging teachers and students to use graphing calculator technology. The National Council of Teachers of Mathematics (NCTM 2000) advocates that problem solving, reasoning, communication, and interdisciplinary connections be woven throughout K-12 mathematics instruction. The National Council of Social Studies (NCSS, 1994) has called for teachers to encourage abstract thought and make use of data related to students' personal experiences and the problems of society. Graphing calculator technology can serve as a catalyst for teachers to create interdisciplinary, real-world projects that engage students in meaningful activities aligned with the goals of the NCTM and NCSS.

Methodology

We used an interpretive case study methodology to focus on how mathematics and social studies teachers understand, approach, and utilize graphing calculators in their classrooms. In addition, we analyzed student work produced in the class for the purpose of uncovering the benefits and limitations of graphing calculator use in mathematics and social studies. We also assessed the problems associated with using graphing calculators. This study aimed to identify and explore resources that helped teachers and students engage in meaningful, not technology driven, mathematics and social studies instruction.

Three economics and two mathematics 11th and 12th grades classrooms from a school in Fayette County, Georgia were utilized for this project. Students from all five classrooms engaged in interdisciplinary lessons in mathematics and social studies over a 2-week period. The lessons were specifically designed for use with Texas Instruments TI-83 graphing calculators. A preliminary interview with one of the social studies teachers indicates that the social studies teachers have seen the devices but have no concept of its related uses. In addition to the lessons, the students in all 5 classes completed 5 integrated social studies and mathematics content assignments over the 2-week period. Students were given pre- and post-tests designed to assess their content knowledge related to the assignments. Interviews were conducted before, during, and after instruction with the five teachers and ten students (two from each class). The purpose of the interviews was to gain a deeper understanding of how teachers and students engaged content knowledge while working with graphing calculators.

Data Collection

In this study, five integrated lessons were used for instruction. As indicated earlier, students were assessed on what they have learned using pre- and post-tests. In addition, pre- and post-interviews were conducted to selected students and five teachers. In brief, data collection for the study consists of: pre-test and post-test, students' work on the individual classroom activities, transcripts of pre- and post-tests interviews of students and teachers, and observation of classroom activities.

An additional benefit of this collaborative inquiry was in the preparation of mathematics and social studies teachers to use graphing calculators in their classrooms. A study of how teachers approach, plan and use graphing calculators in their classrooms enabled us to determine the impact of such activities on teachers' ability to 1) integrate graphing calculators into their classrooms and 2) provide opportunities to teach using interdisciplinary approaches.

Discussion

A change in the teachers and most of the students in their relation to mathematics and social studies and their self-confidence were observed. As expected, the initial stage of the implementation phase posed some discomfort for the teachers, especially the social studies teachers due to their lack of familiarity with graphing calculators. However, the use of graphing calculators provided opportunity for a powerful pedagogical alliance, which led the teachers to think critically about the content they teach as well as led the students to think critically about the content they are learning.

In our preliminary analysis of the data collected, we found that (1) the use of the graphing calculators produced improved performance and deepened students' understanding of mathematics and social studies contents, (2) after the implementation of the integrated activities, the students with low pretest scores gained on the posttest, (3) the students developed a positive attitude toward mathematics and social studies, and (4) the interdisciplinary teaching approach increased collaboration between mathematics and social studies teachers.

On the contrary, we found that the instrumentation process for both teachers and students who were becoming skilled at using the graphing calculators effectively was slow and complex because they require sufficient time to be able to use them effectively. Also, some of the students failed to link graph and symbolic representations. Due to lack of established relationship with the devices, many students accepted the visual image on the calculator screen without considering the context of the task. In the short oral session we will present detailed analysis of the work of the students and the teachers reactions to the integration of mathematics and social studies instruction. In addition, we will present our conclusion and the research implications for both teachers and students.

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RESOLUTION OF ARITHMETIC-ALGEBRAIC WORD PROBLEMS WITH THE USE OF THE CAS (COMPUTER ALGEBRA SYSTEM) AS SYMBOLIC MANIPULATOR

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This paper describes some data obtained in an experimental study with students among 13 and 15 years the ones who are studying the third grade of secondary school in México, when solving arithmetic-algebraic word problems with the use of the CAS like symbolic manipulator, in the context of the algebraic syntax-semantics relationship.

Background

Following the Project of Resolution of Arithmetic-algebraic Problems carried out by Filloy, Rojano and Rubio (Filloy, Rojano and Rubio, 2001) where it was carried out an exploratory study and worked with a group of results, from which ones, empiric derivations were obtained that became in thesis and then, they put on approval in the experimental study of the final phase of the project, serving as analysis instruments; it is accepted in this paper like theoretical framework: the Local Theoretical Models and The Mathematical Systems of Signs (Filloy, 1990). This theoretical framework giv es semiotics tools that allow to analyze the observations of the phenomena, as teaching and learning phenomena, taking in consideration the specific mathematical knowledge, the teaching, the fellows that learn, the observer and the established communication. It investigates this way, the mathematical formal sense of the concepts and the meaning assigned by their use in the problems solve, i.e., their pragmatic sense. This way , it analyzes the strategies and the intermediates codes produced by that learn, while it becomes more competent in the use of the mathematical knowledge. From this perspective, this paper describes the resolution of arithmetic-algebraic word problems with the use of the CAS as symbolic manipulator (symbolic manipulator in the sense of being a "competent user" of the Algebraic System of Signs, within reach of the students) in the context of the syntax-semantics algebraic relationship (Filloy, and Rojano, 2 002).

Regarding the teaching and learning with CAS, MacGregor and Stacey (1996) and Heid (1996) they suggest that this doesn't work alone, in order that the students operate it, they only need not to know how to execute the program, but also that they hope the technology makes for them. Therefore, the use of the CAS can allow the students to develop a more algebraic approach to solve problems, since its use requires a good algebraic understanding.

Data Sources

The students need an elementary algebraic understanding to structure and to transform the equations in order the CAS can help them to find the answer of arithmetic-algebraic word problems. In base of the above mentioned, in this paper some obtained data of the work are shown developed with a group of students between 13 and 15 years, of third grade of secondary school in México, the ones who are studying mathematics through the work of group and individual work, taking a control of their advances.

The teaching model of the CAS included: the resolution of lineal equations with and without the use of the CAS, series of "abbaco" type problems (for example: If to a number, you add twelve and then you multiply it for nine, it gives you hundred seventeen. Which is that number?) to be solved with and without the use of the CAS, as well as of arithmetic-algebraic problems series (organized by family of problems) to be solved by the CAS. Regarding the advance in the explanation of the use of the CAS, the students determined it according to their necessities of resolution of the arithmetic-algebraic word problems.

Results

The following observations are results of the previously described process:

- Still with the use of the CAS, the student returns to his arithmetic strategies, when it is not capable, for example, to synthesize in an equation his analysis strategies. In fact, the students with more algebraic competence, preferred to solve the problem arithmetically, although they could make it with an equation using the CAS.
- Many times the students combine the arithmetic strategies with the use of the CAS, i.e., they can begin with an equation, using the CAS, and if they don't solve the problem, they appeal to an arithmetic posing, returning later to the use of the equation and the CAS to corroborate the result.
- The strategies of the students' analysis regarding the problems, many times they begin with drawings or arithmetic operations, which is useful to them as antecedents to the synthesis of an equation, only the most competent students in the Algebraic System of Signs structure an equation directly of the analysis of the problem and they proceed to "to prove" their equations in the CAS, and once they obtain an answer, they connect it with the problem. In fact, many problems demanded them the interpretation o f the results after obtaining a solution. &n; bsp;
- An adjustment of the intermediate Mathematical System of Signs of the students exists, to have to introduce in the machine an equation through of a Mathematical System of Signs that the CAS interpret correctly. That is, they change their writings to adapt them to the Mathematical System of Signs that it understands the machine: in general terms the Algebraic System of Signs.

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HIGH SCHOOL TEACHERS' USE OF MULTIPLE REPRESENTATIONS WHEN TEACHING FUNCTIONS IN GRAPHING CALCULATOR ENVIRONMENTS

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The NCTM (2000) standards for multiple representations and mathematical connections require teachers to appropriately guide their students on which representations to choose for particular situations. The technology principle encourages students to use technology to support their work with multiple representations of mathematical ideas. This view is supported by numerous research studies, including those that show that classrooms where graphing calculators are used tend to shift from being teacher centered to student centered (Penglase & Arnold, 1996). In this study I examine the relationship between teachers' planned teaching strategies and instructional tasks and the representational forms that emerged during implementation.

This study draws on Vygotsky's (1978) sociocultural theory of learning as well as on a theoretical framework for studying the interaction between technology and the user developed by Salomon, Perkins, and Globerson (1991). I sought to find out how teachers planned their lessons to accommodate multiple representations of functions when teaching with graphing calculators and how the graphing calculators impacted their implementation.

Participants in this study were four high school mathematics teachers drawn from three high schools in a medium sized city school district in the northeastern United States. Sources of data included task-based interviews and classroom observations.

Results show that calculators influenced the teachers' selection and design of instructional tasks while the tasks mediated calculator usage. The teachers said that they did not think that calculators affected their planning. However, further discussions revealed that calculators were indeed central to most of their planning. No two teachers taught the same lesson but the tasks they selected were similar in terms of calculator usage and function representations elicited. Most of the tasks presented equations and required students to obtain tables and graphs. Students would discuss and decide, either in small groups or whole class, window settings for their calculators.

Occasionally the teachers thought their lessons achieved beyond what they had expected but often times they thought the lesson could have been better.

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ANCHORING PROBLEMS IN TECHNOLOGY-RICH ENVIRONMENTS TO ENHANCE MATH SKILLS OF ADOLESCENTS

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1. Topic or issue that your poster will address

The poster will present Fraction of the Cost, the software that we have developed and researched over the past several years. We have used it in math classrooms to help students learn fractions in a more realistic way in the context that is meaningful to the students.

Fraction of the Cost is a video-based software to engage low-achieving students in math classrooms. The short video situates instruction in contexts that students find meaningful in their real life. Through this particular video, students are challenged to build a skateboard ramp with the least amount money possible. After students have figured out the minimum amount of wood they need with spending the least amount of money possible, students actually build the ramp in classroom. Students are learning fractions and other math involved in carrying out the project in a complex, real life context.

2. The elements of the poster presentation

The poster will include the following elements:

A. Introduction: Introduction will address the current issues in math education.

B. Fraction of the Cost: This main part will show the structure and content of the software, how it is used in math classrooms, and the actual research results that we have gained from test scores and classroom observations.

C. Outcomes: This part will highlight the positive outcomes of using the video-based instruction in classrooms. This includes; enabling students to create rich mental models of math problems, uncovering contextual factors that contribute to student achievement, bringing together unique combinations of teachers (math, technology education, and special education) in multiple settings, and sustaining school-based learning with community and workforce supports.

D. Feedback from students and teachers: This final part will display some of the feedbacks we have received from students and teachers who have participated in the study using Fraction of the Cost software.

3. Format of the poster

I will use poster board for presenting the introduction, structure and content of the software, outcomes, and feedbacks. A laptop and a projector will be used to show the video embedded in the software.

MATHEMATICS LEARNING OBJECTS FOR AT-RISK SECONDARY STUDNENT RESULTS OF AN ONLINE SURVEY

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This poster displays database information gathered from a survey of online mathematics learning objects. The survey was the first step in a long-term project to develop electronic resources suitable for at-risk secondary students on fundamental mathematics concepts.

There are studies of how to teach students with learning problems, and how to present particular concepts that students find difficult (cf. Pegg, Graham, Doran, & Bellert, 2003; Raborn, 1995). In addition, research has shown that low achieving secondary students benefit from the use of technology in mathematics (cf. Pijls, Dekker, & Van Hout-Wolters, 2003; Schumann & Green, 1994; Sutherland & Rojano, 1993). The recent proliferation of web-based applets offers the opportunity for researchers and educators to investigate, in light of these studies, whether online learning objects meet the needs of at-risk learners and if so, to develop methods for incorporating these resources in the mathematics program.

The survey focused on identifying and annotating web-based mathematics objects and modules: 1) that are appropriate for adolescent learners, and 2) that address fundamental topics such as fractions, factoring, area, perimeter, volume and probability. The following categories chosen from the Learning Object Metadata database (Suthers, 2001) were used to tag objects: 1) General -Identifier, Title, Keywords, Language, Location, Description; 2) Educational - Interactivity type, Learning resource type, Typical age range, Difficulty, and 3) Annotation – i.e., comments on the educational use of the learning objects as well as information on when and by whom the comments were created.

The categorized learning objects will be analyzed in the next stage with regard to their potential in fostering at-risk students' higher-order thinking and problem-solving strategies.

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ACCESS TO COMPUTER TECHNOLOGIES AND MATHEMATICS OUTCOMES: FINDINGS FROM CANADIAN GRADE 8 MATHEMATICS CLASSROOMS

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One of the major goals of the reform in mathematics education is the use of school computers and Internet access to enhance academic learning. But just how close is this goal being achieved in mathematics classrooms? In this paper, we employed the 1999 Canadian data from the Trends in International Mathematics and Science Study (TIMSS99) to explore students' access to computer technology in learning mathematics. The data include information about access and use of computer technologies in Canadian grade 8 mathematics classrooms. Students responded to questions about access to the Internet at school, and their teachers' use of the Email, world-wide-web (WWW) and classroom computers for instructional practices. Our analysis indicated that while over 85 percent reported that they have access to internet at school, only about 34 percent reported their teachers' use of WWW, and about 18 percent reported that their teachers assigned mathematics projects requiring the use of Email. About 17 percent reported that their teacher use classroom computers to demonstrate mathematics ideas. We carried out further analysis to determine the effect of computer technologies on students' cognitive and affective outcomes. Our analysis could not provide evidence that computer technologies increase students' mathematics achievement. A finding that surprised us most is the negative correlation between teachers' use of technology to demonstrate mathematics ideas and, students' cognitive and affective outcomes. We suspect that the integration of technology in mathematics learning environments is still a challenge to the few teachers who use these technologies in their classrooms. We contend that the reform in mathematics teaching and learning that computers are expected to incite in Canadian mathematics classrooms has progressed slowly. We recommend education policies that provide opportunities for teachers to involve in the planning and implementation of technology innovations in the classrooms through professional development.

DESIGNING EDUCATIONAL SOFTWARE FOR TEACHERS: ANIMATIONS IN MATHEMATICS

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This poster presents software tools that have been designed to help teachers emphasize meaningful mathematical modeling experiences for young children where students can develop conceptual systems that involve three basic math skills that are notoriously difficult to teach and learn:

1. Whole-number addition and subtraction
2. Fraction addition and subtraction
3. Multi-digit addition and subtraction

The theoretical structure that underlies the design of each of these software tools draw on the pool of knowledge about the nature of children's early concepts related to whole number arithmetic, measurement, geometry, chance, fractions, ratios, and rates (Fuson, 1982, Lesh, Post & Behr, 1988). Salient insights from these varied perspectives are integrated into a single unified account that attempts to explain how these skills and their associated conceptual systems develop.

The software tools feature:

- Simulations of "real life" problem solving situations that emphasize abilities that are needed for success beyond school. This includes describing relevant relations, operations, transformations, patterns, and regularities that exist in complex mathematical systems.
- Sense-making abilities that are needed to describe, explain, quantify, coordinatize, and basically mathematize available information into "mathematical objects" so that powerful-but-elementary mathematical tools can be used.
- Deeper conceptual systems that build on children's everyday experiences, and focus on ideas of equivalence and transformation that underlie meaningful computations and estimations.
- Multi-media representational fluency that encourages students to express, test, and revise alternative ways of thinking. This includes fluency across text, symbols, and graphical diagrams involving multiple embodiments.

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Whole Numbers

THE SPIRIT OF FOUR: A CASE STUDY OF METAPHORS AND MODELS OF NUMBER CONSTRUCTION

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Objectives

Researchers have noted the need for investigation of relationships between different types of reasoning and number construction models (Confrey & Smith, 1995; Olive, 2001; Pepper & Hunting, 1998; Steffe, 1994). The goal of this study was to look at the development of number construction through the lens of metaphor.

In particular, the study investigated the interplay between a largely multiplicative environment and the development of reasoning within this environment that was significantly different from scenarios from other studies.

Conceptual framework

Observing young children makes a strong case for viewing mathematical thinking as fundamentally metaphoric (R. Davis, 1984). Metaphor is the recursive movement between a source and a target that are structurally similar, both changing in the dynamic process of learning (B. Davis, 1996; R. Davis, 1984; English, 1997; Lakoff & Johnson, 1980; Lakoff & Nunez, 1997, 2000; Pimm, 1987; Presmeg, 1997; Sfard, 1997).

For analyzing number construction, I used the counting scheme (Olive, 2001; Steffe, 1994) and the splitting conjecture (Confrey & Smith, 1995; Lehrer, Strom, & Confrey, 2002). The metaphor that connects sources of sharing, folding or similarity, and the target of multiplicative one-to-many actions can be considered the basis of splitting as a cognitive scheme. The metaphor that connects the source of counting and the target of the number sequence is the basis of the counting scheme. In the splitting world multiplicative reasoning develops via grounding metaphors with sources such as sharing. In the counting world multiplicative reasoning is based on the linking metaphor which connects interiorized, reversible counting with iterable units (Olive, 2001; Steffe, 1994).

Modes of inquiry

This paper presents a longitudinal case study of reasoning in a child up to the age of five, whose home environment was restructured to incorporate more multiplicative activities. Researchers often consider metaphor to be private, unformulated and difficult to study (Presmeg, 1997). Additional access issues came from the need for a very young subject necessary to trace the beginnings of number concept development, and from the longitudinal nature of the study. These considerations pointed to the necessity of a close relationship between the subject of the study and the researcher, and I invited my daughter “Katya” to be the subject of the study. As a parent, I was in a privileged position of access to the majority of the details of Katya’s day-to-day life, as well as to the meaning of her utterances and gestures.

Data sources and evidence

Data for the study came from fieldnotes of observations as a participant-observer; videotapes and audiotapes of unstructured and semi-structured interviews; photographs of activity settings; and a collection of artifacts used in activities.

Results

The non-sequential order in which conventional number names first appeared in Katya’s speech corresponded to multiplicative, rather than counting, actions. For example, the utterance “two twos” appeared about eight months earlier than the word “four,” and also earlier than the word "three." Appearance of "two threes" in games preceded the use of the words "four," "five" and "six," and appearance of "two fours" preceded the use of numbers greater than four.

In constructing numbers from one to four, Katya used individual (Presmeg, 1997) metaphors based on instant recognition of the quantity. In these metaphors, the source was an image with a quantity intrinsically embedded in it, such as "dog's legs" for "four." Katya mostly used mixed references for multiplicative situations, for example, "two dogs" to signify "two times four." This availability of two systems of signifiers provided a language necessary to address the asymmetrical nature of the multiplication models Katya used. For example, in the case of "two dogs" the words underlined the distinction between sets and set members in the set model of multiplication. Lack of signifiers for this asymmetry of multiplication models may be problematic and may hinder development of multiplicative reasoning. Confrey and Smith (1995) note that "a counting number is typically used to *name* the result or outcome of a split" (p.75, italics mine).

If learners see the splitting and counting worlds as isomorphic (Confrey & Smith, 1995), they can understand structures of one world by making parallels with the corresponding structures of the other world. Children's structure transfer attempts become especially visible when they differ from accepted standards. For example, researchers often focus on children inappropriately applying additive strategies to multiplicative situations (Post, Behr, & Lesh, 1986). Katya frequently tried to use *multiplicative relationships instead of additive*. For example, when asked to continue a pattern of arrays made out of circles: 2 by 1, 2 by 2, 2 by 3, ___ she attempted to iterate the previous array twice, drawing a 2 by 6 array instead of the expected "2 by 4". Upon my explanation that a pair of circles is *added* to the array in each step, Katya said, somewhat angrily, that these pictures "are not real." Multiplicative relationships were more "real" to her.

In another unexpected example, a square was split into four equal squares, and then each of the small squares was split into four tiny squares. Katya used words signifying size gradients, such as "large, small, and tiny," and "babies and adults," consistently across different multiplicative worlds.

This metaphor of "growth" united different multiplicative worlds and allowed Katya to compare their structures, working on what a mathematician would call "powers" or "base systems." Katya used the word "spirit" to denote the action in each world, for example, talking about "the spirit of four" in the split square above. She claimed that if we cut the 4-square piece in four, the result would be zero. Upon cutting, she was surprised that the result was one square. However, in repeated activities with the same picture, or with pictures based on other powers from other split worlds, Katya consistently said that the result of splitting the power base picture would be "zero", or "nothing," even after observing again and again that it turned out to be one.

I hypothesized that these names were expressions of metaphors for the origin, and I told Katya that researchers call the entity in question "the origin." We compared the origins of additive and power-based structures, and Katya felt validated to discover a "real" zero at least at some origin. This instance of isomorphism between additive and multiplicative worlds helped Katya to build her idea of the origin as a "superordinate construct" (Confrey & Smith, 1995), whereas the idea was problematic while she stayed within the multiplicative world.

Deeper understanding of connections between additive and multiplicative reasoning can benefit further theory construction in areas such as number construction, ratio and proportion, or exponential functions. Practitioners can draw on possible uses of metaphors for working with deep mathematical ideas throughout the mathematical curriculum. Since the majority of studies of young children are done in additive environments, research of cases developed in a predominantly multiplicative environment can provide a valuable vantage point for theory development.

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QUANTITIES STRUCTURED IN TENS, FIVES, AND ONES: A CASE OF JAPANESE GRADE 1 STUDENTS

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This study investigated Japanese Grade 1 students' understanding of place values through four different tasks. Compared to previously conducted international studies, the percentage of the students who showed their understanding of tens quantities was higher than that of European-language speaking students. Performance on the tasks indicated that most Japanese early Grade 1 students in the study could use tens, some used fives, but very few knew the names of places before instruction in school. The students also understood 10 + n before 4 + n, as do Chinese-speaking students but not English-speaking students.

In many cultures, we use a 10-based place-value system that structures the ways to use quantities in our lives. By placing a group of 10 in the next left column, the number to the left is 10 times larger, and a digit in a different position represents a different quantity. This place-value understanding can become a foundation for young students' number explorations in the future as they learn to conceptualize and manipulate larger numbers in different ways. Literature shows that the development of this place-value understanding is not a simple task, and many students who have difficulties with mathematics concepts in upper elementary and middle schools may lack this understanding.

Several international studies were conducted to examine children's thinking of quantities in terms of 10s and 1s previously (Ho and Fuson, 1998; Miura, et. al, 1993; Naito and Miura, 2001). They present a picture that Japanese students use 5s and 10s as chunks when they think of numbers, and that thinking is developed through their experiences in the classroom. This chunking of 10s sets the foundation for their place-value thinking as they explore larger numbers in the future. However, no study has been conducted which incorporated the whole range of place value tasks, especially with young Japanese students upon entering school.

Method

Japanese Grade 1 students were interviewed to show their thinking of quantities upon entering school. Interview tasks were taken from previous international studies to compare the results. For Study 1, 16 Grade 1 students participated in the study. For Study 2, 9 Grade 1 students participated in the study. Those two groups of students attended a same school in different years. These studies were conducted at a full-day Japanese school in a midwestern metropolitan city.

Students were interviewed individually in Japanese. All the interviews were audio- or video-taped and transcribed afterwards. For Study 1, there were 3 tasks. For task 1 (cognitive representation of number), students were asked to use base-ten blocks to represent quantities for 13 and 28. For task 2 (place-value understanding), for the same set of numbers (13 and 28), students indicated the digits in the ones place and tens place. For task 3 (representation using 5), students showed their thinking process as they solved problems 2 + 6 and 8 – 2 using 10s, 5s, and unit blocks. For task 4 (hidden object addition task) in Study 2, after interviewer placed cubes in a shoe box (with a lid) over two separate times, students were asked to state the total number of the cubes in the box and explain their thinking. Several 4 + n and 10 + n questions were asked.

Results and Discussion

For task 1, 100% of the answers were correct. Forty-four percent of the answers used canonical 10 representation, 7% used a non-canonical 10, and 50% used one-to-one representation. These data differ from Miura's study (1993), where their sample of Japanese students used canonical 10, non-canonical 10, and one-to-one representations for 72%, 10%, and 18% respectively. Our data are similar to Naito and Miura's study (2001) in Japan that of 45%, 5%, and 50%, respectively. For task 2, students in this study did much worse than students in both other studies. Compared to 60% correct for Miura's study (1993) with Japanese students and 84% for Naito and Miura's students, less than 1% of student responses in this study identified the place value of the digits correctly. For task 3, all the answers students gave were correct (100%). Twenty-five percent of the responses used a 5 block to show number 6 for the addition problem (2 + 6), and 31% to show 8 for the subtraction problem (8 – 2). For the use of 5 in a canonical 10 representation for task 1, 6% used two 5 blocks to show the 10 in 13 and 31% used one 5 block to show the 8 in 28. The reason why the percentage for 13 is lower than 28 is that 13 is more commonly seen as one 10 and three 1s than as two 5s and three 1s. For task 4, after all students answered the first problem correctly (2 + 1), 66% of the students (6 students out of 9) showed their understanding of embedded-10 cardinality understanding as they solved the rest of the problems (10 + n). This result was fairly similar to Ho and Fuson's (1996) Chinese five-year-olds in which 70% of students demonstrated the understanding of embedded-10 cardinality understanding.

The results of the studies suggest that some Japanese Grade 1 students early in the year understood quantities in terms of tens and ones (embedded 10 cardinality understanding), and the percentage of students who had such understanding was higher than that of European-language speaking students reported by Miura, et. al. (1993) and by Ho and Fuson (1996). With this study being conducted during the first month of the school year for both groups of students, it is assumed that the students had developed the understanding without formal instruction on place values but from their informal experiences with quantities at home and in the community.

For the use of 5s in their thinking, the majority of students who participated in the study did not use 5 blocks to solve problems and/or describe their thinking as they solved the problems. Although 5s are used in representations in textbooks, etc., the use of 5 was not directly taught in the classrooms during observations, and when asked, the classroom teacher mentioned that he did not emphasize the 5. Future study is needed to investigate why some students use 5s in their thinking and how such use might facilitate their understanding of place values and quantities.

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TEACHING LONG DIVISION TO LEARNING DISABLED STUDENTS: A PSYCHOLOGICAL ANALYSIS OF TWO ALGORITHMS

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We present a detailed psychological analysis of two different approaches to teaching long division to reading and mathematics disabled students. To our knowledge, there have been no other attempts to systematically explore the suitability of different approaches to teaching long division to this population of students. We analyze the standard algorithm for long division and an alternative algorithm (Fuson, 2003) that is more open-ended, more conceptual and closer to student-invented approaches to long division. We analyze mathematical, cognitive, emotional, and motivational features of these two algorithms and consider learning disabled students' known learning and psychological characteristics. By juxtaposing the results of these two analyses we find that the psychological features of the alternative algorithm make it more suitable for teaching reading and mathematics disabled students.

Our analyses suggest that the standard algorithm (1) masks the operation of division while the accessible algorithm makes it transparent; (2) allows only one way of obtaining the answer while the accessible algorithm allows for choices and use of one's strengths; (3) requires a complex visual display of the page that seems very confusing while the visual layout of the accessible algorithm is less complex and better organized; (4) requires the students to commit more elements to long-term memory during the learning process than the alternative algorithm; (5) requires the students to recall more elements when performing the operation of long division than the alternative algorithm; (6) requires the students to maintain and coordinate more elements in working memory than the alternative algorithm; moreover, the accessible algorithm has built-in spatial and temporal cueing which allows the limited working memory resources of these students to be allocated to other tasks; (7) putts extremely high cognitive demands on students, and thus has a potentiality to elicit a number of negative emotions (such as anxiety, frustration and embarrassment), which in turn could further diminish the access to the limited cognitive resources of learning disabled students; by contrast, the alternative algorithm has a potentiality to elicit positive emotions by making the whole operation meaningful and easy, and thus further facilitate the learning process.

We conclude that mathematical, cognitive and affective/motivational features of the alternative algorithm for long division make it *educationally advantageous* over the standard algorithm for teaching long division to learning disabled students. We suggest that the alternative algorithm is a perfect example of an approach to teaching that meets Case's criteria for an appropriate instruction—it puts minimal processing demands on the learner and provides a "conceptual bridge" from the current level of understanding to the next (Case, Sandieson, & Dennis, 1986).

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