

STUDYING UNDERGRADUATE MATHEMATICS: EXPLORING STUDENTS' BELIEFS, EXPERIENCES AND GAINS

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This study explores undergraduate students' self-reported gains in college mathematics courses and how these relate to the inquiry-based learning (IBL) methods used in their courses. These experiences were studied against the qualities and development of students' beliefs, motivation and strategies for learning and solving mathematical problems. Pre- and post survey data during one semester-long course showed that students found the instructional practices beneficial and reported cognitive, affective and social gains due to the course. Clear positive correlations appeared between students' gains and their experiences of instructional practices as well as between the gains and their beliefs, motivation and strategies. Moreover, positive changes between pre- and post-surveys in beliefs, motivation and strategies indicate the positive impact of the classes on students' perceptions and practices in studying college mathematics.

Background

The important role of beliefs, affect, and motivation in learning mathematics is well acknowledged in mathematics education research. Students' mathematical beliefs and attitudes have powerful impacts on their engagement and achievement, especially on problem solving. Students' beliefs about the nature of mathematical knowledge and skills, about mathematical problem-solving, and about their own mathematical capability, often determine their level of attendance and learning. Negative attitudes and emotions, together with inadequate self-regulatory behaviors, are often connected with students' preventive beliefs and perceptions in mathematics learning situations (DeBellis & Goldin, 2006; Malmivuori, 2001, 2007; McLeod, 1992; Schoenfeld, 1992). Such beliefs and behaviors derive from students' previous classroom experiences, both positive and negative; they are highly stable and difficult to change (e.g., Bishop, 2001; Cobb, Yackel & McCain, 2000).

Students who choose to study college mathematics differ from those studying secondary and high school mathematics, as does their learning context. College students who study mathematics as their major or minor subject usually show positive attitudes towards mathematics and high motivation, but nonetheless have varying goals for their study of mathematics and varying beliefs about mathematics and mathematical problem solving, for example in their beliefs about mathematical proofs (Selden & Selden, 2007; Sowder & Harel, 2003). Other students, required to take mathematics courses for another major, may exhibit less positive attitudes. Pre-service teachers represent a third group of college students with distinct beliefs, goals and attitudes.

All these various personal, social and instructional aspects determine the context in which students develop knowledge, beliefs and attitudes. Active teaching approaches all share goals of engaging students in their own learning processes and activating their responsibility for their own learning (Prince, 2004). Inquiry-based learning (IBL) is one such approach. Closely related to discovery learning or guided discovery (Bruner, 1961; Dewey, 1938) and problem-based learning (e.g., Savin-Baden & Major, 2004), IBL provides opportunities for students to engage in knowledge creation and argumentation (Rasmussen & Kwon, 2007) and promotes problem solving skills, independent thinking and intellectual growth (Buch & Wolff, 2000; Duch, Gron &

Allen, 2001). In addition, pedagogical practices that emphasize cooperative learning are seen to foster student dialogue, build positive interdependence within groups, and promote higher-order thinking (Gillies, 2007; King, 2002). Such instructional approaches offer a context that clearly differs from the lecturing, exams and transmittance model of providing content knowledge that are traditional in college mathematics courses. This study explores undergraduate students' beliefs, experiences and learning gains in this kind of an active instructional context.

Objectives of the Study

This study reports preliminary findings on undergraduate mathematics students' beliefs, motivation, strategies and gains with respect to their experiences of instructional practices during one semester of college mathematics applying IBL instructional methods. The main focus is on students' beliefs, goals and experiences students while studying college mathematics, and how these are related to their achievement, measured in the form of self-reported learning gains. The results also show changes in students' beliefs, motivation, and strategies during their IBL mathematics course, and, further, how these changes are related to their learning gains.

Methodology

Subjects of the Study

Survey data was gathered from undergraduate students studying mathematics in four different US research universities during one term. The students represent mostly general math major and minor students in advanced mathematics courses; three sections of elementary and secondary pre-service teachers were also included. A structured, paper-and-pencil questionnaire was administered in the beginning and end of each course. The data derive from 13 different IBL sections with 192 students responding to the pre- and post-surveys. Four additional IBL sections were included in the analysis of post-survey data, for a total of 233 students.

Instrument

The first part of the survey was constructed on the basis of theory about mathematical beliefs, affect, goals and strategies of learning and problem solving. It was tested and revised using item analysis. The seven sections measured students' interest in and enjoyment of mathematics, goals in studying mathematics, learning and problem solving actions taken while doing mathematics, and beliefs about learning mathematics, problem solving, and proofs. Responses varied on a seven-point Likert-scale (e.g., from "not at all important" to "extremely important").

The post-survey included the same items, plus six additional sections. Four sections asked about students' experiences of instructional practices: how much various practices helped their learning, on a five-point scale from "no help" to "a great help." Two sections measured students' learning gains in understanding, attitudes, confidence and capabilities, on a five-point scale from "no gain" to "great gain". These sections were based on the SALG instrument (Student Assessment of their Learning Gains; SALG, 2008), which was developed to enable faculty and program evaluators to gather formative and summative data on classroom practices. Both the pre- and post-surveys gathered information on students' personal and mathematical background.

Variables and Data Analysis

New composite variables were constructed on the basis of the designed scales and exploratory factor analyses: 17 measures of beliefs, motivation, affect and strategies; 5 measures of instructional practices; and 4 measures of gains (see Table 1). Scores varied between 1 and 7 (where 4 points to a neutral or average view) or between 1 and 5 (where 3 refers to moderate

help or gain). Reliability scores for these scales varied between 0.64 and 0.96. Statistical analysis included descriptive statistics, correlation analysis, and parametric tests (T-tests, ANOVA).

Table 1. *Composite Variables Measuring Student Beliefs, Experiences and Learning Gains*

Variable	items	scale	Emphasis of items
Motivation			
<i>Interest</i>	3	7	Interest in learning and discussing mathematics
<i>Math major</i>	1	7	Desire to graduate with a math major
<i>Math future</i>	2	7	Desire to pursue math in future work or education
<i>Teaching</i>	1	7	Desire to teach math
Enjoyment	7	7	Pleasure in doing and discovering mathematics
Goals for studying math			
<i>Intrinsic</i>	4	7	Learning new ways to think & to apply math
<i>Extrinsic</i>	4	7	Meeting requirements; degree, good grades
<i>Communicating</i>	2	7	Communication of mathematical ideas to others
Beliefs about learning			
<i>Instructor-driven</i>	5	7	Exams, lectures, instructor activities
<i>Group work</i>	3	7	Whole-class or small group work
<i>Exchange of ideas</i>	3	7	Active verbal interaction with other students
Beliefs about problem-solving			
<i>Practice</i>	2	7	Repeated practice, remembering
<i>Reasoning</i>	5	7	Rigorous reasoning, flexibility in solving
Beliefs about proofs (Yoo & Smith, 2007)			
<i>Constructive</i>	4	7	Process view; revealing mathematical ideas
<i>Confirming</i>	4	7	Product view; recall and confirming conjectures
Strategies			
<i>Independent</i>	4	7	Finding one's own way to think & solve problems
<i>Collaborative</i>	4	7	Seeking help, actively sharing with others
<i>Self-regulatory</i>	6	7	Planning, organizing, reviewing one's own work
Experience of classroom practices (what helped me learn)			
<i>Overall</i>	7	5	Teaching approach, atmosphere, pace, workload
<i>Active participation</i>	5	5	Personal engagement in discussion & group work
<i>Individual work</i>	4	5	Studying & problem-solving on one's own
<i>Assignments</i>	8	5	Nature of tests, homework, other assigned tasks
<i>Personal interactions</i>	6		Interaction with peers & instructor, in/out of class
Learning gains			
<i>Mathematical thinking</i>	4	5	Understanding concepts, how mathematicians think
<i>Application</i>	3	5	Applying ideas elsewhere, understanding others' ideas
<i>Empowerment</i>	10	5	Confidence to do math, appreciation, persistence
<i>Working with others</i>	3	5	Working with others, seeking help

Results

Experiences of Instructional Practices and Gains

Table 2 shows the average ratings of students' experiences of the helpfulness to them of various classroom practices and of their self-reported gains due to participating in a college IBL mathematics course. These are reported separately for each campus with more than one section and advanced mathematics students (CM1-CM3) are distinguished from pre-service teachers at one campus (CT4).

Table 2. Average Ratings of Experiences of Classroom Practices and Self-reported Learning Gains

		CAMPUS			
		CM1	CM2	CM3	CT4
Classroom practices (5-point scale)	<i>overall</i>	3.99	3.43	3.56	2.56
	<i>active participation</i>	4.18	3.84	4.05	3.65
	<i>individual work</i>	4.09	3.94	3.77	3.32
	<i>assignments</i>	3.87	3.36	3.42	2.63
	<i>interactions</i>	4.14	3.89	3.95	3.57
Learning gains (5-point scale)	<i>mathematical thinking</i>	4.37	3.89	4.01	2.72
	<i>application</i>	3.19	3.03	3.45	2.26
	<i>empowerment</i>	3.72	3.45	3.59	2.41
	<i>working with others</i>	3.66	3.39	3.92	2.86

* $N = 34-77$ for each campus.

The averages in Table 2 reflect that students on each campus found IBL instructional practices helpful to their learning. Averages near or above 4.0 further indicate that many students experienced great help due to their participation in the IBL class. Students reported the most help to their learning from active participation and interaction during class work, while the particular assignments were least helpful.

Table 2 further shows that students' experiences slightly varied between the campuses. Pre-service teachers (CT4) found less benefit in the instructional practices than other undergraduate mathematics students. Lower ratings on the overall approach to teaching and learning and class atmosphere may reflect what students reported (in separate interviews) to be high workloads. This was valid especially for pre-service teachers, but also appeared among the advanced mathematics students. Campus 1 students (CM1) seemed to find their classroom experiences slightly more helpful than students at other campuses, but the differences were not large and probably reflected real variation in the actual instructional practices among classes.

Averages for the four learning gain variables show that students reported moderate or good gains due to their course work. Again, averages near and above 4.0 indicate that many students felt they made great gains in their IBL class. Among all groups, the highest gains were reported in understanding mathematical thinking and concepts, with lower gains in understanding how the course ideas were applied outside mathematics or how to make mathematics understandable for other people. Again, pre-service teachers reported weaker gains than did advanced mathematics students; relatively speaking, they reported stronger gains in learning to work well with others.

Campus 2 students (CM2) reported lower gains than other students, consistent with their slightly less positive reports of benefits from the instructional practices in their courses. On the other hand, students at Campus 3 (CM3) had the highest gains in working with others. Unlike Campus 2 students, they found interaction and active participation more helpful than individual work. These variations among student groups may thus reflect the differences in actual instructional practices, consistent with our (separate) observations of classroom sessions. For example, individual work may have been more emphasized at Campus 2 than elsewhere.

Analysis of the correlations among the instructional practices and learning gains for all students revealed statistically significant positive correlations between all the variables (from 0.297 to 0.749). That is, those students who experienced various class practices and interactions as clearly helpful also reported higher gains from their class, and vice versa. These correlations

indicate that students found IBL instructional approaches beneficial to their learning.

Beliefs, Motivation, and Strategies

Table 3 lists the averages on scales for mathematical beliefs, motivation and strategies for learning and problem solving. The results concern advanced mathematics students (not pre-service teachers) in 10 IBL sections at four campuses who took both pre- and post-surveys.

Table 3. *Averages for Measures of Beliefs, Motivation, and Strategies for Pre- and Post Surveys*

VARIABLE		AVERAGES (7-point scale)		
		Pre survey	Post survey	Sig. level
Motivation	<i>interest</i>	4.68	5.08	**
	<i>math major</i>	4.49	5.12	*
	<i>math future</i>	6.20	6.19	
	<i>teaching</i>	3.70	4.27	*
Enjoyment		5.34	5.56	
Goals	<i>intrinsic</i>	5.78	5.79	
	<i>extrinsic</i>	5.28	5.36	
	<i>communicating</i>	5.18	5.50	*
Beliefs about learning	<i>instructor-centered</i>	5.14	4.90	*
	<i>group work</i>	4.67	4.98	*
	<i>interaction</i>	5.26	5.35	
Beliefs about problem solving	<i>practice</i>	4.88	4.83	
	<i>reasoning</i>	5.27	5.45	*
Beliefs about proofs	<i>constructive</i>	5.65	5.85	*
	<i>confirming</i>	4.96	4.77	
Strategies	<i>independent</i>	5.23	5.55	**
	<i>collaborative</i>	4.46	4.97	**
	<i>self-regulatory</i>	5.10	5.20	

* $N = 112-184$; $p < .05$ *, $p < .01$ **.

Table 3 indicates that these students had rather strong motivation and promotive beliefs that emphasized rigorous reasoning, flexibility and construction in problem solving. They stressed the value of interaction in learning and were motivated both by intrinsic and extrinsic goals and an ability to communicate about mathematics. In their preferred strategies for learning and problem solving, students emphasized both individual work and collaboration and reported a high level of self-regulatory activities.

Despite these rather high initial averages, some general changes between the pre- and post-surveys could nonetheless be observed in students' beliefs, motivation and strategies. Most of these involved increases in the strength of students' beliefs about the importance of collaboration and group work in studying mathematics, and in their motivation and their use of effective problem-solving strategies. Females showed increases from pre- to post-surveys more often than males. Students reported more use of both individual and collaborative ways to learn after the IBL course. They also reported higher interest in mathematics in general, in graduating with a college math major, and a slightly higher likelihood that they would teach mathematics in the future. Moreover, the observed changes showed some growth in a constructive view of proving and in seeing the importance of rigorous reasoning and multiple approaches in solving math problems—views more consistent with mathematicians' views. Finally, students showed

declines in a transmittal view of learning that emphasized instructor explanation and seeing similar examples to their homework. All these changes suggest a positive impact of IBL classes on students' perceptions of mathematics and on their practices in studying college mathematics. *Connections Between Beliefs, Motivation, Strategies and Gains*

Correlations were computed between the constructed post-survey variables and gains for all four campuses. Table 4 displays the strongest correlations between the four gain scales and the post-survey scales on beliefs, motivation, and strategies for learning mathematics.

Table 4. *The Strongest Correlations between Learning Gains and Post-survey Beliefs, Motivation, and Strategies, among all Students*

VARIABLE		GAINS			
		Mathematical thinking	Application	Empowerment	Working with others
Motivation	<i>interest</i>	0.515	0.338	0.466	0.220
Goals	<i>intrinsic</i>	0.505	0.371	0.469	0.261
Enjoyment		0.564	0.346	0.489	0.182
Beliefs about learning	<i>interactions</i>	0.272	0.307	0.343	0.296
Beliefs about problem solving	<i>reasoning</i>	0.421	0.319	0.327	0.247
Beliefs about proofs	<i>constructive</i>	0.412	0.289	0.289	0.222
Strategies	<i>independence</i>	0.452	0.249	0.351	0.226
	<i>self-regulation</i>	0.359	0.320	0.415	0.355

* All correlations are significant at the level $p < .01$

** $N = 197-222$

Table 4 shows clear positive connections between students' cognitive, affective, and social gains and their beliefs, motivation and learning strategies. The strongest correlations related to students' gains in understanding mathematical concepts and thinking, and to their gains in empowerment: confidence, positive attitude, persistence, and ability to improve their own mathematical capacity. High interest, enjoyment and intrinsic goals for learning college mathematics were the most positively related to learning gains. Interestingly, these motivational connections were clearest among pre-service teachers—among whom interest and enjoyment were initially lower. Students' preference for independent thinking strategies was most strongly linked to higher gains in mathematical thinking, while strategies of active self-regulation in solving problems were most clearly linked to higher empowerment.

Students' gains in mathematical thinking were linked to beliefs about the importance of rigorous reasoning and flexibility in solving problems, and to constructive views of mathematical proof. The belief that it is important to have active interaction with other students when learning mathematics was also clearly related to cognitive, affective and social gains. Unsurprisingly, this relation was strongest for students' gains in willingness and ability to work with others.

Comparisons between Tables 3 and 4 indicate that the strongest correlations to gains were represented by those variables that also showed increases from pre- to post-surveys. This applied in particular to students' interest and independence of learning but also to the importance they attached to rigorous reasoning and flexibility in solving mathematical problems and to constructive views of proofs. Positive changes in these views and approaches may have a critical role in affecting students' learning gains, not only among these students in IBL classes but also among college mathematics students more generally.

Discussion

The role of beliefs, affect, and motivation has been widely studied in secondary school contexts, but less so among college mathematics students. Teaching methods applying inquiry-based learning with active collaboration create a context different from traditional college mathematics instruction. Active engagement of students in their own learning processes, with responsibility, collaboration, and creative use of personal resources, is seen to enhance growth of thinking and problem solving (Prince & Felder, 2007) and social skills (Duch et al., 2001; Jordan & Metais, 1997). Promotion of cognitive, affective and social skills in such learning contexts may then be reflected in students' beliefs, experiences, and activities (e.g., Kwon, Rasmussen, & Allen, 2005; Smith, 2006). Our preliminary findings point to such positive impacts.

The advanced math students in this study began with high motivation and adequate beliefs about mathematics learning and problem solving. However, participation in an IBL mathematics course seemed to further promote these views and approaches. Students gained interest in and motivation to study mathematics and exhibited less belief in rote learning methods. They attached greater importance to group work and active collaboration, and to communicating mathematical ideas. Their choice of problem-solving strategies emphasized both independent thinking and collaboration. After taking an IBL course, they saw rigorous reasoning in solving problems as more important and reflected a more process-based view of mathematical proofs.

Students reported rather high cognitive, affective and social gains due to their participation in an IBL course and reported IBL classroom practices as helpful, especially their own active participation and interaction during the class work. Pre-service teachers reported lower gains and less benefit from IBL learning approaches than advanced math students.

The results also showed important, direct connections between these cognitive, affective, and social gains and students' beliefs, motivation, and use of particular learning and problem solving strategies. Higher interest, enjoyment, and intrinsic goals for learning college mathematics were most clearly connected to higher gains. The strongest connections to gains appeared in the beliefs, motivation and strategies that also showed increases during an IBL course.

Limitations to the study include a rather low sample size and large variation in the instructional practices, nature of courses, and in student backgrounds within and between the campus sections. More data are needed to confirm these preliminary findings. However, the results indicate that use of active instructional methods with inquiry and collaboration represents a learning context that may have powerful effects on students' learning and positive attitudes toward college mathematics. Future study will show how these gains and experiences vary between different student groups, especially with respect to gender and in comparison with students experiencing more traditional college mathematics teaching.

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