WORKING GROUP

MODELS AND MODELING

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The Models and Modeling Working Group at PME-NA has successfully continued its work since 1999. The purpose of this Working Group is to discuss and enrich different views in which models are used in the learning of mathematics and applied science. That is, models are considered conceptual and representational tools that allow us to better understand how students, teachers, researchers, and other educators learn, develop, and apply relevant mathematical concepts (Lesh & Doerr, 2003; Lesh, Doerr, Carmona, & Hjalmarson, 2003). To this workshop we would like to invite participants to begin or continue the development of the greatly needed communities of researchers and practitioners to expand our focus of research on the ways in which models are used in Problem Solving, Curriculum Development, Student Development, and Teacher Development. This year, a special focus will also be given to Models and Modeling as it applies to Assessment and Research Design, and its relation to Complexity Theory.

In this workshop, we will continue to reflect on *a Models and Modeling Perspective* to understand how students and teachers learn and reason about real life situations encountered in a mathematics classroom. We will discuss the idea of a model as a conceptual system that is expressed by using external representational media, and that is used to construct, describe, or explain the behaviors of other systems. We will reflect on the characteristics that are elicited, including the complexity, dynamic, and iterative features of model-development. We will consider the types of models that students, teachers, and researchers develop (explicitly) to construct, describe, or explain mathematically significant systems that they encounter in their everyday experiences, as these models are elicited through the use of model-eliciting activities (Lesh, Hoover, Hole, Kelly, & Post, 2000). During the workshop we will continue to explore these aspects of learning, teaching, and research by continuing our work in panels and smaller groups focusing in: Student Development, Teacher Development, Curriculum Development, Problem Solving, and a strong emphasis on Research and Assessment Design, and Complexity Theory.

A models and modeling perspective has proven to be a rich context for research and development. During past workshops, we have discussed and continued to work on innovative designs for research and assessment that can help answer questions involving the understanding of complex situations that are dynamic and iterative. There are several characteristics that need to be sustained by the types of research design needed. These include:

First, it is important to radically increase the relevance of research to practice, involving many levels and types of participants (students, teachers, researchers, curriculum designers, policy makers, and others) (Lesh & Kelly, 2000). Second, it is necessary to understand that the educational phenomena that are researched are complex systems, in the sense that they are dynamic, interacting, self-regulating, and continually adapting. Third, it is necessary for educational decision-makers to rely on reports that involve more than simple-minded uni-dimensional reductions of the complex systems that characterize the thinking of students,

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teachers, and researchers. Recent advances in mathematics and other scientific fields have made available the use of technologies that are capable of using graphic, dynamic, and interactive multimedia displays to generate simple (but not simple minded) descriptions of complex systems (for example, weather, systems, traffic patterns, biological systems, dynamic and rapidly evolving economic systems) (Lesh & Lamon, 1993). And fourth, research is about knowledge development; and not all knowledge is reducible to a list of tested hypotheses and answered questions. In particular, in mathematics and science education, the outcome products that are needed from our research often focus on the development of models (or other types of conceptual tools) for construction, description, or explanation of complex systems. Thus, distinctions need to be made between: (a) model development studies and model testing studies; (b) hypothesis generating studies and hypothesis testing studies; and (c) studies aimed at identifying productive questions versus those aimed at answering questions that practitioners already consider to be priorities.

From these assumptions, many participants from the Models and Modeling Working Group have been working on a research design first described by Collins (1990) and Brown (1992) called *Design Studies*. This type of research design explicitly focuses on the development of constructs and conceptual systems used by students, teachers, researchers, and other educators. Principles applying to Design Research, the types of research questions it allows to answer, appropriate methodologies involved in the design of these types of studies, and examples of Design Research Studies are some of the discussion topics that will be considered in our working sessions.

The Models and Modeling Working Group at PME-NA Mérida

The Models and Modeling Working Group at PME-NA XVIII has the following goals**:**

- To disseminate and contribute to the research on the use of models and modeling in school mathematics, with a focus on students, teachers, researchers, and policy makers.
- To create and support collaborations among researchers to build international communities of practice.
- To extend the field of mathematics education towards new directions on assessment, problem solving, research design, learning environments and complexity; as it relates to the use of models and modeling in school mathematics.

For the PME-NA XXVII Models and Modeling Working Group, several sessions will be organized throughout the Conference. In particular, there will be two main working group sessions. For each session, after a general introduction on different topics is provided, participants will be invited to select one, and smaller groups will be formed. Each sub-group will have a panel of discussants, and a discussion leader, who will approach the selected theme. In addition, participants will be encouraged to attend to other sessions that will be offered throughout the Conference, and that will further support and enrich the discussion that will take place during the two Working Group sessions.

These panels and smaller groups will be guided by topics related to models and modeling and: Student Development, Teacher Development, Curriculum Development, and Problem Solving. More particularly, for this year we would like to extend our work by placing an emphasis on Design Research Studies (Collins, 1990; Brown, 1992) as a framework for Research and Assessment Design, and Complexity (Hills, Hurford, Stroup, & Lesh, in press; Lesh & Yoon, 2004).

Participants will participate in the sessions according to their interests to discuss these issues more in depth, as well as to outline a plan of action for future collaboration for those who are interested in continuing their work through out the year.

Some accomplishments of the Models and Modeling Working Group

Some of the publications and other accomplishments of the participants of this working group. The *Handbook of Research Design in Mathematics and Science Education* (Kelly & Lesh, 2000) describes a variety of innovative research designs that have been developed by mathematics and science educators to investigate interactions among the developing knowledge and abilities of students, teachers, and others who influence activities in mathematics and science classrooms. The book *Beyond Constructivist: A Models & Modeling Perspective on Mathematics Teaching, Learning, and Problems Solving* (Lesh & Doerr, 2003) includes chapters written by many of the participants of this working group, where the authors give a fuller description of a Models and Modeling Perspective.

A special issue on *Mathematical Thinking and Learning: An International Journal* edited by Lyn English explicitly dedicated to a Models and Modeling Perspective, as a theoretical perspective (Lesh & Lehrer, 2003; Lesh, Doerr, Carmona, & Hjalmarson, 2003), and how it applies to student (Petrosino, Lehrer, & Schauble, 2003), teacher (Schorr & Koellner-Clark, 2003) and problem solving (Lesh & Harel, 2003).

A Models and Modeling perspective has proven to be rich context for research and development. Nevertheless, we have found the need to innovative research designs that can better help us answer the types of questions we are mostly interested in. A research design that has proven to be very useful for conducting research from a Models and Modeling Perspective are *design experiments* or *design research studies* (Collins, 1990; Brown, 1992). One of the works in progress of many participants of this working group is the development of a book on this type of research design, and how it can be used to conduct useful research to better understand students', teachers', researchers', and other educators' development of relevant mathematical ideas. Not only will the new book focus on design research methodologies, but it will also describe on new types of dynamic and iterative assessments that are especially useful in design research –where rapid multi-dimensional feedback is needed about the behaviors of complex, dynamic, interacting, and continually adapting systems.

Finally, a new publication is soon to be released, focusing on *Real-World Models and Modeling as a Foundation for the Future of Mathematics Education.* Some of the questions that are answered in this book, and that will also be a focus for discussion during our working group include: How can research investigate systems of interacting systems –in situations where students interact with one another, students interact with teachers and students, teachers interact within continually evolving learning communities, and the learning activities are themselves continually evolving situations? What steps can be taken to develop a research community that is more than just a community of isolated individuals?

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