TEACHERS' MATHEMATICAL AND PEDAGOGICAL AWARENESS IN CALCULUS TEACHING

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The focus of the study

Teachers' mathematical and pedagogical knowledge have received increased research attention in recent years (Ball & Bass, 2000). However, most work in this area has focused on primary or early secondary education. Our study attempts to explore teachers' mathematical and pedagogical awareness in higher secondary education and most specifically calculus teaching and the concept of derivative. It needs to be noted that although there is a large amount of research on calculus education this looks into students' learning and not the actual teaching practices and the way that this affects students understanding learning. In particular, the main research questions that this study aims to answer are: a) what is the nature of teachers' mathematical knowledge concerning derivative? b) what are the teachers' pedagogical practices and views about teaching and learning calculus? and c) what kind of interrelationships can be identified between teachers' mathematical and pedagogical activity.

Theoretical background

The notion of teacher knowledge has been recognized as an increasingly complex phenomenon (Cooney, 1999). A number of studies have attempted to describe this knowledge and it seems that there is some consensus in regard to three of its most important elements: mathematical knowledge, knowledge of students and knowledge of mathematical pedagogy (Lappan & Lubienski, 1994; Even & Tirosh, 1995). Different labels have been used to refer to these elements such as subject matter knowledge, pedagogical knowledge, pedagogical content knowledge (Shulman, 1986), knowledge about mathematics (Ball, 1991), or mathematical know-how (Boaler, 2003). Ball, Lubienski and Mewborn (2001) emphasize the need to investigate how teachers' mathematical understanding affects their practice.They suggest that this should be investigated through the observations and analysis of actual teaching. Mason (1998) elaborates further the notion of teacher knowledge and talks about awareness in action, in discipline and in counsel both in mathematics and in mathematics teaching. In addition to this, mathematical and pedagogical knowledge constitutes not only knowing- that, knowing- how, knowing –why but also knowing to act and knowing to act in the moment (Mason and Spence, 1999). Although research on teachers' knowledge of mathematics as it is extrapolated through actual teaching practice is gaining ground, examples from the area of advanced mathematical thinking are very limited. This is in striking contrast to the large amount of research to calculus education concerning students' learning.

Methodology

The study is a qualitative research within an interpretative framework. The data was collected from three different schools in Cyprus. The data is comprised by classroom observations, informal discussions before and after teaching and audio-taped semi-structured interviews with

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each teacher after the school visits (their duration was about one hour). The researchers observed and took field notes from three teaching sessions conducted by each of the nine teachers. Field notes were taken by two researchers and summaries were constructed immediately after each observation from a combination of the researchers' field notes. The summaries included a general description of the lesson and important issues that emerged. Specific examples from the field notes were given as evidence to the identified issues. The interviews focused on a) teachers' experience concerning learning and teaching mathematics b) teachers' views about teaching and learning mathematics in general and calculus and derivative in particular and c) teachers' interpretations of specific pedagogical actions that were identified during the observations.

The data collected was analyzed systematically based on the grounded theory approach (Strauss & Corbin, 1998). The analysis of the summaries aimed at identifying elements of teachers' knowledge as they emerged from their practice according to Shulman's (1986) three categories: subject matter knowledge, pedagogical knowledge and pedagogical content knowledge. These categories comprised the general framework for the analysis but they started to get a special meaning for the specific content area under investigation. The analysis of the transcribed interviews was initially done vertically for each particular teacher and then horizontally across the nine teachers in order to identify general patterns and relations among the different elements of knowledge.

Results

In this paper, we focus on two teachers who were not pleased from their teaching of calculus in high school but they could not see any other alternatives. Their teaching was mostly teacher-centered while students' participation was limited to the performance of routine exercises or to their responses to teachers' close questions. Below, we briefly discuss how they introduced the concept of the tangent of a curve and we attempt to identify mathematical and pedagogical aspects of their knowledge both from the observations and the interviews. The teachers started with the tangent of a circle and discussed its critical property, that it has only one common point with the circle. The teachers gave two examples of curves that exemplified the inappropriateness of the above geometrical property as a definition for the tangent of a curve. However, the image of the tangent of the circle seemed to dominate students' responses. The teachers did not seem to build on these responses and they continued the lesson by introducing on the board the formal definition of the tangent of a curve. In this part of the lesson the teachers presented the new concept while the students did not actively participate. In some cases where the teachers asked some questions to encourage students' participation, the students seemed to have difficulties to make sense of the situation. A question that emerges is "why the teachers did not develop ways to face students' difficulties and how this is related to their mathematical and pedagogical knowledge?". A first analysis of the interviews allowed us to approach this question and offer possible interpretations.

Teachers' mathematical knowledge about the relationship between the tangent of a circle and the tangent of a curve seemed to be rather fragmented. For example, teacher A stated that "the concept of the tangent in a circle is not the same concept as in a curve…in a curve it is the tangent to a certain point". Teacher B was wondering about this relation not only in the final interview but also in the informal discussion before teaching: "Can we give a global definition for the tangent of a curve like in the case of circle?... I looked for a definition, as we say this is… but I did not find one in the textbooks..". Teachers had difficulties in seeing the tangent of a circle as a special case of the tangent of a curve because they expected to find a "general"

definition for the curve of the same type as the one for the circle. In the case of the tangent of a circle they recognized a global characteristic property- exactly one common point – while in the general case of the curve such global characteristic does not exist. This fragmented nature of teachers' mathematical knowledge was also identified through their explanations why teaching the concept of tangent is essential for students' mathematical development. The main reason they offered was that it would help them in their future exams. The fact that the two teachers could not identify epistemological differences and commonalities between the tangent of circle and curve, and they could not also see the importance of the concept of tangent in mathematics and in science, is possibly an indication that their mathematical awareness had not reached the level of awareness in discipline.

In terms of teachers' pedagogical awareness both teachers seemed to realize that students had difficulties in understanding the particular concept: "Four or five students understood it. The others cannot understand… but these (concepts) need to be taught even for those students."(teacher A). However, even when they were asked to give specific reasons for these difficulties they mostly described a number of external factors (eg. the curriculum, lack of interest, tests, private lessons) and sometimes their own teaching: "they might have not understood yet the concept of tangent but I had not particularly analyzed it in my teaching."(teacher B). We could argue that their knowledge about students' mathematical understanding of the concept of tangent remained at a superficial level. They needed to integrate their practical knowledge with the theoretical- research based knowledge of Mathematics Education in order to be able to develop interpretive tools for teaching and learning.

Coming back to our initial questions about teacher knowledge and its role on teacher's practices, the first findings indicate that both have an effect on actual teaching as the teacher has to go deeply in mathematical and pedagogical aspects of her teaching in order to take "effective" decisions.

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