

A "**Proof Story**" Across the Grades:
Beginning a Conversation on the Learning of Proof in Grades K-16

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Abstract:

This working group explores the topic of proof as a common thread across the grades K-16. The group aims to bring together people with a focus on the learning and teaching of proof to provide a forum to begin to identify the common themes and researchable questions and to establish an agenda for future work. It aims to foster a collaboration between people with a focus on K-12 or college level mathematics. Specifically, participants in this group will focus on two issues that will form the basis of the subsequent work of the group: (a) identifying the current status of research on proof and generating specific goals for the group, and (b) generating a common line of research questions regarding the learning and teaching of proof that have the potential of involving intense collaborative work by groups of mathematics educators and practitioners at the K-16 levels.

“The concept of proof is one which not only pervades work in mathematics but is also involved in all situations where conclusions are to be reached and decisions to be made. Mathematics has a unique contribution to make in the development of this concept, and [...] this concept may well serve to unify the mathematical experiences of the pupil” Harold P. Fawcett (1938)

In the sixty-five years since the statement above was written, the assumptions about proof as *a logical argument that one makes to justify a claim and to convince oneself and others*, and its role in mathematics and mathematics education, have not changed. Mathematicians and mathematics educators agree on the importance of proof in mathematics and the necessity for students to develop both the understanding of concepts related to proof and the skills to read and write proofs. As such, there have been calls for the learning of proof to become a central goal of mathematics (AMATYC, 1995; MAA, 2000; NCTM, 2000; RAND Mathematics Study Panel, 2002; Royal Statistical Society, 1995). As such, it is crucial that we provide ways for students to think about proof throughout their education in mathematics, beginning in the elementary grades and extending through their post-secondary experience.

We see a larger need to detail a connected K-16 “story” of mathematical proof. The *Principles and Standards for School Mathematics* (NCTM, 2000) has emphasized that proof should be a part of all pre-college students’ mathematical experiences in order to deepen and extend learning and to democratize access to these ideas to a broader population of students. Such a story would include understanding how the forms of proof, including the nature of argumentation and justification as well as what counts as proof, evolve chronologically and cognitively and how content and instruction can support this. Building a “proof story” across grades K-12 would inform how to integrate mathematical proof in instruction at all levels, thus building a habit of mind in students that would support their transition to higher mathematical thinking at the tertiary level.

An important step in this direction is to build a strong, research-based framework to characterize mathematical proof across different cognitive domains and to detail how

content and instruction can support the integration of mathematical proof into students' experiences. This requires the building of a broad research effort on the learning, teaching and assessing of proof across grades. Currently, a number of researchers are focusing on proof at different age/grade levels, using a variety of methodological perspectives. The outcomes of these efforts are now beginning to appear in journals and in conferences such as PME (e.g., Blanton, Stylianou & David 2003; Harel & Sowder, 1998; Herbst, 2002; Knuth, et al., 2002; Raman, 2003; Weber, 2001; Yackel, 2001). But, while these studies are providing a significant contribution to our understanding of students' capacity for proof and some of the impediments to that understanding, they are often happening in isolation and do not capture the evolution of student learning over significant periods of time nor the interplay between different domains of learning. The ability to read and do proofs in mathematics is a complex one that depends on a wide expanse of beliefs, knowledge, and cognitive skills and that is uniquely shaped by the social realm in which learning occurs. It is not at all clear, however, which of these factors are the most salient for students nor how these factors interact with one another (Moore, 1994). Even less is known as to how students overcome difficulties throughout their study of mathematics and how college mathematics students' conception of mathematical proof progresses over a long period of time. Indeed, the development of one's capacity for proof is a long-term effort (hence, we characterize it here as a K-16 endeavor), and understanding this development will require us to coordinate efforts and perspectives..

This working group aims to bring together people with a focus on the learning and teaching of proof to provide a forum to begin to identify the common themes and researchable questions and to establish an agenda for future work. It aims to foster a collaboration between people with a focus on K-12 or college level mathematics. Specifically, participants in this group will focus on two issues that will form the basis of the subsequent work of the group:

- (a) Identifying the current status of research on proof and generating specific goals for the group.
- (b) Generating a common line of research questions regarding the learning and teaching of proof that have the potential of involving intense collaborative work by groups of mathematics educators and practitioners at the K-16 levels.

Each of the two issues is explained further below.

Group Goals and Future Directions

Several researchers (many of whom are active members of PME-NA) are currently involved in research on proof. Our goal is to bring this group together to share their visions and goals, as well as give short presentations on the status of their current work on proof. Subsequently, the group will be asked to discuss the ways in which these independent studies can inform each other and form a continuum of research.

Generating Research Questions

The main body of the existing studies on proof has focused on one side of the story, namely, the learning of proof. Researchers have sought to identify the characteristics of student-constructed proofs, including misconceptions and difficulties that students face at different levels of mathematics instruction. There have been few efforts to study the

instruction of proof. The group will be asked to examine these current trends in the study of proof and to discuss ways in which we can expand our research foci to form an overall view of the topic of proof in mathematics education, and, at the same time to search for a common focus in our efforts.

General Direction

It is our goal to assist the members of the group to collaborate in order to generate and share research on the development of student learning of proof and the teaching of proof. It is important that we use the time in this working group session to organize and prioritize future work for the group. The session leaders will take the responsibility to summarize the discussions of the group and provide focused directions for the future. We have already secured funding from the NSF that will allow us to continue the work of the group throughout the year through an electronic discussion forum to form the basis for future meetings of the group (PME-NA 2005). For more information on this project please visit our website: www.theproofproject.org

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