

**Models and Modeling  
PME-NA XXVI Working Group**

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The Models and Modeling Working Group at PME-NA has successfully continued its work since 1999. The purpose of this Working Group is to discuss and enrich different views in which models are used in the learning of mathematics and applied science. That is, models are considered conceptual and representational tools that allow us to better understand how students, teachers, researchers, and other educators learn, develop, and apply relevant mathematical concepts. To this workshop we would like to invite participants to begin or continue the development of the greatly needed communities of researchers and practitioners to expand our focus of research on the ways in which models are used in Problem Solving, Curriculum Development, Student Development, Teacher Development, Assessment, and Research Design.

In this workshop, we will continue to reflect on *a models and modeling perspective* to understand how students and teachers learn and reason about real life situations encountered in a mathematics and science classroom. We will discuss the idea of a model as a conceptual system that is expressed by using external representational media, and that is used to construct, describe, or explain the behaviors of other systems. We will reflect on the characteristics that are elicited, including the complexity, dynamic, and iterative features of model-development. We will consider the types of models that students, teachers, and researchers develop (explicitly) to construct, describe, or explain mathematically significant systems that they encounter in their everyday experiences, as these models are elicited through the use of model-eliciting activities

(Lesh, Hoover, Hole, Kelly, & Post, 2000). During the workshop we will continue to explore these aspects of learning, teaching, and research by continuing our work in smaller groups focusing in: Student Development, Teacher Development, Curriculum Development, Problem Solving, and a strong emphasis on Research Design and Assessment Design.

A models and modeling perspective has proven to be a rich context for research and development. During past workshops, we have discussed and continued to work on innovative designs for research and assessment that can help answer questions involving the understanding of complex situations that are dynamic and iterative. There are several characteristics that need to be sustained by the types of research design needed. These include:

First, it is important to radically increase the relevance of research to practice, involving many levels and types of participants (students, teachers, researchers, curriculum designers, policy makers, and others) (Lesh & Kelly, 2000). Second, it is necessary to understand that the educational phenomena that are researched are complex systems, in the sense that they are dynamic, interacting, self-regulating, and continually adapting. Third, it is necessary for educational decision-makers to rely on reports that involve more than simple-minded uni-dimensional reductions of the complex systems that characterize the thinking of students, teachers, and researchers. Recent advances in mathematics and other scientific fields have made available the use of technologies that are capable of using graphic, dynamic, and interactive multimedia displays to generate simple (but not simple minded) descriptions of complex systems (for example, weather, systems, traffic patterns, biological systems, dynamic and rapidly evolving economic systems, to name a few) (Lesh & Lamon, 1993). And fourth, research is about knowledge development; and not all knowledge is reducible to a list of tested hypotheses and answered questions. In particular, in mathematics and science education, the outcome

products that are needed from our research often focus on the development of models (or other types of conceptual tools) for construction, description, or explanation of complex systems. Thus, distinctions need to be made between: (a) model development studies and model testing studies; (b) hypothesis generating studies and hypothesis testing studies; and (c) studies aimed at identifying productive questions versus those aimed at answering questions that practitioners already consider to be priorities.

From these assumptions, many participants from the Models and Modeling Working Group have been working on a research design first described by Collins (1990) and Brown (1992) called *Design Studies*. This type of research design explicitly focuses on the development of constructs and conceptual systems used by students, teachers, researchers, and other educators. Principles applying to Design Research, the types of research questions it allows to answer, appropriate methodologies involved in the design of these types of studies, and examples of Design Research Studies are some of the discussion topics that will be considered in our working sessions.

### **The Models and Modeling Working Group at PME-NA Toronto**

At PME-NA Toronto, our sessions will consist of a series of brief 5-minute “elevator speeches” (so called because they’re similar to what a researcher might be able to say about “the essence of his or her work” if asked about it on an elevator) followed by discussions related to the following issues: Student Development, Teacher Development, Curriculum Development, and Problem Solving. More particularly, for this year we would like to extend our work by placing an emphasis on Design Research Studies (Collins, 1990; Brown, 1992) as a framework for Research Design and Assessment Design.

As in past Working Groups, speakers will give highlights of their research, and then smaller groups will be formed according to participants interests to discuss these issues more in depth, as well as to outline a plan of action for future collaboration for those who are interested in continuing their work through out the year.

### **Some accomplishments of the Models and Modeling Working Group**

Some of the publications and other accomplishments of the participants of this working group include the book *Beyond Constructivist: A Models & Modeling Perspective on Mathematics Teaching, Learning, and Problems Solving* (Lesh & Doerr, 2003). This book including chapters written by many of the participants of this working group, where the authors give a fuller description of a Models and Modeling Perspective.

A special issue on *Mathematical Thinking and Learning: An International Journal* edited by Lyn English explicitly dedicated to a Models and Modeling Perspective, as a theoretical perspective (Lesh & Lehrer, 2003; Lesh, Doerr, Carmona, & Hjalmarson, 2003), and how it applies to student (Petrosino, Lehrer, & Schauble, 2003), teacher (Schorr & Koellner-Clark, 2003) and problem solving (Lesh & Harel, 2003).

The *Handbook of Research Design in Mathematics and Science Education* (Kelly & Lesh, 2000) describes a variety of innovative research designs that have been developed by mathematics and science educators to investigate interactions among the developing knowledge and abilities of students, teachers, and others who influence activities in mathematics and science classrooms. This handbook has helped in setting the foundations to identify several characteristics that distinguish the type of research design needed to answer they types of questions we are most interested in, which at the same time, lead to the need of new research designs in mathematics and science education.

A Models and Modeling perspective has proven to be rich context for research and development. Nevertheless, we have found the need to innovative research designs that can better help us answer the types of questions we are mostly interested in. A research design that has proven to be very useful for conducting research from a Models and Modeling Perspective are *design experiments* or *design research studies* (Collins, 1990; Brown, 1992). One of the works in progress of many participants of this working group is the development of a book on this type of research design, and how it can be used to conduct useful research to better understand students', teachers', researchers', and other educators' development of relevant mathematical ideas. Not only will the new book focus on design research methodologies, but it will also describe on new types of dynamic and iterative assessments that are especially useful in design research –where rapid multi-dimensional feedback is needed about the behaviors of complex, dynamic, interacting, and continually adapting systems.

## References

- Brown, A. L. (1992). Design Experiments: Theoretical and Methodological Challenges in Creating Complex Interventions in Classroom Settings. *The Journal of the Learning Sciences*, 2(2), 141-178.
- Collins, A. (1990). *Toward a design science of education*. New York, NY: Center for Technology in Education.
- Kelly, A. E., & Lesh, R. A. (2000). *Handbook of research design in mathematics and science education*. Mahwah, N.J.: Lawrence Erlbaum.
- Lesh, R., Doerr, H. M., Carmona, G., & Hjalmarson, M. (2003). Beyond Constructivism. *Mathematical Thinking and Learning: An International Journal*, 5(2/3), 211-234.
- Lesh, R., & Harel, G. (2003). Problem Solving, Modeling, and Local Conceptual Development . *Mathematical Thinking and Learning: An International Journal*, 5(2/3), 157-190.
- Lesh, R., Hoover, M., Hole, B., Kelly, E., & Post, T. (2000). *Principles for developing thought-revealing activities for students and teachers*. Mahaway, NJ: Lawrence Erlbaum.
- Lesh, R., & Lehrer, R. (2003). Models and Modeling Perspectives on the Development of Students and Teachers. *Mathematical Thinking and Learning: An International Journal*, 5(2/3), 109-130.

- Lesh, R. A., & Kelly, E. A. (2000). Multitiered Teaching Experiments. In R. A. Lesh & E. A. Kelly (Eds.), *Handbook of Research Design in Mathematics and Science Education*. Mahawah, NJ: Lawrence Erlbaum.
- Lesh, R. A., & Lamon, S. J. (1992). *Assessment of authentic performance in school mathematics*. Washington, D.C.: American Association for the Advancement of Science.
- Petrosino, A.J., Lehrer, R., & Schauble, L. (2003). Structuring Error and Experimental Variation as Distribution in the Fourth Grade. *Mathematical Thinking and Learning: An International Journal*, 5(2/3), 131-156.
- Schorr, R. & Koellner-Clark, K. (2003). Using a Modeling Approach to Analyze the Ways in Which Teachers Consider New Ways to Teach Mathematics. *Mathematical Thinking and Learning: An International Journal*, 5(2/3), 191-210.