Knowledge of Algebra for Teaching

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Common wisdom suggests that students' knowledge is influenced by teachers' knowledge. For more than three decades, researchers have been trying to identify critical components of mathematics teachers' knowledge (see, for example, the reviews by Begle, 1972; Fennema & Franke, 1992; Ball, Lubienski & Mewborn, 2001). Shulman (1986) suggested that there are at least three components of knowledge for teaching: subject-matter content knowledge, curricular knowledge, and pedagogical content knowledge. Ma (1999) describes in detail the profound understanding of fundamental mathematics among some teachers of elementary school mathematics. The highly structured knowledge packages these teachers have seem to exhibit characteristics of all three types of knowledge Shulman hypothesized. Ball & Bass (2000a, 2000b) suggest that mathematical knowledge for teaching is different from the mathematical knowledge used by other specialists, just as the mathematical knowledge for engineering is different from the mathematical knowledge needed for chemistry.

Until recently most of the work about knowledge for teaching mathematics was confined to the lower grades. Recent work at the secondary level includes the development of a course about high school mathematics from an advanced standpoint (Usiskin, 2000; Usiskin, Peressini, Marchisotto & Stanley, 2002) and the development of a provisional framework for teaching algebra by Kahan, Cooper & Bathea (2003). The RAND Mathematics Study Panel (2003) also notes the need for clarification of the knowledge demands of teaching, and singles out algebra as a key area for further research.

The project, A Study of Algebra Knowledge for Teaching at the Secondary Level, was funded by the National Science Foundation to examine the mathematical knowledge needed for teaching algebra and to develop a framework to describe algebra knowledge for teaching. By “algebra”, we refer to both algebra as a strand in school mathematics and algebra as a mathematics course. To start to develop a framework, project personnel first reviewed the research on pedagogical content knowledge (e.g. Shulman, 1986, 1987; Wilson et al., 1987), knowledge for teaching mathematics at elementary school (e.g.,Ball, 1990; Ball & Bass, 2000a, 2000b; Ball & Hill, 2003), and the learning and teaching algebra (e.g. Fey, 1989; Usiskin, 1988; Wagner & Kieran, 1989; Kieran, 1992; Bednarz et al., 1996; Nathan and Koedinger, 2000), and on their own experiences teaching algebra. From this review a provisional framework was proposed for "algebra knowledge for teaching." The central components of that framework, which will be elaborated on in during the session, are as follows:

1. Substantive Knowledge of Mathematics
   a. decompressing (unpacking)
   b. trimming
2. Knowledge of the Nature and Practice of Mathematics
   a. Knowing conventions (in language, notation, etc.)
   b. Coordinating mathematical and everyday language
   c. Constructing and judging arguments
3. Knowledge of Connections

a. Understanding intersections
b. Translating and Coordinating representations

As the provisional framework was being developed other project personnel conducted several empirical studies, including interviews with teachers and curriculum developers, analyses of videotapes of algebra lessons, and analyses of high school algebra textbooks. To focus and unify the work of the research, all empirical studies examined the same topics in algebra. The topics chosen are central mathematically, appear across various conceptions of school algebra, pose difficulties for students to learn, raise questions that suggest teachers might need additional knowledge of students or algebra for teaching, and are the subject of some research. Two topics seem to fit these criteria: (1) variables, expressions, and equations and (2) linearity. The research team consists of members from departments of Teacher Education, Mathematics, Counseling and Psychology, Curriculum, and Measurement and Quantitative Analysis; the methodology and analyses reflect the diverse perspectives on approach and process. Analyses and syntheses of the various empirical studies are currently underway as is the compilation of a data-base of examples tied to the framework.

Issues that arose during the work. The study was designed to begin with an initial framework as described above. This initial framework was then informed by the empirical work of the researchers and modified and adjusted according to our findings. This approach created a tension for the researchers, making it difficult to focus the empirical work with a nebulous framework, and difficult to revisit the framework with data that was not always specifically tied to the framework. A second issue that arose was the lack of substantive work done in this area at the secondary level in contrast to that at the primary level and the tendency to think the same about secondary teacher knowledge as about elementary teacher knowledge, when in fact, the situations are clearly different with respect to the amount and depth of content knowledge teachers are expected to have and a difference in constructing the work of teaching at the two levels. A third issue was how to clearly traverse the path between research on interesting theoretical aspects that emerged and research clearly linked to helping inform the practice of those teaching teachers about the algebra knowledge needed for teaching. A fourth issue was finding language to describe the categories within the framework in a way that would be clear to those who might benefit from the study, even to the difference in stating the problem - algebra knowledge for teaching or knowledge of algebra for teaching. And a central issue is consideration of the proposed framework as a model, how it might contribute to the discussion about content knowledge, pedagogical content knowledge, and mathematical knowledge needed for teaching at the secondary level, and the role of the examples in this discussion.

Session Plan: This would be the first meeting of a discussion group on the topic, mathematical knowledge for teaching algebra. The session would consist of a brief overview of the project by the researchers, including its goal and a brief description of the methodology and some results of the four empirical studies: interviews with teachers, analysis of videotapes of algebra lessons, analyses of algebra textbooks, interviews with developers of those algebra textbooks. (45 minutes), after which the audience would

divide into four groups, each of which would address one data collection aspect of the project (30 minutes). Each group will be led by one of the researchers who will provide additional examples from that study and lead a discussion of the algebra knowledge examples of the algebra knowledge needed for teaching involved in those examples. The session will conclude with a presentation to the whole group (20 minutes) by one of the principal investigators about the revised framework that was developed. The session would conclude with a whole group discussion of the issues identified by the researchers as well as those suggested by the participants in light of their discussion of the examples in their small groups (25 minutes).

Anticipated follow up activities include posting of papers related to the project on a project website along with notes from the session; a targeted on-line discussion related to the framework and the examples; an open forum for examples offered by participants with a discussion of how they fit into the framework, and a proposed session at the next PME building on the work of the project and the discussion that follows this session.

Implications for theory. Offering a framework to organize the mathematical knowledge needed for teaching for secondary teachers can help the community conceptualize avenues to think about what teachers should know about algebra for teaching and to design experiences into both pre-service and in-service teacher education programs to bring this knowledge to the forefront. Indeed, the provisional framework as written above suggests ways of thinking more broadly about all mathematical topics and in each case, the component of the framework can be interpreted specifically in terms of knowledge of algebra

Implications for practice. The findings of the project can help us gain a deeper and better understanding of teaching and learning mathematics, in particular algebra, at the secondary level. We contend that the converging lines of work on pedagogical content knowledge and mathematical knowledge for teaching are rich and intriguing, with potential implications for more coordinated research, for improving the design of pre-service and professional development opportunities for teachers, and for communicating more effectively with mathematicians, mathematics teacher educators, and policy makers about the nuances of teachers' subject matter knowledge.